Electrical Science

Rose-Hulman Institute of Technology

R.J. Marks II Class Notes

(1969-1970)

ELECTRICAL SCIENCE I

WEEK-CLASS		CH.	PAGES	TOPIC	PROBLEMS
Oct. 1	1-1	1	1-17	Voltage	
	1-2		17-21	Current	5,8
	1-3				12,19
Oct. 7	2-1	2	29-43	V-I Curves	1,3,4,6
	2-2		43-55	Sources-Connections	7,9,10
	2-3				15,16,19,21
Oct. 14	3-1	3	61-67 Hank	Kirchoff's Laws	2,4,5
	3-2		67-73	Node-Datum	6,7,8
HOMECOMING	3-3				9,11,12
Oct. 21	4-1		74-79	Mesh Currents	TODAY – 13,14,15,16
	4-2				17,19,20
	4-3		79-85	Non-Linear Elements	22,24,25
Oct. 28	5-1				26,28,29 FRIDAY
	5-2		86-91	Dep.Sources-Superpos.	32,33,34
	5-3				35,36,38
Nov. 4	6-1	4	105-116	Equivalence	1,2,3,5
	6-2		116-133	Thevinins-Norton	6,7
Mid-Terms	6-3				10,11,12,14
Nov. 11	7-1				15.18
	7-2		133-142	3-Node Equiv.	19,21,22,24
	7-3			Slush Day	
Nov. 18	8-1	6	195-203	Capacitance	1,2,3,4
	8-2		208-223	Inductance	7,8,10,11
THANKSGIVING	8-3		205-208	Impulse	12,14,15
Dec. 2	9-1			Impulse	
	9-2		224-225	Equivalence	16
	9-3	7	233-247	R-C, R-L	1,3,4,5
Dec. 9	10-1				8,9,12,13
	10-2		247-256		15,17,20
	10-3				21,22,25,27
NEXT	11-1	8	265-274		1,2,4,5
QUARTER?	11-2		275-287		6,7,8,10
	11-3		Slush Day		

ELECTRICAL SCIENCE I
FORMULAS

I. CHAPT I CIRCUIT THEORIES & VARIABLES

A) DEFINITIONS
1) Voltage - measure of energy necessary to move charge from one terminal to the other
2) Current - Measure of rate charge is moving
3) Power = Voltage (Current)

B) Coulomb's law & such
1) $F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$
2) $E = \frac{dF}{dQ} = \frac{Q_1}{4\pi\varepsilon_0 R^2}$ = electric field intensity (Pt. Charge) in $\frac{Nt}{COUL} = \frac{V}{M}$
   a) $\varepsilon_0 = 8.85 \times 10^{-12} \frac{COUL^2}{Nt\ m^2}$
   b) $\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9 \frac{Nt\ m^2}{COUL^2}$

C) E between 2 plates $= \frac{Q}{\varepsilon_0 A}$

D) $F_{FIELD} = QE$

E) $\Delta W = \frac{1}{2} m u^2 = QEL$ (independent of path in cons. field)
1) Voltage (DEF) $\equiv V_{21} = \frac{W_{P2} - W_{P1}}{Q} = -\int_1^2 E_L\, dL$ = potential difference
2) $V_{21} = -E_L L_{12}$

F) Current (DEF)
$i = \frac{dq}{dt}$ in Amps $= \frac{coul}{sec}$

G) Current density J in $\frac{A}{m^2}$
1) $J = \frac{di}{dA}$
2) $i = \iint J_A\, dA$

H) Charge density ρ (rho) in $\frac{COUL}{m^3}$
1) $\rho = \frac{dq}{dV}$
2) $J_L = \rho U_L$

J) Cathode Ray Tube
1) $\frac{1}{2} m u^2 = eV_0$ (EMISION $V = 0$)
2) $E_L = \frac{V_{01}}{d_0}$

II. CHAPT II - VOLTAGE - CURRENT RELATIONSHIPS

A) Energy of a Photon - $W = \frac{hc}{\lambda}$
1) h = Planck's constant $= 6.63 \times 10^{-34}$ J·sec
2) λ = wavelength
3) c = speed of lite

B) $i(T) = aAT^2 e^{-\frac{b}{T}}$ ($T = °K$, A = surface area, a & b material constants)

C) Child-Langmuir Law - $i(V) = 2.33 \times 10^{-6} \frac{A}{d^2} V^{\frac{3}{2}}$ (d = + and - spacing)

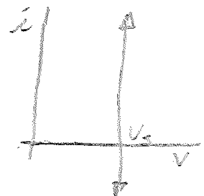
(OVER)

D) SOURCES & CIRCUITS

1) Voltage Source

a) a lumped, 2 terminal circuit, with ~~constant~~ V or

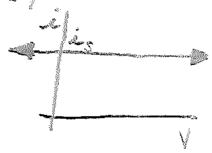
b) $V = V_S$ for all i



2) Current Source

a) a lumped, two terminal circuit, with constant i, or

b) $i = i_s$ for all V



3) May be dependent or independent ◇

4) Open circuit ⇒ $i = 0$ for all V (special current source)

5) Short circuit ⇒ $V = 0$ for all i (" voltage ")

E) OHM'S LAW

1) $V = Ri$

2) $R = \frac{dV}{di}$ (reciprical of V-i slope)

3) R has same sign as P (power)

F) An ideal DIODE

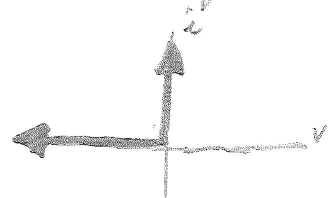
1) allows no (+) voltage or (-) current

2) ie) $V = 0, \; i > 0$

$i = 0 \quad V < 0$

$Vi = 0$ for all V-i

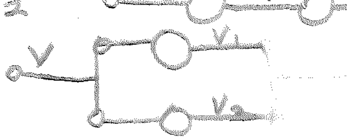
3)



G) Parallel & Series Connections

1) Series ⇒ $i = i_1 = i_2$

2) Parallel ⇒ $V = V_1 = V_2$



III. CHAPT. III - NETWORK SOLUTIONS

A) DEFINITIONS

1) NODE - a point at which 2 or more circuit elements have a common connection

2) BRANCH - a part of a network extending from one node to another $\geq$ circuit element

3) LOOP - a sequence of circuit elements forming a closed path that doesn't pass thru any node > 1

4) MESH - a loop with no circuit elements

5) PLANAR NETWORK - a network with no crossing wires

6) ONE PORT NETWORK - Only 2 terminals available for connection

B) KIRCHHOFF'S LAW

1) VOLTAGE LAW

ΣE around a closed path $= 0$

$\therefore \sum_{i=1}^{M} V_i = 0$ around any closed path

2) CURRENT LAW

Σi entering any node $= 0$

$\therefore \sum_{j=1}^{M} i_j = 0$

OCT. 1 1969

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CLASS

Mass ... measured indirectly

$\vec{F} = k \frac{m_1 m_2}{r^2} \vec{a}$

Coulombs Law

$$\vec{F} = \frac{q_1 q_2}{(4\pi\varepsilon_0) r^2} \vec{a}_r$$

q = elec. charge

ε_0 = *permittivity of freespace* $= \frac{10^{-9}}{36\pi} \frac{\text{FARAD}}{\text{M}}$

gravitational field

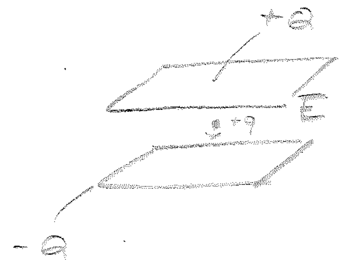
electric field

$\vec{E} = \lim_{q \to 0} \frac{\vec{F}_e}{q}$ $\qquad$ $\vec{g} = \lim_{m \to 0} \frac{\vec{F}_G}{M}$

CHAPT. I
5, 8, 12, 19

Can't measure field directly, must be ...

(EX)



must take into account charge (+q)

BOOK

VOLTAGE - *Energy to transmit electric charge from one terminal to another*

CURRENT - *Measure of rate at which an electric charge is moving thru an element*

(VOLTAGE)(CURRENT) = POWER

COULOMB'S LAW:

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2} \; : \; \varepsilon_0 = 8.85 \times 10^{-12} \frac{\text{FARAD}}{\text{METER}}$$

FIELD - *Region of influence or source of influence*

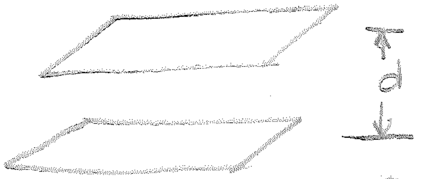
ELECTRIC FIELD INTENSITY:

$$E = \lim_{\Delta Q \to 0} \frac{\Delta F}{\Delta Q} = \frac{dF}{dQ}$$

MAGNITUDE OF THE FIELD OF A PT. CHARGE Q_1

$$E = \frac{Q_1}{4\pi\varepsilon_0 R^2} \qquad E = \frac{\text{NEWT}}{\text{COUL}} = \frac{\text{VOLT}}{\text{M}}$$

$\frac{V}{m} \, C = N$



$F = 9.65 \times 10^{4}\ C$

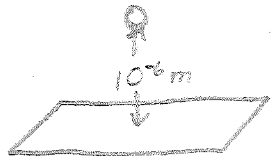
$\varepsilon_0 = 8.54 \times 10^{-7}\ \frac{C}{m}$

$E = \frac{Q}{\varepsilon_0 A}$ $\qquad$ $\frac{C}{F} = V$

Q in COLOMBS

TRIVIA ↓ ↓

Pg 8 ~~EX)~~ XI-1



$$E = \frac{Q_1}{4\pi\varepsilon_0 R^2}$$

$$= \frac{10^{-3}\ \text{COUL}\ \cancel{m}}{4\pi(8.85\times10^{-12}\ \text{FAR})\ 10^{-12}\ m^{2}}$$

$$= 8.98 \times 10^{19}\ \frac{\text{COUL}}{\text{FARADAY}}$$

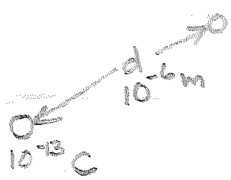
$$= 8.98 \times 10^{19}\ \frac{\text{COUL}}{\text{FARAD}}\ \frac{\text{FAR}}{\text{COUL}\ 9.65\times10^{4}}$$

$$E = 9.41 \times 10^{14} = F \times d$$

$$F = \frac{9.41 \times 10^{14}}{10^{-6}\ m} =$$

~~EX)~~ XI-1

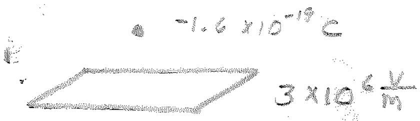
a) $F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 d^2}$



$$F = \frac{(10^{-13}\ \cancel{C})(-1.6\times10^{-19}\ C)\ \cancel{m}\ \ \cancel{F}\ \text{FAR}}{4\pi(8.85\times10^{-12}\ \text{FAR})\ 10^{-12}\ m^{2}\ 9.65\times10^{4}\ \cancel{C}}$$

$$F = 1.490 \times 10^{14}\ \frac{C}{m}$$

~~b)~~



$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}$$

$$E = \frac{Q}{\quad}$$

$$=$$

Pg (a)

TRIVIA (CONT.)

OCT. 2 1969

1-X-1)

a) $-1.6 \times 10^{-19} C$, $10^{-5} m$, $10^{-13} C$

$$F = \frac{Q_1 Q_2}{4\pi \epsilon_0 R^2} = \frac{-1.6 \times 10^{-22}}{4\pi\ 8.85 \times 10^{-5}\ 10^{-10}}$$

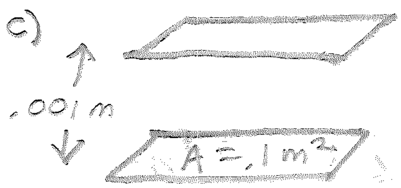
$$= 1.44 \times 10^{-24}\ NT$$

b) $E = 3 \times 10^{6} \frac{V}{M}$

$\frac{V}{M} C = N$

$(3 \times 10^{6} \frac{V}{M})(-1.6 \times 10^{-19}) = 4.8 \times 10^{-13}$

c)



$$E = \frac{Q}{\epsilon_0 A} = \frac{10^{-6}\ C\ \ M}{8.85 \times 10^{-5}\ FAR\ .1 M^2}$$

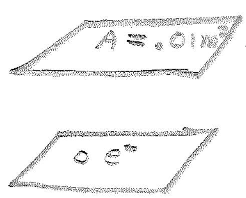
$$= .113 \frac{C}{FAR\ M} = .113 \frac{V}{M}$$

$$(.113 \frac{V}{M})(-1.6 \times 10^{-19} C) =$$

$$.181 \times 10^{-19}\ NT =$$

$$1.81 \times 10^{-20}\ NT$$

~~1-X-2)~~



$E_0 = \frac{Q_0}{\epsilon_0 A_0}$

$F_e = E_0 Q_e$

$F_e = m_e a$

$a = V_0 t \quad V_0 = d t$

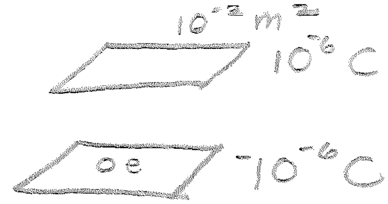
$\frac{a}{V_0} = \frac{V_0}{d_0}$

$d_0 = \frac{V_0^2}{a} = \frac{V_0^2 m_e}{F_e} = \frac{V_0^2 m_e \epsilon_0 A}{Q_0 Q_e}$

$$d = \frac{(8.85 \times 10^{-5})(25 \times 10^{14} \frac{m}{s})\ 9.11 \times 10^{-31} kg\ F\ m^2}{5 M\ 10^{-6} C\ -1.6 \times 10^{-19}}$$

$$= 1.25 \times 10^{-5} m$$

1-X-2) ~~a)~~



$$E = \frac{Q}{\epsilon_0 A}$$

$$= \frac{10^{-6}\,C}{8.85 \times 10^{-7}\,F\,M^2}\ \ \frac{M}{}$$

$$= .113 \times 10\,V = 1.13\,\frac{V}{M}$$

$$F = 1.13\,\frac{V}{M}\left(-1.6 \times 10^{-19}\,C\right) = 1.81 \times 10^{-19}\,Nt$$

$$a = \frac{1.81 \times 10^{-20}\,NT}{9.11 \times 10^{-31}\,\frac{Kg}{S}} = 1.99\,\frac{M}{SEC^2} \times 10^{11}$$

$$\frac{v^2}{a} = d$$

$$d = \frac{25 \times 10^{14}}{1.99 \times 10^{11}}\,m = 12.6 \times 10^{4}\,m$$

b) $E = 10^{-15}\,J = 10^{-5}\,VC$ $\frac{J}{C}$

E =

TRIVIA

BOOK

$$F_{field} = QE$$

$\frac{1}{2}mV^2 = W = QE_L L_{12}$ (IN JOULES) E_L = COMPONENT OF E IN L DIRECTION

Work is independent of the path in a conservative field

$$V_{21} = \frac{W_{P2} - W_{P1}}{Q} \qquad \text{Voltage = Potential difference}$$

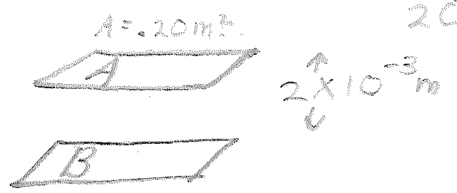
$$V = \frac{J}{C}$$

$$= E_L L_{12}$$

If $E_L = 0$, (locus of pts = equipotential surface)

MORE TRIVIA

X1-3)



$$20\,cm^2 \cdot \frac{1\,m^2}{100\,cm^2} = .20\,m^2$$

✓a) $Q_A = 10^{-8}C$

$Q_B = -10^{-8}C$

$$E = \frac{Q}{\varepsilon_0 A}$$

$$= \frac{10^{-8}C}{8.85E\ .20\,m \times 10^{-5}}$$

$$= .565 \times 10^{-3}\,\frac{V}{M} = .565\,\frac{V}{M}$$

$$V_{AB} = E_L L_{BA}$$

$$= (.565 \times 10^{-3}\,\tfrac{V}{M})(2 \times 10^{-3}m)$$

$$= 1.13 \times 10^{-6}\,V$$

✓b) $V = \frac{J}{C}$

$$= \frac{8 \times 10^{-18} J}{-1.6 \times 10^{-19} C} = -50\,V$$

✓c) $E_{AB} = 2 \times 10^{6}\,\frac{V}{M}$

$$V_{AB} = E_L L_{BA}$$

$$= 2 \times 10^{6}\,\frac{V}{M}\,(2 \times 10^{-3} M)$$

$$= 400\,V$$

X1-4)

a) $V = \frac{J}{C}$

$$= \frac{3\,J}{2\,C} = 1.5\,V$$

b) $\Delta P_{12} = 9.6 \times 10^{-19}$

$$V = \frac{9.6 \times 10^{-19}}{-1.6 \times 10^{-19}} = -6\,V$$

c) $|\vec{Z}|$

BOOK

AMPHERE = UNIT OF CURRENT = A

$$A = \frac{c}{s}$$

Current is represented by j

CURRENT DENSTINTY = J

$$j = \frac{dq}{dt}$$

$$J = \frac{A}{M^2} = \frac{dj}{dA}$$

FOR UNIFORM CURRENT DENSITY

$$j = JA$$

CHARGE DENSITY = ρ

$$\rho = \frac{dq}{dV} \quad (V = \text{VOLUME})$$

$$\boxed{J_L = \rho u_L \quad u = \text{VELOCITY}}$$

POWER

$$P = Vj$$

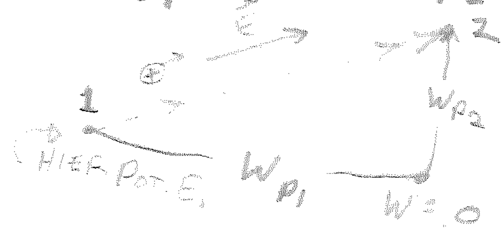
LAB

OCT. 3 1969

DEF $$\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q}$$

OCT. 3 1969

DEF $$W = -\int_1^2 \vec{F} \cdot d\vec{l} = -\int_1^2 q\vec{E} \cdot dl = W_{p_2} - W_{p_1}$$

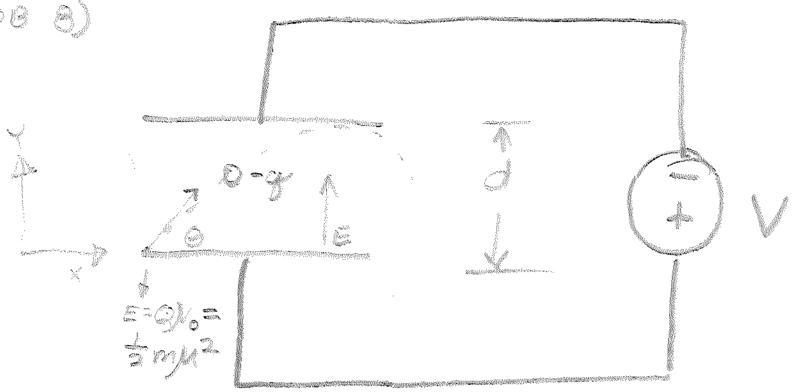


$$q\left[-\int_1^2 \vec{E}\,dl\right] = q\,V_{21}$$

Must memorize:
Fundamental Laws (No derivation)
And Definitions (Mathematical)

CH. 1.

PROB 8)



$W = qV_0 = -eV_0$ $\quad e = +1.6 \times 10^{-19}\,C$

$\mu_Y = \mu \sin\Theta$ $\quad F_Y = q\frac{V}{d} = m\frac{d\mu_Y}{dt}$

$\mu_X = \mu\cos\Theta$ $\quad E_{TOTAL} = QV_0$

$V = V_{21} = -\int_1^2 E \cdot dl = -(-Ed \Rightarrow V = ED$

$\int_{t_1}^{t_f} q\frac{V}{d} = m\int_{V_0}^{0} dU_y$

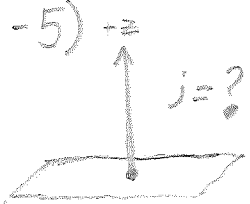
$\Delta K_Y = \left(\frac{1}{2}mv_{Y0}^2 - 0\right) = qV = eV$

5/2/9

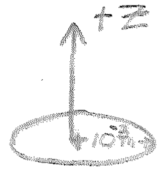
TRIVIA

OCT. 4 1969

X1-5)



a)



$J = 5000\frac{A}{m^2}$

$j = JA = 5000\frac{A}{m^2}\pi 10^{-6} = 15{,}700 \times 10^{-6} A =$
$1.57 \times 10^{-2} A$

b) $V_e = 10^7\frac{m}{s}$
$D = \frac{10^8 e}{m}$

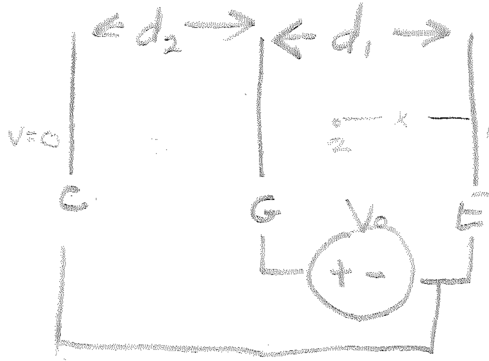
$J = \rho u$
$= \left(\frac{10^8 e}{DA}\right)\left(\frac{-1.6 \times 10^{-8} C}{e}\right)10^7\frac{m}{s}$
$= -1.6 \times 10^5 \frac{C}{SEC}$

c)

$$J = \frac{0.2}{r} e^{-10,000 r} \ \frac{A}{m^2}$$

CLASS

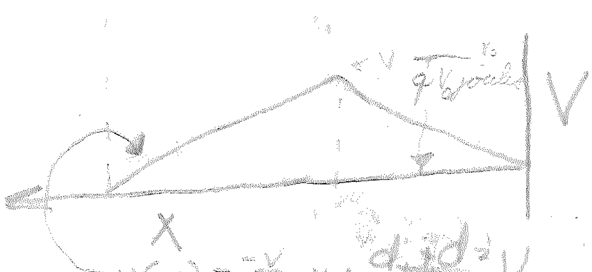
5 a)



$\hat{E}$ is uniform in closely ∥ plane

$$V_{21} = -\int_1^2 E \cdot dl$$

$$= -E\int_1^2 dl = -Ex$$



$$V(x) = \frac{V}{d_2}x + \frac{d_1 + d_2}{d_2} V_0$$

a) $V_C = 0$

(The difference of the voltage is the same)

$$qV_0 = \frac{1}{2} m u^2$$

$$u = \sqrt{\frac{2qV_0}{m_e}}$$

b) $V_G = ?$

$$v = \frac{dx}{dt}$$

$$dx = v\,dt$$

$$\int_0^{d_1} dx = \int_0^{t_1} v\,dt$$

$$d_1 = \int_0^t \frac{qE}{m} t\,dt$$

$$= \frac{qE}{2m} t_1^2$$

$$t_1 = \sqrt{\frac{2md_1}{qE}}$$

$$F = qE = m\frac{dv}{dt}$$

$$qE\int_0^t dt = \int^{v(t)} m\,dv$$

$$v(t) = \frac{qE}{m} t$$

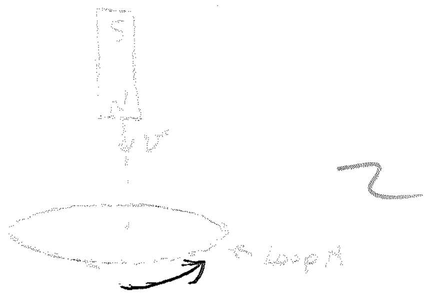
$$|V F \,|\int E dx|$$

$$\left|\frac{dv}{dx}\right| = |E|$$

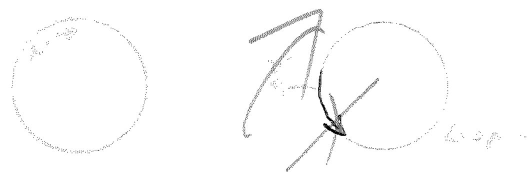
Bob Marks

6

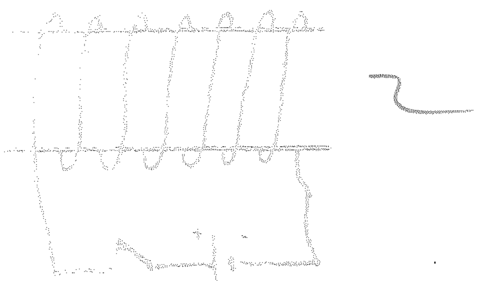
... FOLLOWING SKETCHES INDICATE THE DIRECTION ...
... CURRENT WILL FLOW IN LOOP A



magnet moving
toward coil

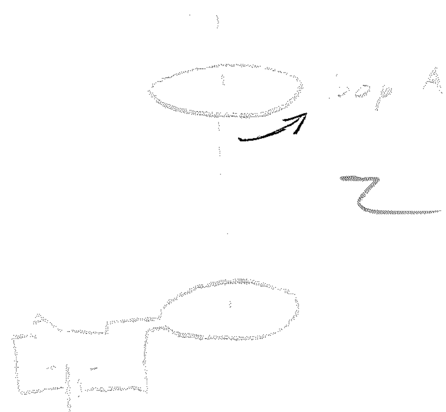


2) both coils in same plane. first
loop carrying current i and loop A
moving toward 1st loop.



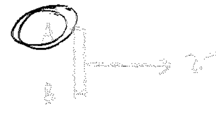
V

when switch is closed

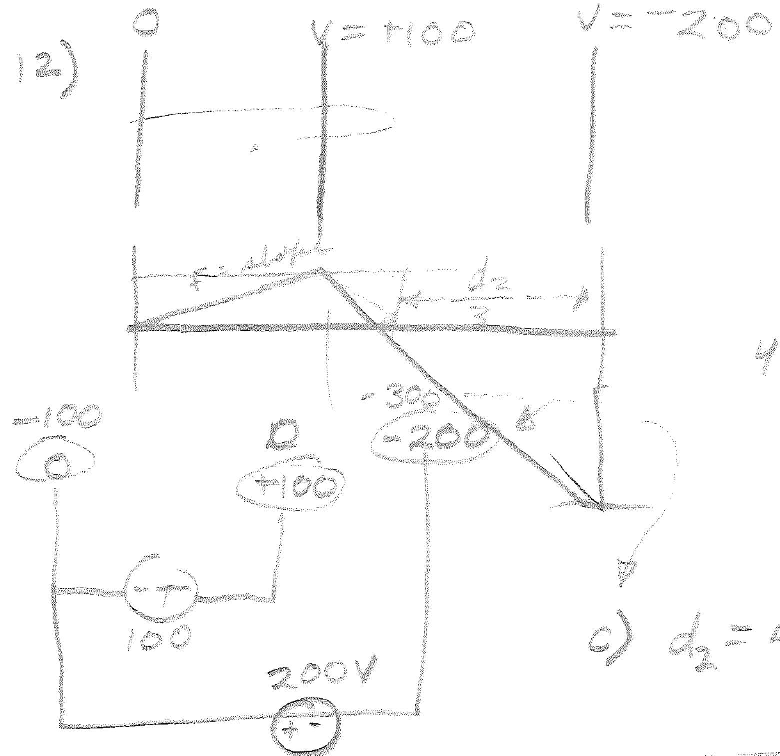


4) When switch is closed

... END A or B IS AT THE HIGHER POTENTIAL IF
METAL ROD AB is moving to right in the magnetic
... produced by current i in a long wire



12)



$y = \left(\frac{-300}{d_2}\right)x + 100$

if $y = 0 \Rightarrow x = \frac{d_2}{3}$

c) $d_2 = 4d_1$

(15, 13)

CLASS
CURRENT

$amp = \frac{coul}{sec}$

$i = \frac{dQ}{dt}$ - def.

ρ = charge density in $\frac{c}{m^3}$

$= \bar{J} A$

$= \rho \mu A$

$= (\rho A)\mu \quad \frac{coul}{m} \frac{m}{sec}$

BOOK
POWER (WATT)

$W = \frac{J}{S}$

$F = \frac{QV}{d}$

$E = \frac{V}{d}$

(13) $j = 8\int_0^{\frac{10^{-3}}{2}}\int_{y=x}^{\frac{10^{-3}}{2}} \left[2\times10^{8}\, x\right] dy\, dx$ $\qquad dA = dy\,dx$

$J = 2\times10^{8}$

$$j = 8\int_0^{\frac{10^{-3}}{2}} \left[10^{5}\ \frac{x}{\frac{10^{-3}}{2}}\right]\left[\frac{10^{-3}}{2} - x\right] dx$$

$$= 8\left[10^{5}\,\frac{x^{2}}{2} - \frac{10^{5}}{\frac{10^{-3}}{2}}\,\frac{x^{3}}{3}\right]_0^{\frac{10^{-3}}{2}}$$

$$= .2\left(\tfrac{1}{6}\right) = 33\ \text{mA} = .033\,\text{A}$$

$W = -\int \vec{F}\cdot d\vec{l} = -q\int \vec{E}\cdot dl$

joules = volt meter = coul meter

= amps volt

= watt sec

$qV = \frac{1}{2} m u^{2} \quad \frac{J}{e} \times \frac{10^{11} e}{m}$

$$i = \oint J \cdot dl = -\frac{dQ}{dt} = \frac{d}{dt}\int \rho \, dv$$

$\oint$ means integrate over a closed surface

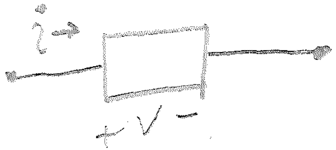
Kerchoff's Current Law

CLASS

PROB 1,3,4,5
CHPT. 2

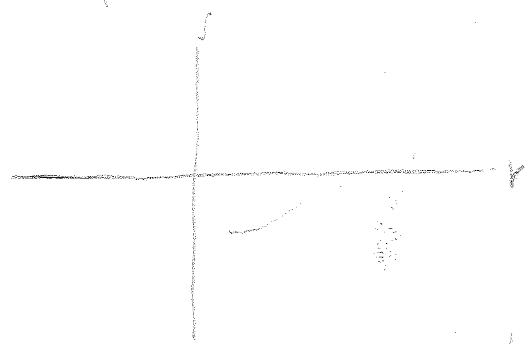
2 Terminal device

i always enters + terminal

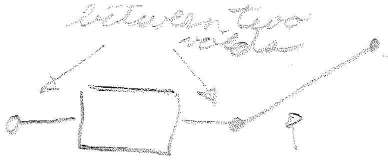


Most electronic circuits don't have handy mathematical equations

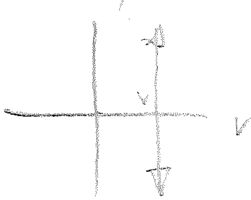
Restrictions
2 terminal device
v-i are time independent



V volts between two nodes

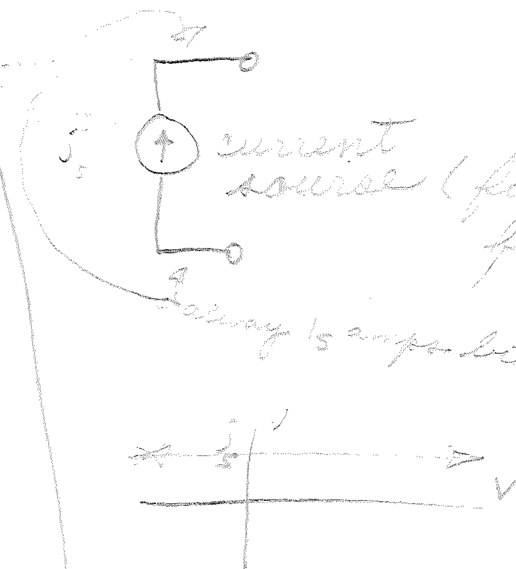


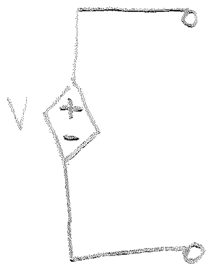
assume no resistance (perfect conductor)



current source (forcing function)

always I_s amps between them





$V = kV_g$, where V_g (i_g etc.) is some other V in current
$V = k i_g$ etc.

∴ V is dependent on some other measurement (V, i, etc.) in the circuit

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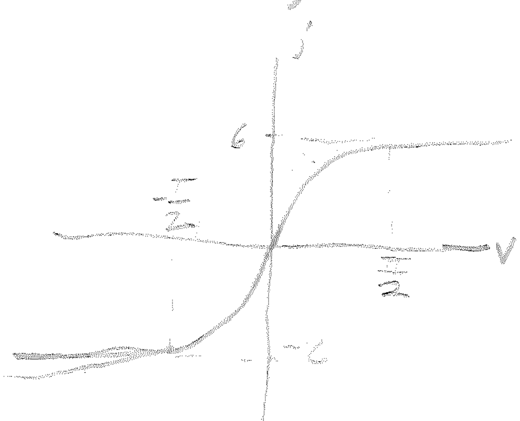
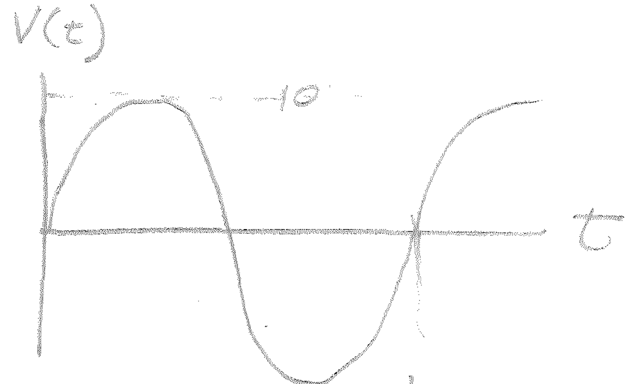
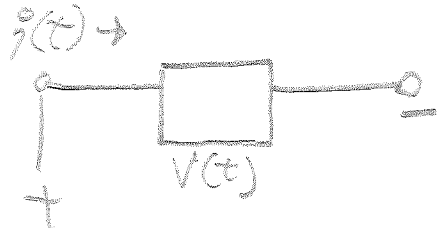
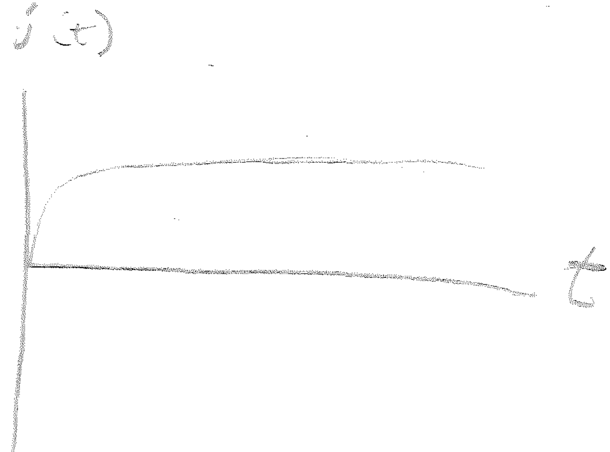
CHAPT 2, PROB. 2



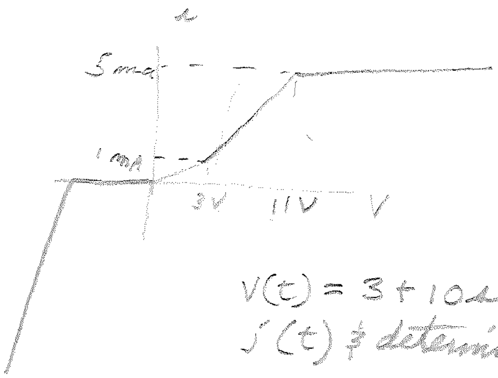


$V(t) = 10\left[\sin 60(2\pi)\boxed{t}\right]$ $\frac{1}{60}$ sec



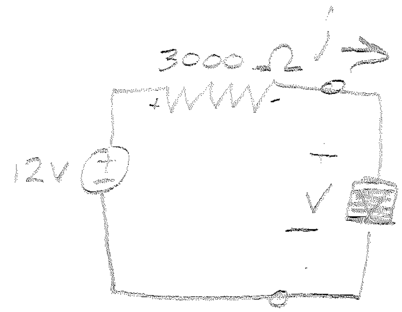


can coincide axes & use projections



$V(t) = 3 + 10 \sin \omega t$

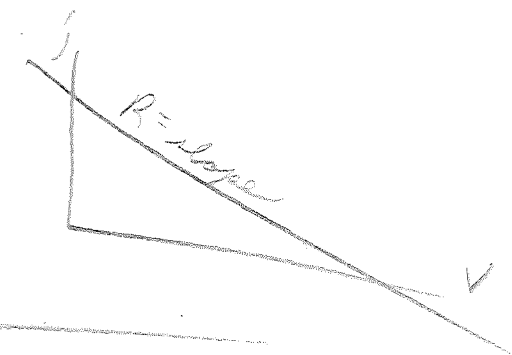
$i(t)$ & determine max & min –

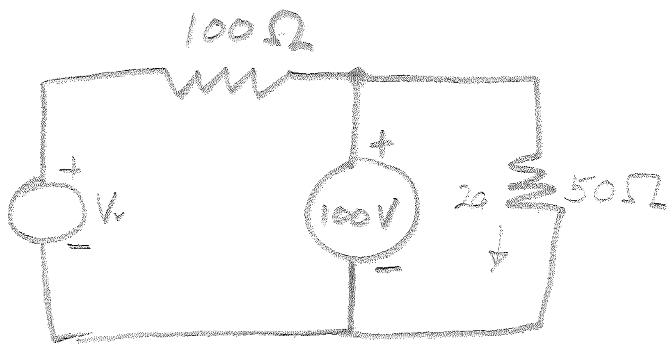


$V = ?$

$i = ?$

$V = 12 - 3000 i$





$$\oint J \cdot dA = -\frac{dq}{dt}$$

$$W = -\int F \cdot dl = -q \int E \cdot dl = qV$$

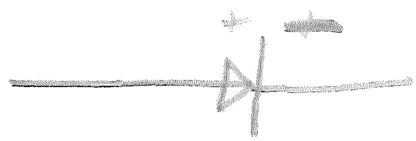
$$P = \frac{dw}{dt} = V\frac{dq}{dt} + q\frac{dv}{dt} = Vi$$

$$V = iR \qquad P = Vi = i^2 R = \frac{V^2}{R}$$

1,8,11,14 or 17

1) Isolate each part of the system
2) Observe if the voltage across a line is the Voltage across the same line but connected through a different source
3) Compare both sources by simultaneous equations or intersections of graphs to see common pt.

Diode

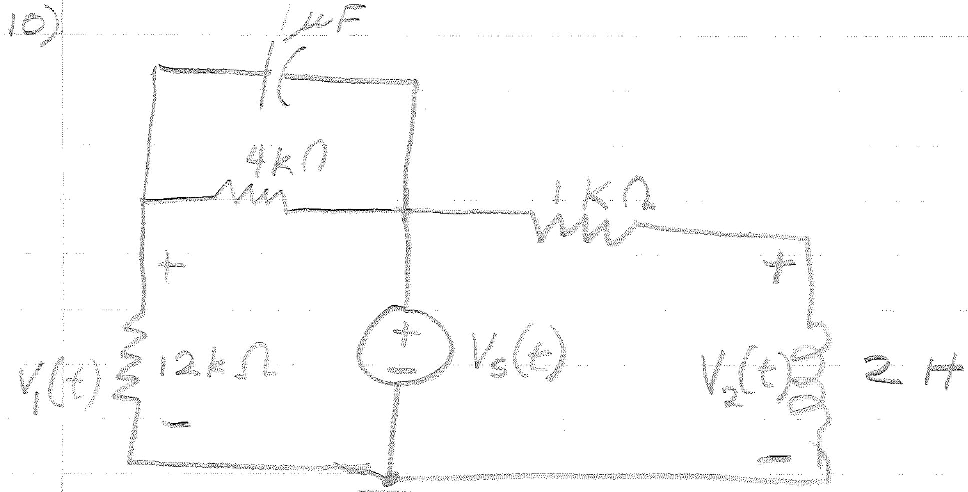


① When current is flowing with arrow, there's a direct short.
② If current flows in opposite direction, got an open circuit

a diode is sensitive to what is going on in the circuit, like an electric switch

Pp. 259-60

10)



$V_s(t) = 100 + 200\mu(t)$

N TO D

$\frac{V_1}{12{,}000} + \frac{V_1 - V_s}{4000} + 10^{-6}\left(\frac{d}{dt}(V_1 - V_s)\right) = 0$

$\frac{1}{2}\int V_2 dt + \frac{V_2 - V_s}{1000} = 10$

$\frac{V_1}{12} + \frac{V_1}{4} + 10^{-3}\frac{dV_1}{dt} = \frac{V_s}{4} + 10^{-3}\frac{dV_s}{dt} = 25 + 50\mu + .2\delta(t)$

$\frac{V_1}{3} + 10^{-3}\frac{dV}{dt} = 25 + 50\mu(t) + .2\delta(t)$

TO FIND DIFF EQ

$\frac{V_1}{3} + 10^{-3}\frac{dV_1}{dt} = 0$

$V_1 = Ae^{\frac{-t}{3\times10^{-3}}}$

$V_P = V_{11} + V_{12} + V_{13}$

$\frac{V_{11}}{3} + 10^{-3}\frac{dV_1}{dt} = 25$

$V_{11} = 75$

$\frac{V_{12}}{3} + 10^{-3}\frac{dV_{12}}{dt} = 50 \quad t \geq 0 \qquad \frac{V_{12}}{3} + 10^{-3}\frac{dV_1}{dt} = 0 \; t<0$

$V_{12} = 150 \qquad V_{12} = 0$

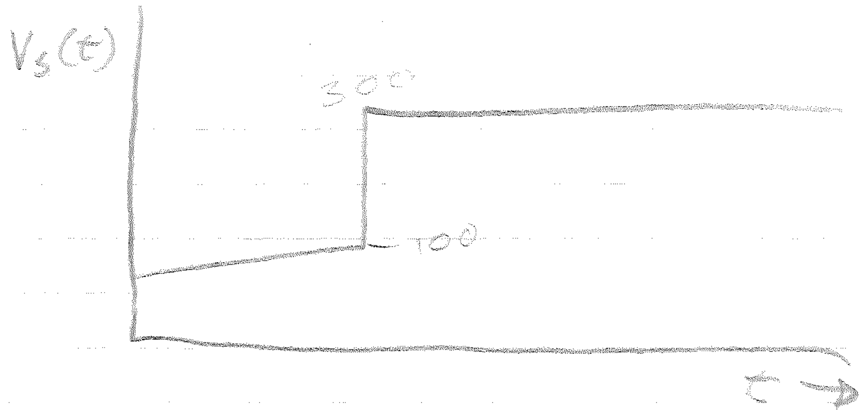
$V_{12} = 150\mu(t) + ??$

$V_{13} = 0$ at $t \neq 0$

SOLUTION

$=0$ at $t<0$

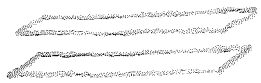
$$V = 9e^{-\frac{t}{3\times10^{-3}}}\cancel{\mu(t)} + 75 + 150\,\mu(t) \qquad \begin{matrix} t>0 \\ t<0 \end{matrix}$$



Pg 12, CHAPT. 1

X1-3

$.2m^2$



$20\,cm^2 \; \frac{1\,m^2}{100\,cm^2} = .2\,m^2$

a) $q_1 = -q_2 = 10^{-8}C$

$E = \frac{q}{\epsilon_0 A} = \frac{10^{-8}}{(8.85\times10^{-12})(.2\,m^2)} = .565\times10^4 \frac{V}{m}$

$V = Ed$

$= (5.650)\frac{V}{m}(2) = 1.13V$

b) $W = Fd$

$\frac{Nt}{m} = \frac{Nt}{COUL}$

$Nt\,m = V\,COUL$

$JOULE = V\,COUL$

$W = Vq$

$\frac{8\times10^{-18}}{1.6\times10^{-19}} = V$

$V = -50$

c) $\vec{E} = 2\times10^6 \frac{V}{M}$

$V = Ed$

$= (2\times10^6)(2\times10^{-4})$

$= 400V$

X1-4

a) $W = Vq$

$V = \frac{W}{q}$

$= \frac{3J}{2} = -1.5V$

b) $E = Vq$

$V = \frac{9.6\times10^{-19}}{1.6\times10^{-19}} = -6\,V$

c) $\Sigma V = 4$

Pg 16

X1-5) a) $J = 5000 \frac{A}{m^2}$

$i = JA$

$= (5\times10^3)\pi(10^{-8})$

$= 15.7\times10^{-5}A$

b) $J = \rho u$

$i = \rho u A$

$= \frac{C}{m^3}\frac{m}{s}m^2$

$\frac{q}{L} = \frac{10^5 e}{m} \; \frac{1.6\times10^{-19}C}{e} = 1.6\times10^{-14}\frac{C}{m}$

$\frac{q}{L}\frac{L}{t} = i = 1.6\times10^{-14}\;10^7 = 1.6\times10^{-7}\frac{C}{sec}$

$\cancel{c)}$ $J = \frac{.2}{r}e^{-10,000r}$

$\int J\,dr = \quad u = \frac{.2}{r} \quad dv = e^{-10,000r}$

$du = \frac{-.2}{r^2} \quad v = \frac{e^{-10,000r}}{-10,000}$

$=$

1-6) a) $q = .02\cos(100t + 1.5\pi)$

$i = \frac{dq}{dt} = -2\sin(100t - 1.5\pi)$

$= -2\ A$

b) $q = 4\times10^{20} e^{-2t}(1.6\times10^{-19}\ \frac{C}{e})$

$q = 6.4\, e^{-2t}$

$i = \frac{dq}{dt} = 12.8\, e^{-2t}$

$i = 12.8\ A$ at $t = 0$

c) $V = 10(t - .3)$ $\quad p = Vi$

$i = \frac{p}{V}$

$= \frac{4}{10(t-.3)}$

at $t = 0,\ i = -1.33\ A$

CHAPT. II Pg 34

X2-2 $V = [3.93\times10^{-2}T - 1.5]\, i$

$\frac{dV}{di} = 3.93\times10^{-2}T - 1.5$

a) at $T = 200$, $\frac{dV}{di} = 6.4\ \frac{V}{A}$

b) at $T = 300$ " $= 10.3\ \frac{V}{A}$

c) " " 400 " " $14.2\ \frac{V}{A}$

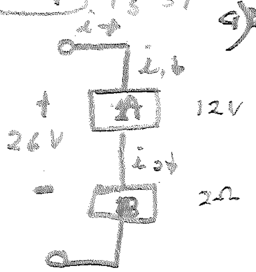
X2-3 pp 40-1

$i = aAT^2 e^{-\frac{b}{T}}$

$= (600)(2.5\times10^{-5})\pi(10^6)\, e^{-121}$

e^{-121}

X2-4 pg 51



a) A = Voltage source = 12V

B = 2Ω

$i = i_1 = i_2$

$V_{2B} = 14$

$V = iR$

$i = \frac{V}{R} = \frac{14}{2} = 7A$

b) $i_1 = i = i_2$

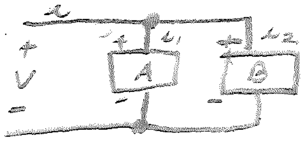
$\Sigma R = 13\Omega$

$V = Ri$

$i = \frac{V}{R} = \frac{26}{13} = 2\ A$

c) $i = \frac{26}{8} = 3.25\ A$

X2-5



$i = i_1 + i_2$ $\quad i_1 = 3A$

a) $A = V_{s_1} = 10V$

$B = i_{s_2} = 5A$

$i = 8A$

b) $A = V_s = 10V$

$B = R = 5\Omega$

$i = i_1 + i_2$

$i_2 = 2A$

$i = 5A$

c) $\frac{1}{R} = \frac{1}{6} + \frac{2}{6} = \frac{3}{6}$

$R = 2$

$V_1 = 18V$

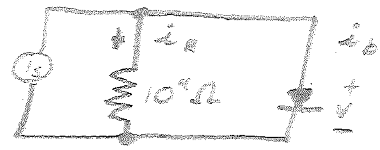
$i_2 = \frac{18}{3} = 6$

$i = 9$

Pg 55
X 2-6)

a) $i_s = .02 \cos 1000\pi t$

$.25\,ms \frac{1}{10^3} = .25 \times 10^{-3} s$



$i_s = .02 \cos .25\pi$

$= .02 \frac{1}{\sqrt{2}}$

$14.1 \times 10^{-3} A$

b) $i_s = i_R + i_b$

$i_b = 0$

$i_s = i_R = .02 \cos \frac{1}{3}\pi = \frac{.02}{2} = 10^{-2} A$

CHAPT 3 Pg 67

X 3-2)

$60V - V_2 - V_7 = 0$ c) $V_7 + 40 - 15 - 12 = 0$

$V_7 = -13V$

a) $60 - V_2 = -13$

$V_2 = 73$

d) $V_6 - 12 - 15 = 0$

$V_6 = 27V$

~~X 3-3)~~ $i_3 + i_2 - 12 = 0$

$i_1 - 2 - 1 - i_8 = 0$

$4 + i_8 + 12$

$\oint \vec{D} \cdot dA = q$

$\vec{D} = \epsilon \vec{E}$

$V = -\int \vec{E} \cdot d\ell$

$q = CV$

FAR. LAW.

$\oint \vec{E} \cdot d\ell = -\frac{d}{dt} \int \vec{B} \cdot dA = \frac{-d\Phi}{dt}$

$\vec{B} = \mu \vec{H}$

AMPERES LAW

$\oint \vec{H} \cdot d\ell = I$

$L = \frac{N\Phi}{I}$ = inductance

$\vec{E} = 0$ IN PERFECT CONDUCTOR

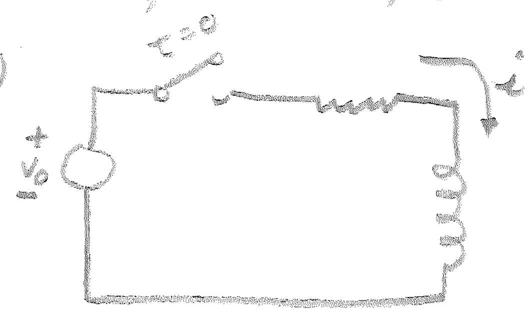
$\therefore V = -\int \vec{E} \cdot d\vec{\ell} = \frac{d\Phi}{dt}$

FUNDAMENTAL LAW

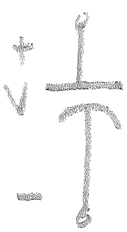
$$\boxed{V = N\frac{d\Phi}{dt} = L\frac{di}{dt}} \qquad i = C\frac{dv}{dt}$$

for inductor, talk about Φ, i relationship

EX)



$V = L\frac{di}{dt}$



$i = C\frac{dv}{dt}$

$V_0 = Ri + L\frac{di}{dt}$

$dt = \frac{L\,di}{V_0 - i}$

$i = \frac{V_0}{R} + Ie^{-\frac{R}{L}t}$

$I = \frac{-V_0}{R}$

CHAPT. 2, Pg 55-6

1) a) $(v, j) = (1, 2), (-6, -1)$

$m = \frac{3}{7}$

$j - 2 = \frac{3}{7}(v - 1)$

$j - 2 = \frac{3}{7}v - \frac{3}{7}$

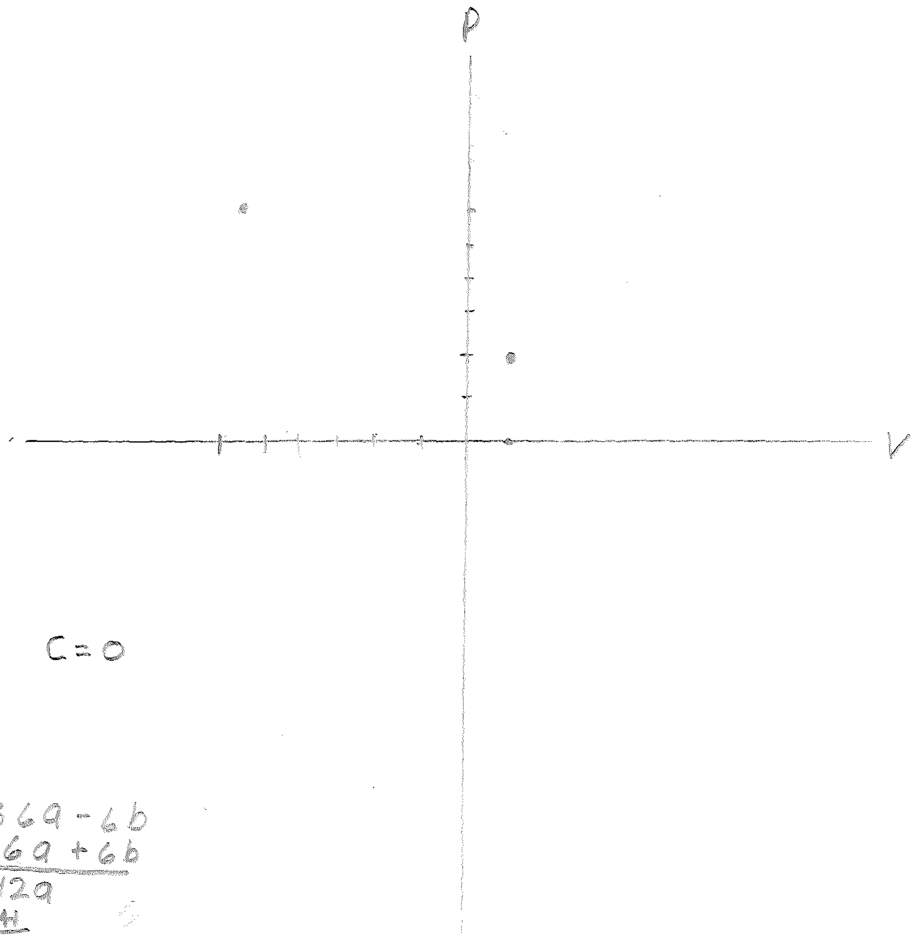
$j = \frac{3}{7}v + \frac{11}{7}$

j inter. $= \frac{11}{4}$

$\frac{3}{7}v = -\frac{11}{7}$

$v = -\frac{11}{3}$

v inter $= -\frac{11}{3}$



b) one

$0 = a\,0^2 + b0 + c$ $\quad c = 0$

$2 = a(1)^2 + b(1) + c$

$-1 = a36 - 6b +$

$72 = 36a + 3b$

$-1 = a36 - 6b$

$73 = 9b$

$b = \frac{73}{9}$

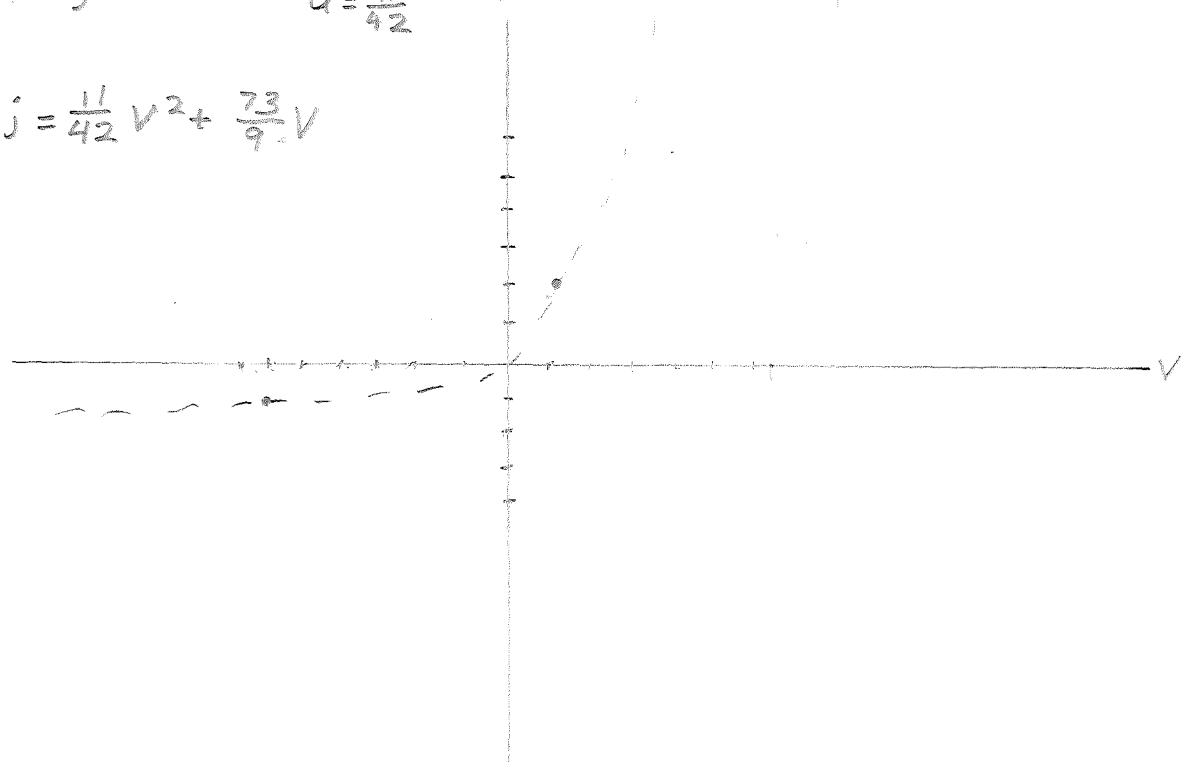
$-1 = 36a - 6b$

$12 = 6a + 6b$

$11 = 42a$

$a = \frac{11}{42}$

$j = \frac{11}{42}v^2 + \frac{73}{9}v$



3) (2, 3), (4, 5) (0, 0), (8, 8) (v, j)

$3 = 8a + 4b + 2c$
$5 = 64a + 16b + 4c$
$8 = 512a + 64b + 8c$

$6 = 16a + 8b + 4c$
$\underline{5 = 64a + 16b + 4c}$
$7 = 48a + 8b$
$-4 = 192a + 32b$
$\underline{-2 = 384a + 32b}$
$2 = 192a$
$a = \frac{1}{81}$
$-2 = 31{,}304 + 32b$
$b = 78.3$
$c = -185.6$

$8 = 512a + 64b + 8c$
$\underline{10 = 128a + 32b + 8c}$
$-2 = 384a + 32b$

$8a + 4b + 2c = 3$
$64a + 16b + 4b = 5$
$512a + 64b + 8c = 8$

$$a = \frac{\begin{vmatrix} 3 & 4 & 2 \\ 5 & 16 & 4 \\ 8 & 64 & 8 \end{vmatrix}}{\begin{vmatrix} 8 & 4 & 2 \\ 64 & 16 & 4 \\ 512 & 64 & 8 \end{vmatrix}}$$

Chapt. 2

pg 56)

4) $j = av^3 + bv^2 + cv + d$

(0,0) (2,3) (4,5) (8,8)

$d = 0$

$3 = 8a + 4b + 2c$

$5 = 64a + 16b + 4c$

$1 = 64a + 8c + c$

$$D = \begin{vmatrix} 8 & 4 & 2 \\ 64 & 16 & 4 \\ 64 & 8 & 1 \end{vmatrix} = 8\begin{vmatrix} 16 & 4 \\ 8 & 1 \end{vmatrix} - 4\begin{vmatrix} 64 & 4 \\ 64 & 1 \end{vmatrix} + 2\begin{vmatrix} 64 & 16 \\ 64 & 8 \end{vmatrix}$$

$$= {}^-128 + 768 - 1024 = {}^-384$$

$$D_1 = \begin{vmatrix} 3 & 4 & 2 \\ 5 & 16 & 4 \\ 1 & 8 & 1 \end{vmatrix} = 48 + 16 + 80 - 96 - 20 - 32 = {}^-4$$

$$a = \frac{4}{384} = \frac{1}{96}$$

$$D_2 = \begin{vmatrix} 8 & 3 & 2 \\ 64 & 5 & 4 \\ 64 & 1 & 1 \end{vmatrix} = 8\begin{vmatrix} 5 & 4 \\ 1 & 1 \end{vmatrix} - 3\begin{vmatrix} 64 & 4 \\ 64 & 1 \end{vmatrix} + 2\begin{vmatrix} 64 & 5 \\ 64 & 1 \end{vmatrix}$$

$$= 8 + 576 - 492 = 92$$

$$b = \frac{-92}{384} = \frac{-23}{96}$$

$$D_3 = \begin{vmatrix} 8 & 4 & 3 \\ 64 & 16 & 5 \\ 64 & 8 & 1 \end{vmatrix} = 8\begin{vmatrix} 16 & 5 \\ 8 & 1 \end{vmatrix} - 4\begin{vmatrix} 64 & 5 \\ 64 & 1 \end{vmatrix} + 3\begin{vmatrix} 64 & 16 \\ 64 & 8 \end{vmatrix}$$

$$= {}^-192 + 1024 - 1536 = {}^-704$$

$$c = \frac{-704}{-384} = \frac{11}{6}$$

$$j = \frac{v^3}{96} - \frac{23v^2}{96} + \frac{11v}{6} \cong .0104v^3 - .230v^2 + 1.83v$$

b) $v = 1 \Rightarrow j = 1.61$

$$\frac{1.61 - 1.75}{1.75} \times 100\% = 8.0\%$$

$v = 3 \Rightarrow j = 3.70$

$$\frac{4.1 - 3.7}{3.10} = 12.9\%$$

$v = 5 \Rightarrow j = 5.20$

$$\frac{5.2 - 5.8}{5.2} 100\% = 11.5\%$$

$v = 7 \Rightarrow j = 5.1$

$$\frac{7.3 - 5.1}{7.3} = 30\%$$

4) $j = aAT^2 e^{-\frac{b}{T}}$

$(.2) = 100(.01)T^2 e^{\frac{-11,600}{T}}$

$.2 = T^2 e^{\frac{-11,600}{T}}$

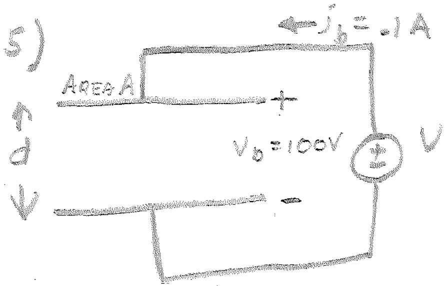
$\ln(.2) = 2\ln T - \frac{11,600}{T}$

-1.6

T	$2\ln T$	$\frac{-11,600}{T}$	$2\ln T - \frac{11,600}{T}$
300	11.4	−38.7	−27.3
500	12.4	−23.2	−10.8
750	13.3	−15.5	−2.1
775	13.2	−15.0	−1.8

$T \cong 775^\circ K$

5)



$j(V) = 2.33\times10^{-6}\frac{A}{d^2}V^{\frac{3}{2}}$

$.01 = 2.33\times10^{-6}\frac{A}{d^2}100^{\frac{3}{2}}$

$\frac{A}{d^2} = \frac{10\times10^{-3}}{(2.33\times10^{-6})(1.0\times10^{3})}$

$= 4.39$

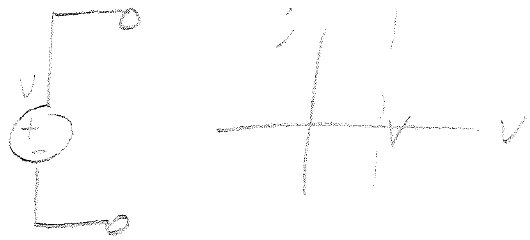
$j(V) = (2.33\times10^{-6})(4.38)(8.0\times10^{3})$

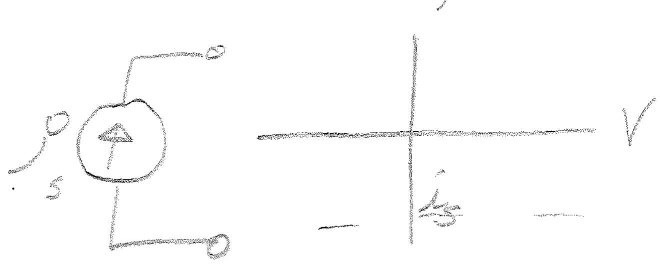
$j(V) = 81.5\times10^{-3}A = \frac{\Delta q}{\Delta t}$

$\Delta q = 8.15\times10^{-2}\Delta t$

$\Delta q = 8.15\times10^{-2}C$

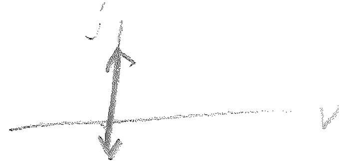
$8.15\times10^{-2}C \ \frac{1e}{1.6\times10^{-19}C} = 5.1\times10^{17}e$

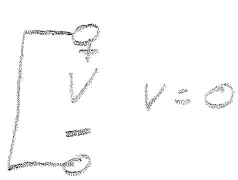




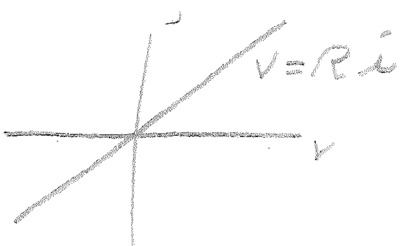
Sht. Circuit force V to 0

Open circuit

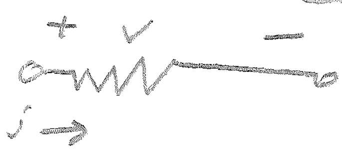




Linear v-i



R = resistance



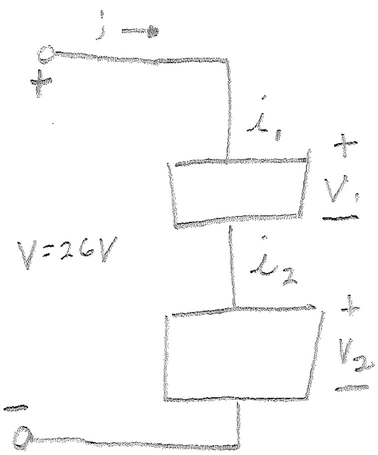
$i = \frac{v}{\infty} = 0$ open cir

$V = 0 i = 0$ short circuit

$R \Rightarrow \frac{\text{Volt}}{\text{amp}} \Rightarrow$ Ohm

$\frac{1}{R} \Rightarrow \frac{\text{Amp}}{\text{Volt}} \Rightarrow$ mho

x2-4



a) $\cancel{V_1 = 12V}$
$\cancel{V_2 = 2\Omega}$
$\cancel{i = i_1 = i_2}$
$\cancel{38V}$
$\cancel{R = \frac{V}{i}}$
$\cancel{2 = \frac{38}{i}}$
$\cancel{\frac{1}{2} + \frac{1}{12} = \frac{1}{V}}$

a) $\frac{(26-12)V}{i} = 2$

$i = 7A$

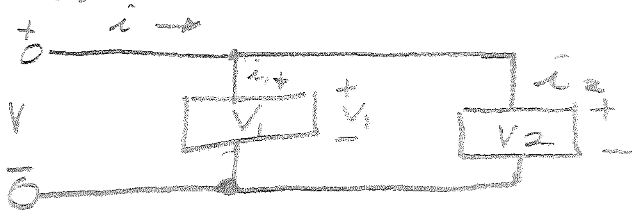
b) $13\Omega = \frac{26V}{i}$

$i = 2A$

c) $8\Omega = \frac{26V}{i}$

$i = 3.25A$

x2-5)



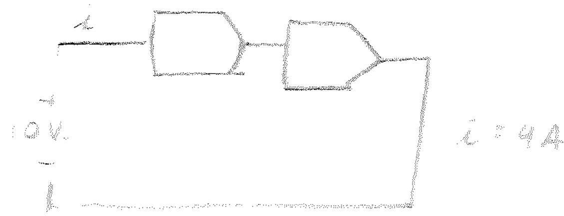
$i_1 = 3A$

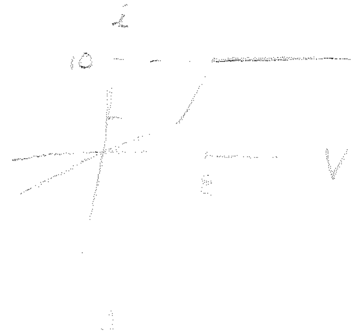
$V_{s_1} = 10V$
$i_{s_2} = 5A$

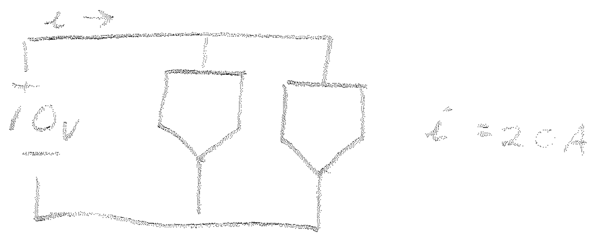
a) 5A

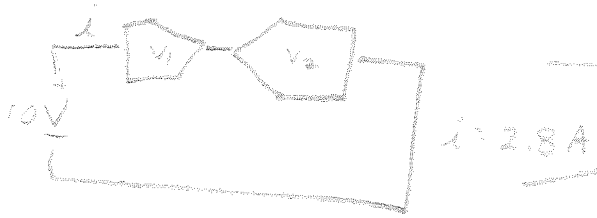
b) 8A

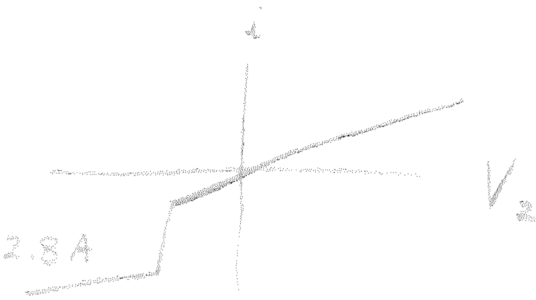
c) $\cancel{\frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2}}$



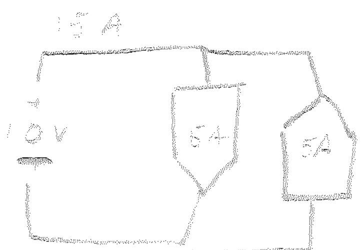


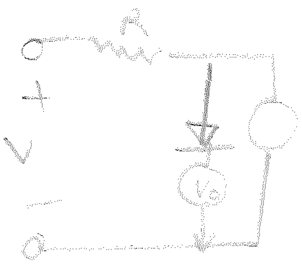


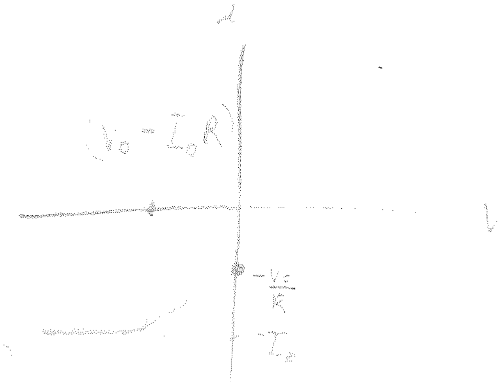




COMPOSIT OF 2 GRAPHS





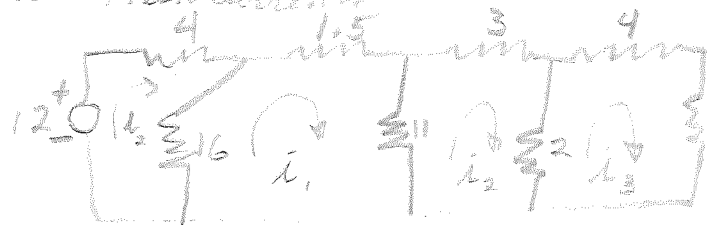


$i + I_0 > 0$ (mode)

$i > -I_0$

$i = \frac{V - V_0}{R}$

LAB Mesh currents



$$\begin{bmatrix} 12 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 20 & -16 & 0 & 0 \\ -16 & 28.5 & -11 & 0 \\ 0 & -11 & 16 & -2 \\ 0 & 0 & -2 & 14 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

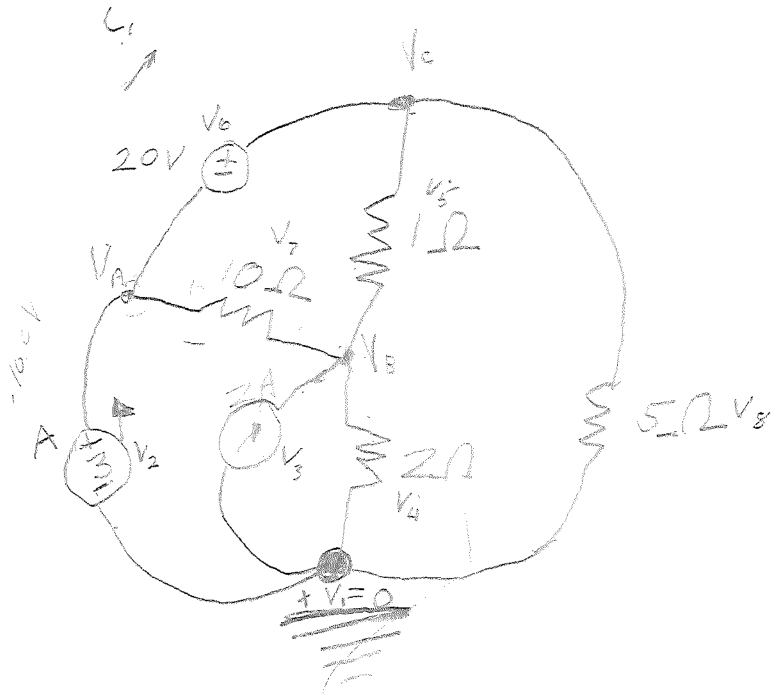
Mesh current matrix is symmetrical about the diagonal. All off diagonal are ≤ 0

circuit from equations

$$\begin{bmatrix} -6 \\ 5 \\ 0 \end{bmatrix} \begin{bmatrix} 5 & -2 & -1 \\ -2 & 10 & -3 \\ -1 & -3 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

Diagonal is sum of resistances around loop

7)



$3A+2A=(i_4+i_5)$

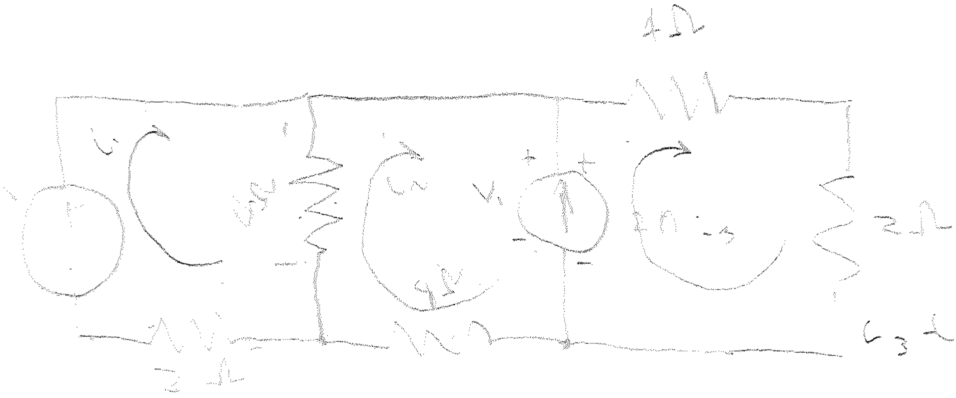
$V_C - V_A = 20$

V_A $\quad i_1 + \frac{V_A - V_B}{10} - 3 = 0$

V_B $\quad -2 + \frac{V_B - V_A}{10} + \frac{V_B - V_C}{1} + \frac{V_B}{2} = 0$

V_C $\quad -i_1 + \frac{V_C}{5} + \frac{V_C - V_B}{1} = 0$

$\frac{V_A - V_B}{10} - 3 + \frac{V_C}{5} + V_C - V_B = 0$



$i_3 - i_2 = 2$

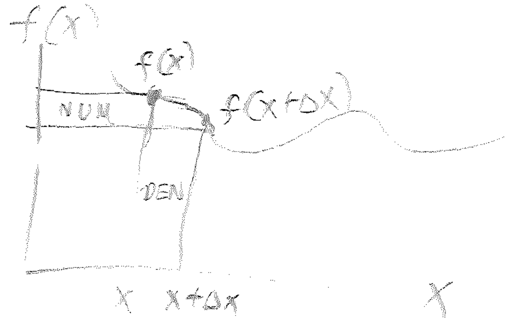
$-10 + 4(i_1 - i_2) + 2i_1 = 0$

$4(i_2 - i_1) + V_1 + 8i_2 = 0$

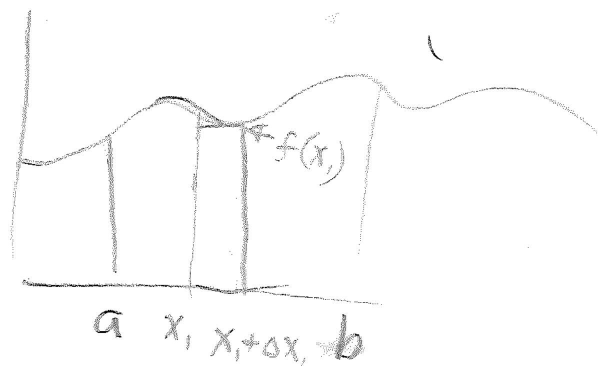
$-V_1 + 4i_3 - 2i_3 = 0$

$$\frac{d\,f(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

TEST
ON TEST
integrals & deriv
definitions
combining not
linear curves
Chapt. 3 as
far as we've gone



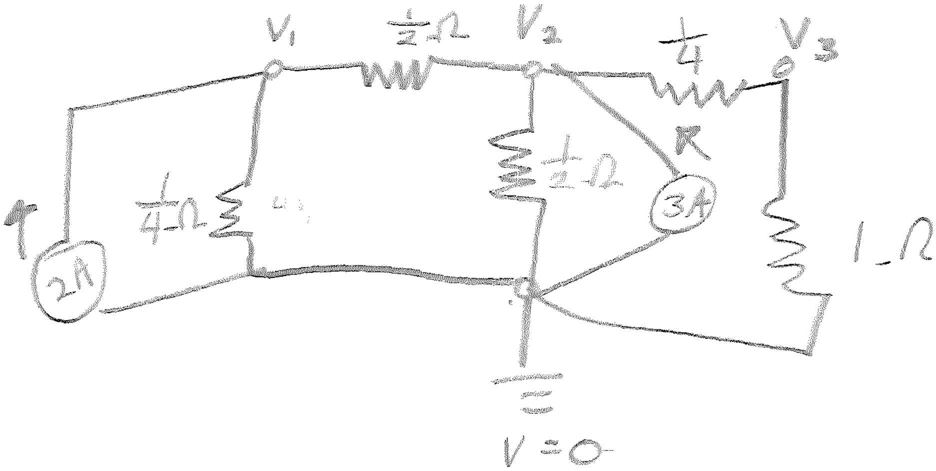
$$\int_a^b f(x)\,dx = \lim_{\Delta x_i \to 0} \sum_{i=1}^{\left(\frac{b-a}{\Delta x}\right)=N\to\infty} f(x)_i\,\Delta x_i$$



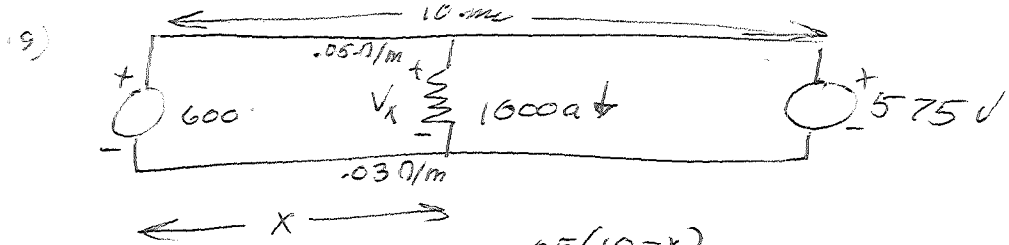
Chapt. 3, #12

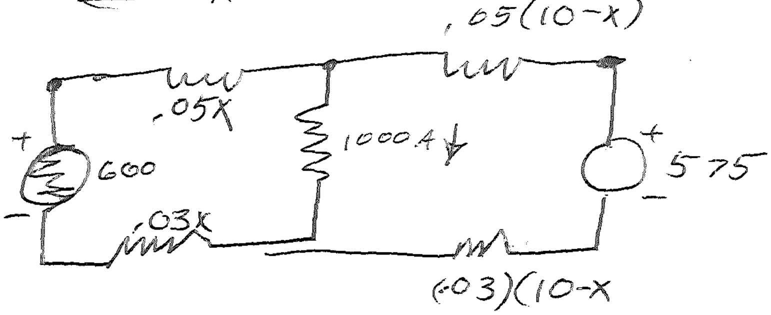
$$6V_1 - 2V_2 = 2$$
$$-2V_1 + 8V_2 - 4V_3 = 3$$
$$-4V_2 + 5V_3 = 0$$

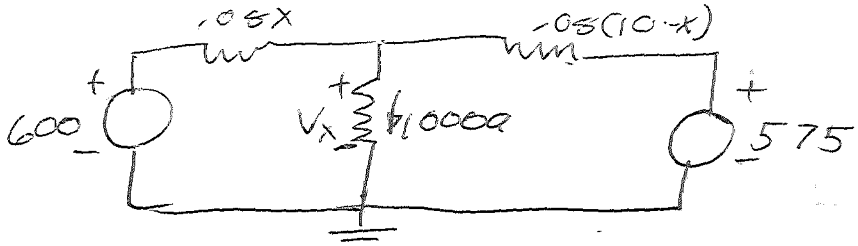
$$\begin{bmatrix} 6 & -2 & 0 \\ -2 & 8 & -4 \\ 0 & -4 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$



9)





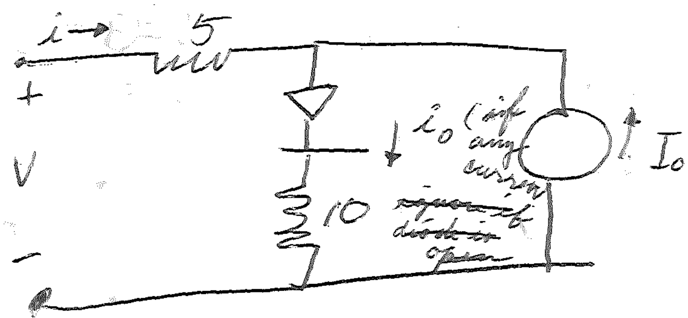


current at a node: $\dfrac{600 - V_x}{.08X} + \dfrac{575 - V_x}{.08(10-x)} = 1000$

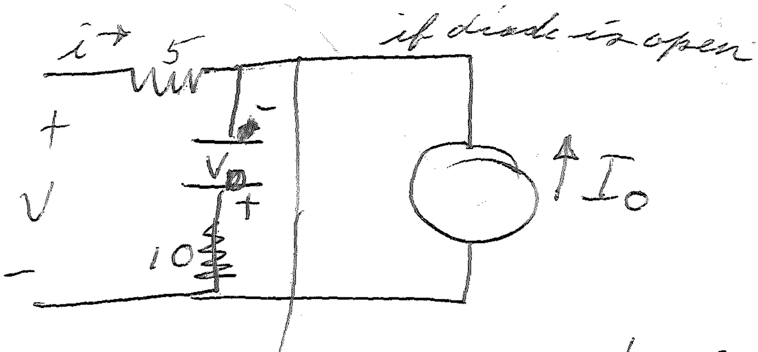
$V_x = 8x^2 + 82.5x + 600$

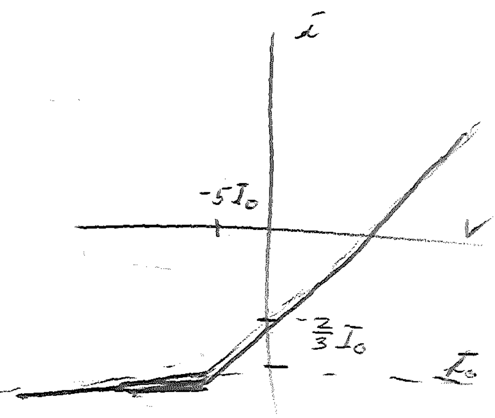
$\dfrac{dV_x}{dx} = 0 \Rightarrow x = 5.16$ mi

Diode problems



$i_0 = I_0 + i > 0$





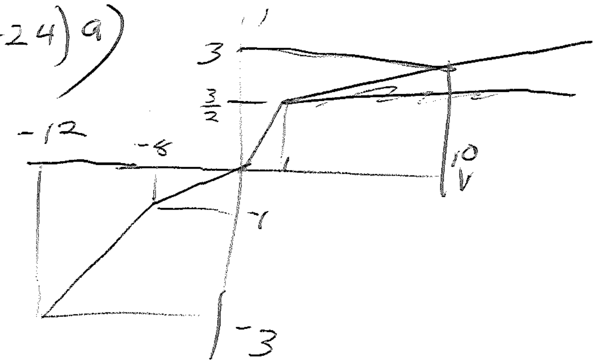
$i = \frac{1}{15}V - \frac{2}{3}I_0$

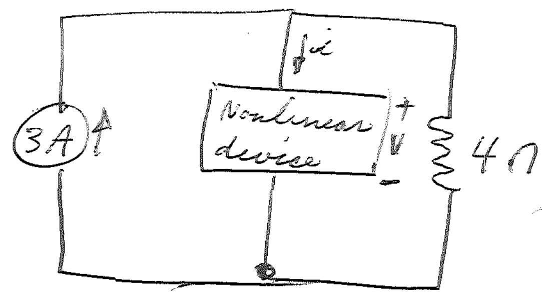
$V_D = 5i - V > 0$

$V < 5i = -5I_0$

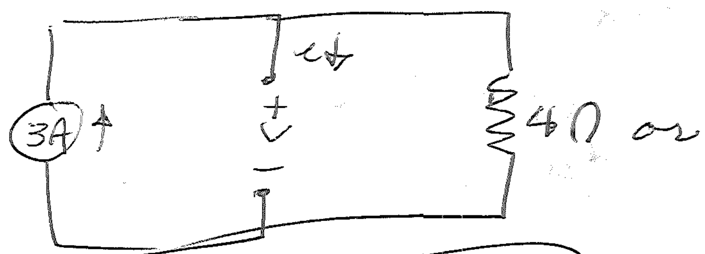
$i = -I_0$

3-24) a)

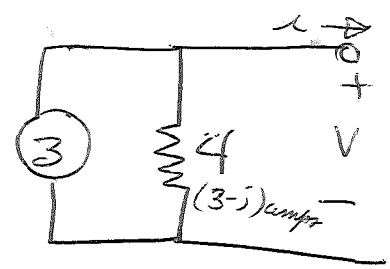




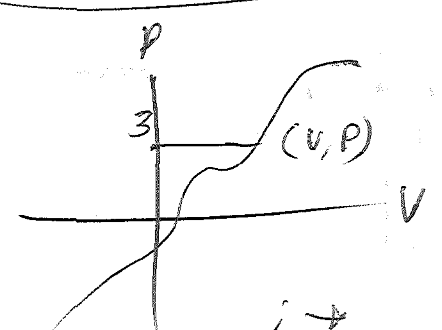
take it out



or

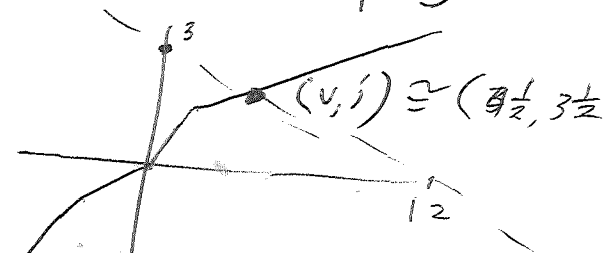


b)



$V = (3 - i)\,4$

$i = 3 - \frac{v}{4} = -\frac{v}{4} + 3$



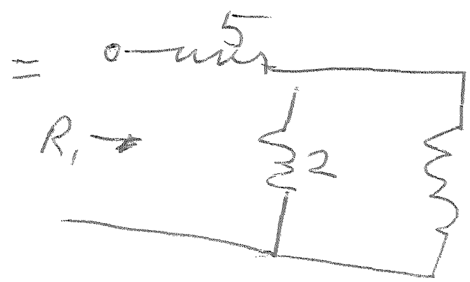
$r = -\frac{v}{4} + I_0$

~~1.5 < i < 20~~

i = 2 Amps

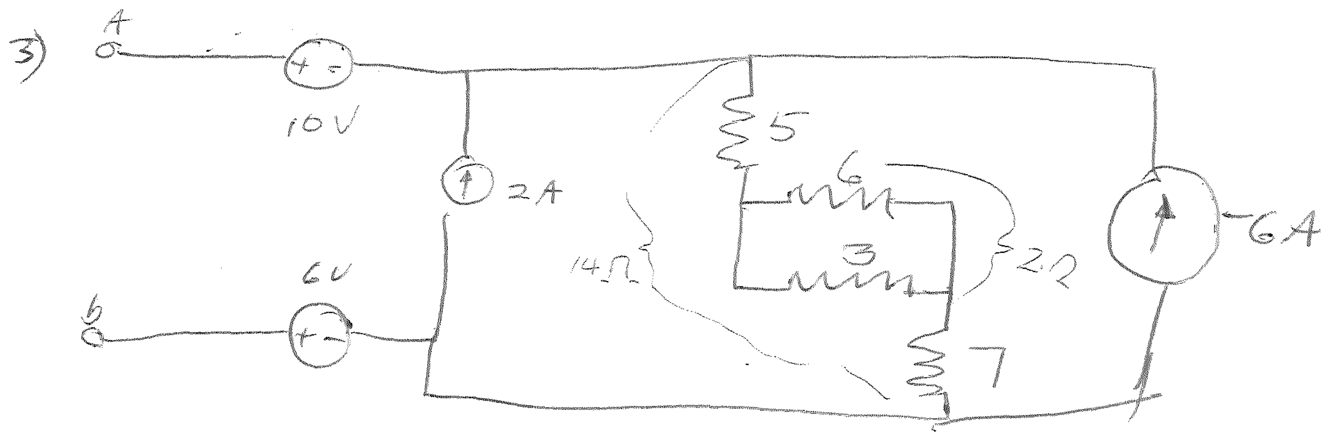
Chapter 4

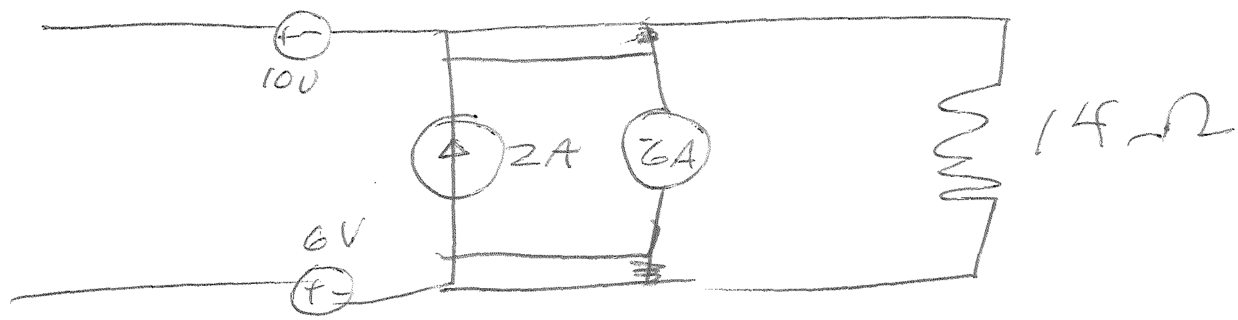
2)

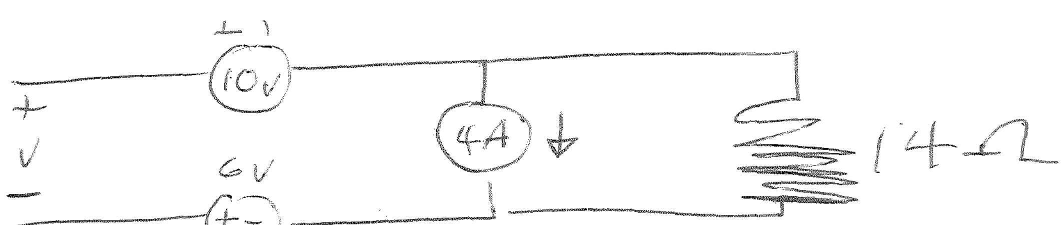


$$= \frac{2R_1}{2+R_1} + 5 = R_1$$

3)

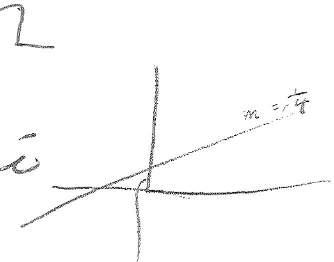


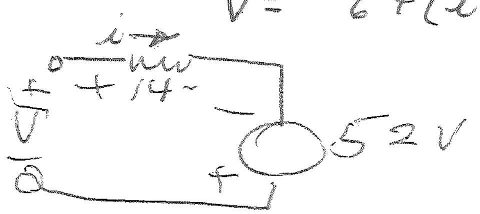


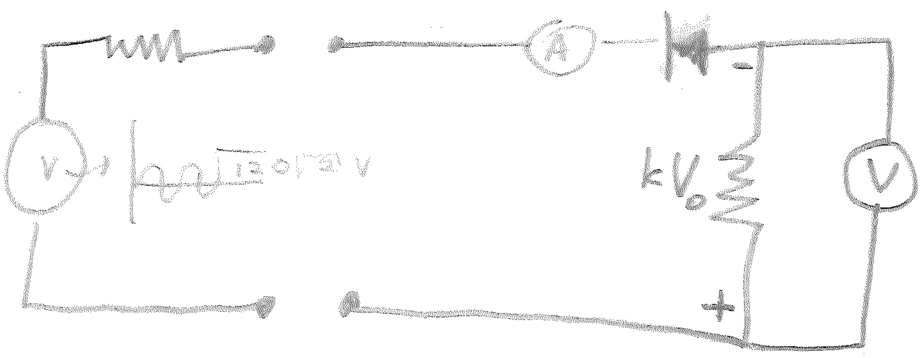


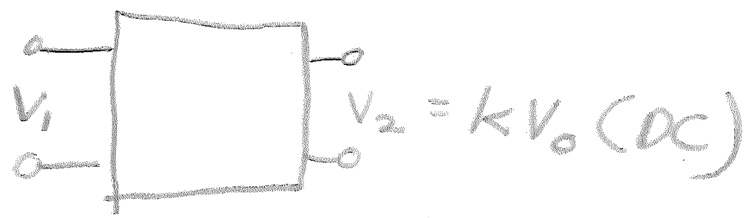
$$V = -6 + (i-4)14 + 10 = -52 + 14i$$

$$i = \frac{1}{14}V + \frac{52}{14}$$

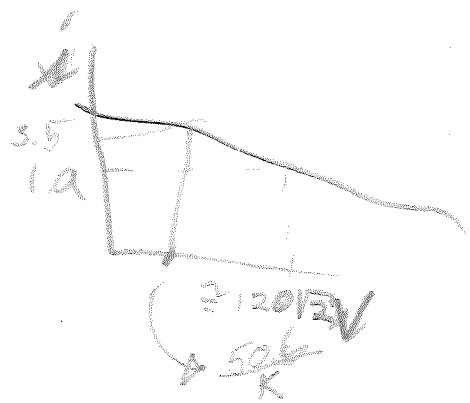




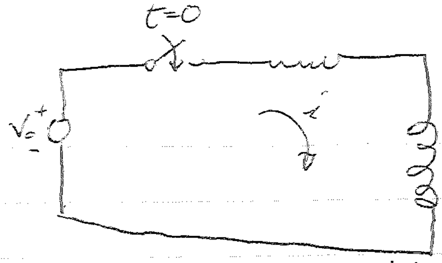




But wave can't be changed in shape



$$K = \frac{51.2}{12\sqrt{2}}$$



$$V_0 = Ri + L\frac{di}{dt}$$

$$i_p = \frac{V_0}{R}$$

$$0 = Ri + L\frac{di}{dt}$$

$$i = I_0 e^{Dt}$$

$$0 = R\cancel{I e^{Dt}} + DL\cancel{I_0 e^{Dt}}$$

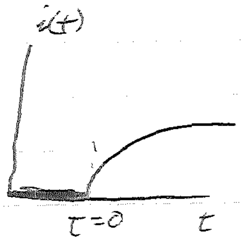
$$0 = R + DL$$

$$D = -R/L$$

$$i_c = I_0 e^{-\frac{R}{L}t}$$

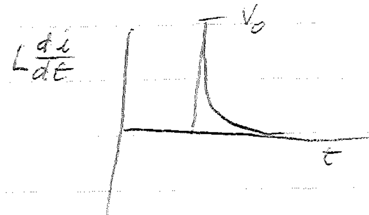
$$i(t) = i_p + i_c = \frac{V_0}{R} + I_0 e^{-\frac{R}{L}t}$$

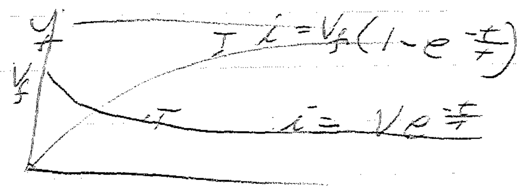
$$i(0+) = 0 = \frac{V_0}{R} + I_0 \Rightarrow I_0 = -\frac{V_0}{R}$$



$$i(t) = \left(\frac{V_0}{R}\right)\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\tau = \frac{L}{R}$$





CHAPT 6, # 7

Bob Marks

Electrical Science I

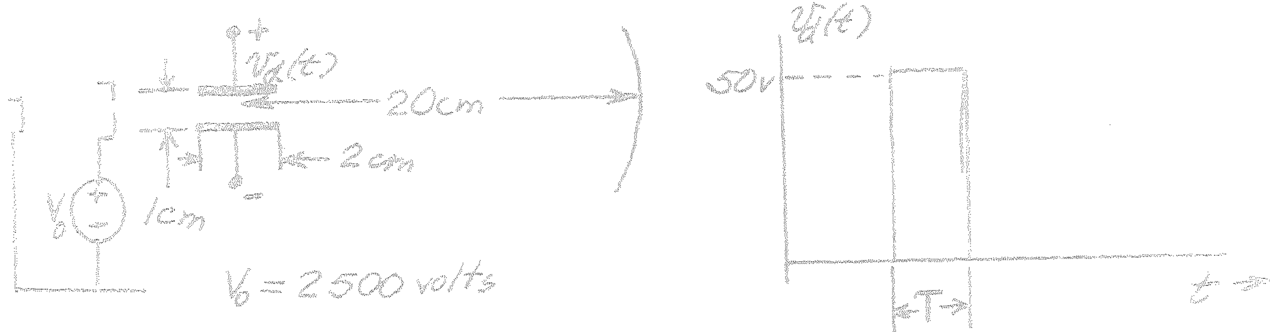
Brain Tickler #1

20

EE-RP1
Oct 69
CCR

HIGH 110
AV 54
LOW 4
FLUNK ≤35

Do all work in the space provided. Place your answers in the spaces indicated.



1. For the cathode ray tube sketched above, a "pulse" of voltage $v_d(t)$ of duration T seconds is applied to the deflection plates. (If you wish, you may leave your answers to this problem in algebraric form or numerical form.)
(a) What is the minimum value of T which will cause a maximum deflection of the beam?

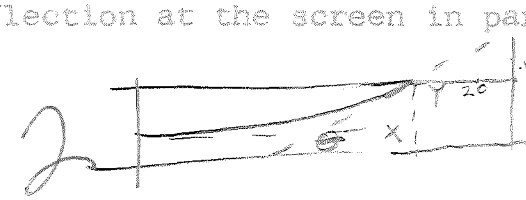
$\frac{2\,cm}{V_0}$

V_0 = beam's initial velocity

1

T = $\frac{2\,cm}{V_0}$ seconds

(b) What will the deflection at the screen in part (a) be?



$\frac{dY}{dX} = \frac{eE}{2mV_0^2}X$

Let $P = \frac{dY}{dX}$ at $X = 2cm$

$P = \tan\theta$

$D = 20cm \tan\theta$

$U_X = U_0$

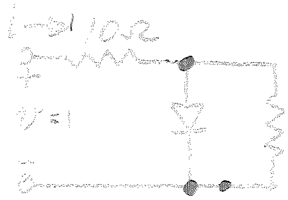
$U_Y = \frac{1}{2}at^2$

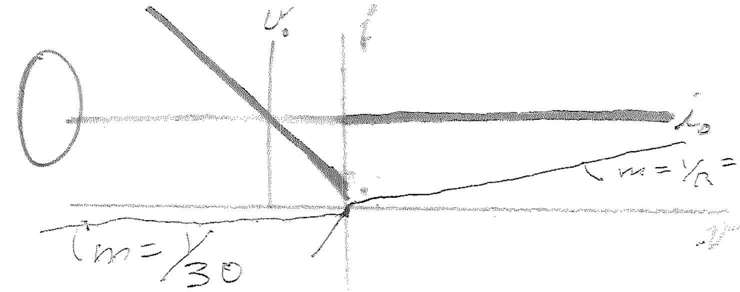
$Y = \frac{eE}{2mV_0^2}X^2$

Defl. = $\boxed{(20cm)\frac{eE}{2mV_0^2}(2cm)}$ cm

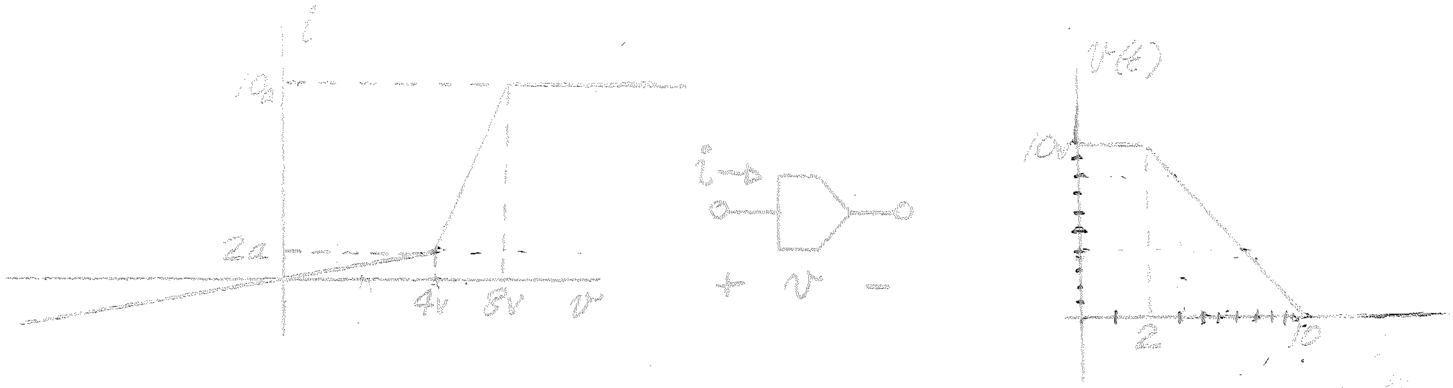
$V_0 = 2500V$
m = mass of el.
e = charge on e
E = ~~[illegible]~~ of field between plates = $\frac{V_0}{d}$

2. Determine and accurately sketch the v-i characteristic of the circuit shown.



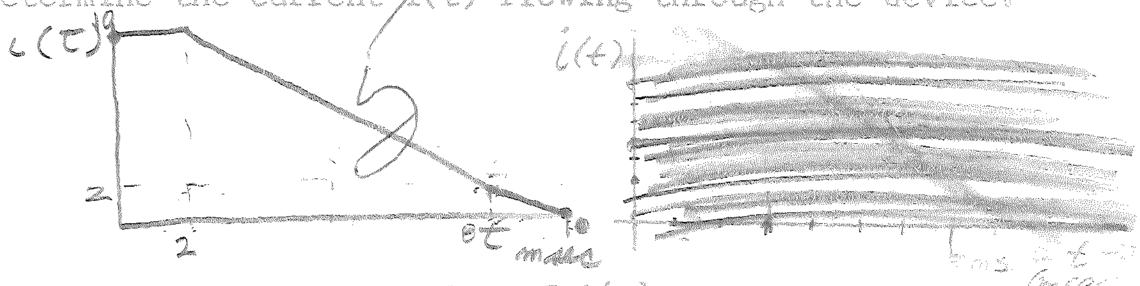


3



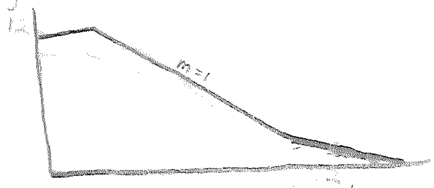
3. The voltage v(t) is applied to the two-terminal device which has the v-i characteristic indicated.

   (a) Determine the current i(t) flowing through the device.



5

   (b) Determine the averave value of i(t).

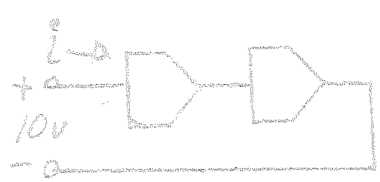


5

area = 47 amp-ms
i(t) = A/10ms

$\bar{i}(t) \cong$ 4.7 amps

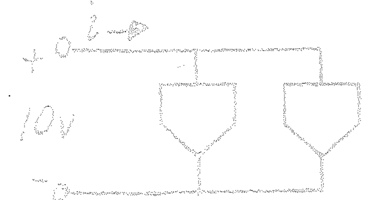
   (c) Two identical devices are connected as shown. Determine i, the current flowing through them.



0

≅ -4.7 amps
i = -(Ans. to b) amps
(-ī(t) amps)

   (d) Two identical devices are connected as shown. Determine i, the current indicated.



2

≅ -9.4 amp
i = -2(Ans. to b) amps
[2 ī(t) amps

12

4. In problem #1, a pulse of voltage of amplitude 50 volts and duration 2T/3 (T = time determined in problem 1-(a)) is applied to the deflection plates. What will the deflection of the beam at the screen be?

$d_N = \frac{2}{3}(2) = \frac{4}{3}$

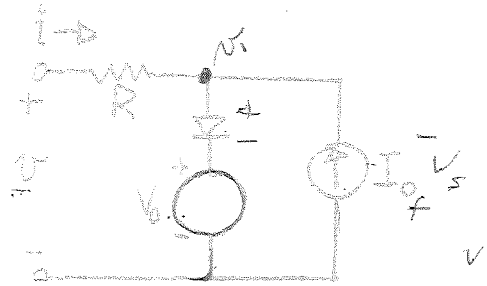
$= 40\,cm^3 \frac{2e}{3 m_e V_0}$

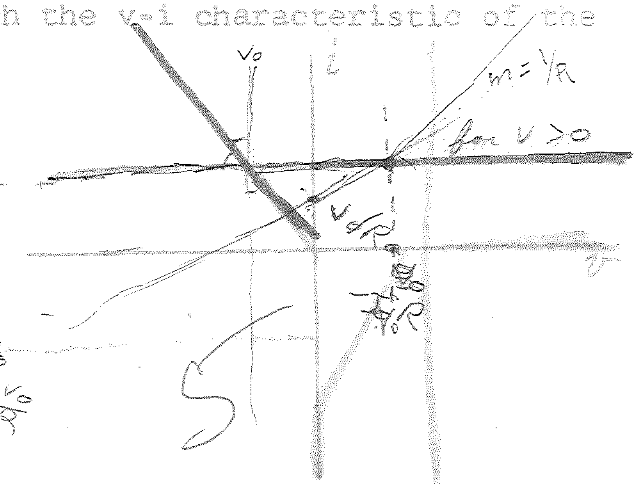
$40\,cm^2 \left(\frac{eE3}{2(m_e)V_0^2}\right) = 40\,cm^2 \frac{e2}{3m_eV_0} = \cancel{\frac{20e}{3m_eV_0}} = \frac{1.6\times10^{-19}}{(9\times10^{-31})50V}$

$\Rightarrow V = Ed$

Deflection = 0 cm

5. Determine and accurately sketch the v-i characteristic of the circuit shown.

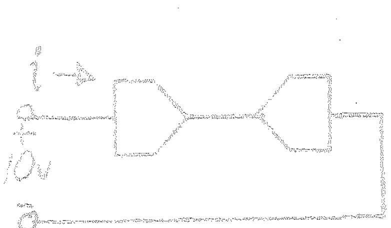


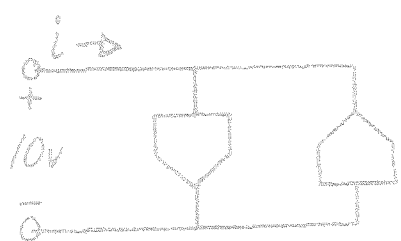


$v = iR + V_0$

$i = \frac{v}{R} - \frac{V_0}{R}$

6. The devices of problem #3 are connected as shown. Determine the current in each case.





(ans to 3b)

i = 4.7 amps

i = 0 amps

0

0

5

You have not been in to see me yet — I'm expecting you —

BOB MARKS

HI - 127
82 - AVER.
LOW - 40

73

Electrical Science I

EE, RPI
Oct 69

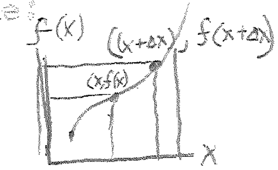
Halloween Trick or Treat

NOTE: In the following definitions and statements, you may use equations, mathematical symbols, graphs, and/or circuit diagrams as you see fit. Words, sentences, and/or phrases are NOT permitted. 2 points will be deducted for each word used.

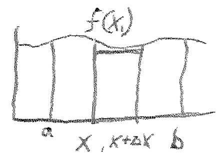
You may use any letters or symbols without defining them if (a) they are utilized in the text in chapters 1, 2, or 3, or (b) they have been used by Dr. Rogers in class. Any other letters or symbols you wish to use you must define:

I. Define:

(a) $\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{\Delta f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$



(b) $\int_a^b f(t)\, dt = \lim_{\Delta x \to 0} \sum_{i=1}^{\frac{b-a}{\Delta x}} F(x)\, \Delta x$



(c) w(energy or work)

(d) $\vec{E}$ (electric field) $= \lim_{\Delta Q \to 0} \frac{\Delta \vec{F}}{\Delta Q} = \frac{d\vec{F}}{dQ}$

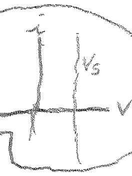
(e) $V_{21} = \cancel{\lim_{\Delta t} \frac{\Delta q}{\Delta t}}$ $\frac{W_{P2} - W_{P1}}{Q} = -\int \vec{E} \cdot d\vec{l}$

(f) $i = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t}$

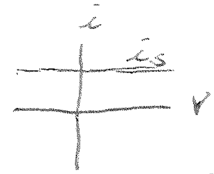
25

~~(g)~~ p (power) $= vi$

(h) voltage source $V = V_s$

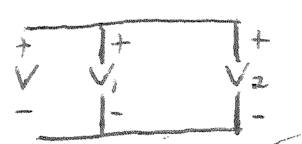


(i) current source $i = i_s$

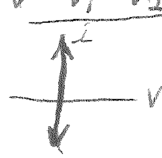


(j) series connection $i = i_1 = i_2$

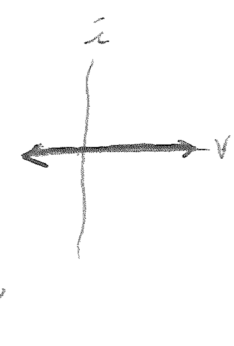
(k) parallel connection $V = V_1 = V_2$



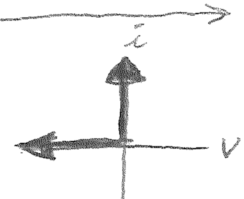
(l) short circuit $V = 0$



(m) open circuit $i = 0$

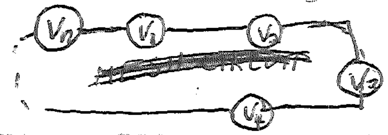


(n) diode $v < 0 \rightarrow i = 0$
$i > 0 \rightarrow V = 0$



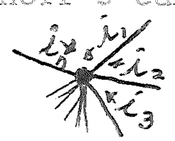
2. State:

(a) Kirchoff's voltage law



$$\sum_n V = 0$$

(b) Kirchoff's current law



$$\sum_n i = 0$$

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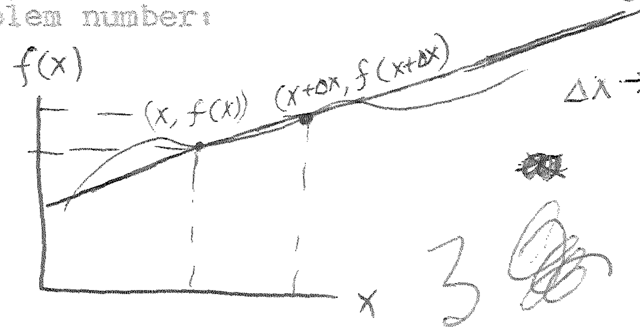
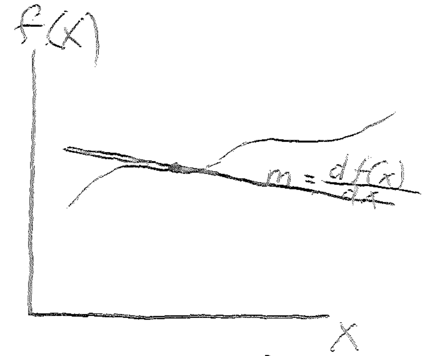
(c) The law of conservation of charge

3. Draw a picture to illustrate the meaning of the definitions in problem number:

1(a)

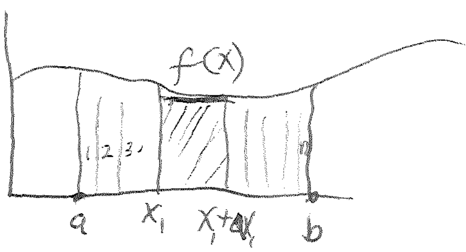


$\Delta x \to 0$



3

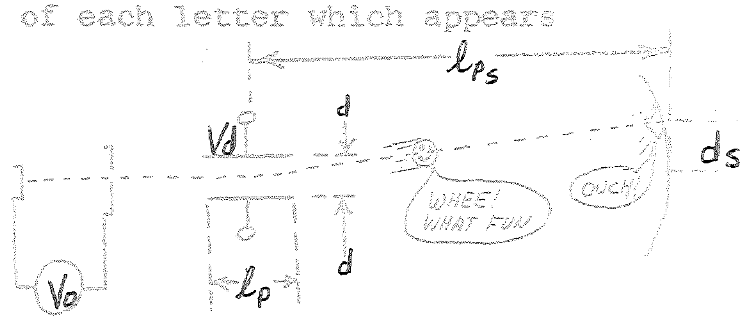
1(b)



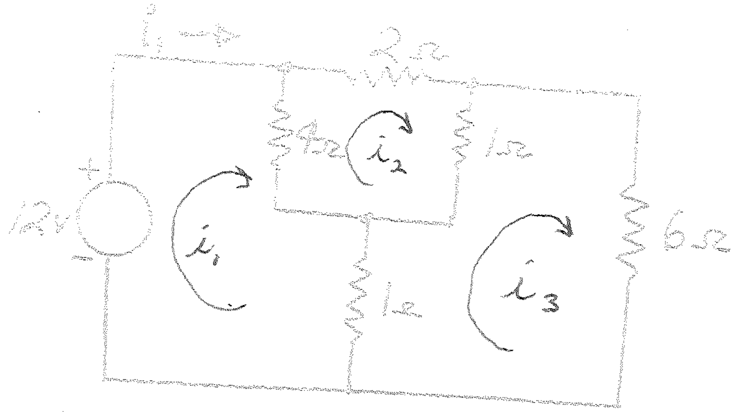
3

4. Derive the "plug" for a cathode ray tube and illustrate on the diagram the meaning of each letter which appears in the "plug".

$$d_s = \frac{V_d l_p l_{ps}}{2V_o d}$$



10



5. Determine i_1 using mesh-current analysis.

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}\begin{bmatrix} 5 & -4 & -1 \\ -4 & 7 & -1 \\ -1 & -1 & 8 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

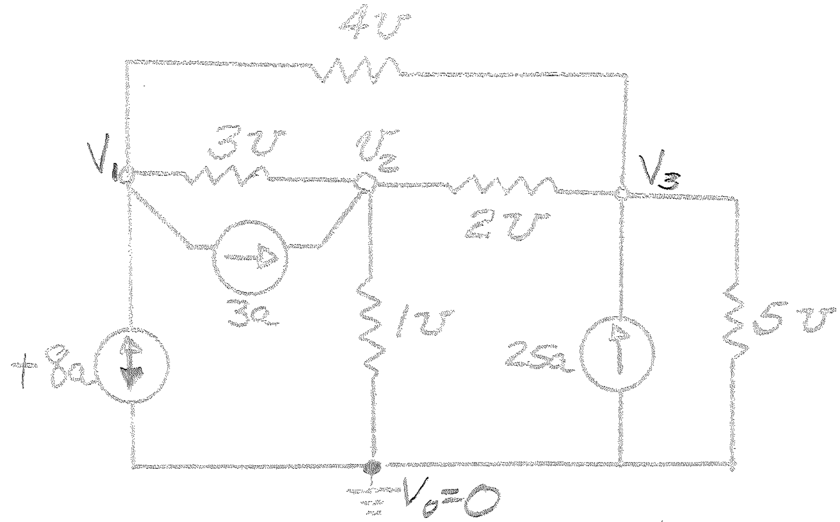
15

$$-12 + 4(i_1 - i_2) + 1(i_1 - i_3) = 0$$
$$5i_1 - 4i_2 - i_3 = 12$$
$$4(i_2 - i_1) + 2i_2 + 1(i_2 - i_3) = 0$$
$$-4i_1 + 7i_2 - i_3 = 0$$
$$1(i_3 - i_1) + 1(i_3 - i_2) + 6i_3 = 0$$
$$-1i_1 - i_2 + 8i_3 = 0$$

$$i_1 = \frac{\begin{vmatrix} 12 & -4 & -1 \\ 0 & 7 & -1 \\ 0 & -1 & 8 \end{vmatrix}}{\begin{vmatrix} 5 & -4 & -1 \\ -4 & 7 & -1 \\ -1 & -1 & 8 \end{vmatrix}} = \frac{672 - 12}{280 + 4 + 4 - 5 - 7 - 128}$$

$$= \frac{660}{288 - 140} = \frac{660}{148} = 4.46 \text{ AMPS}$$

$$\begin{array}{r} 128 \\ 12 \\ \hline 140 \end{array} \quad \begin{array}{r} 16 \\ 8 \\ \hline 128 \end{array} \quad \begin{array}{r} 56 \\ 5 \\ \hline 280 \end{array}$$



6. Determine v_2 using node-voltage analysis.

$$8 + 3 + \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{4} = 0$$

$$\frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{4} = -11$$

$$7V_1 - 4V_2 - 3V_3 = -132$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \begin{bmatrix} 7 & -4 & -3 \\ -2 & 6 & -3 \\ -5 & -10 & 19 \end{bmatrix} = \begin{bmatrix} -132 \\ 18 \\ 500 \end{bmatrix}$$

(NOT SYMETRICAL BECAUSE OF SIMPLIFYING CO-EFFICIENTS)

$$\frac{V_2 - V_1}{3} + \frac{V_2 - V_3}{2} + \frac{V_2}{1} = 3$$

$$-2V_1 + 6V_2 - 3V_3 = 18$$

13

$$\frac{V_3 - V_2}{2} + \frac{V_3}{5} + \frac{V_3 - V_1}{4} = 25$$

$$10V_3 - 10V_2 + 4V_3 + 5V_3 - 5V_1 = 25$$

$$-5V_1 - 10V_2 + 19V_3 = 500$$

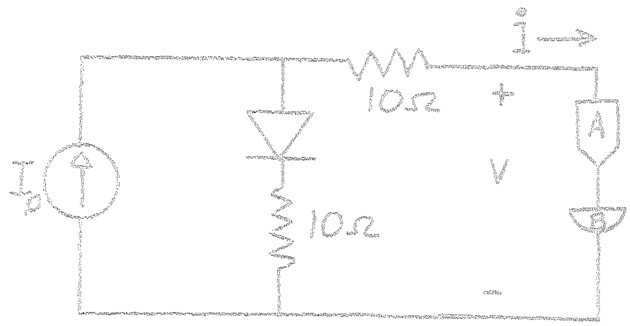
$$V_2 = \frac{\begin{vmatrix} 7 & -132 & -3 \\ -2 & 18 & -3 \\ -5 & 500 & 19 \end{vmatrix}}{\begin{vmatrix} 7 & -4 & -3 \\ -2 & 6 & -3 \\ -5 & -10 & 19 \end{vmatrix}} = \frac{(7)(18)(19) + (132)(3)(5) + \cancel{6(500)} + 27(500) + 15(18) - 38(132)}{42(19) - 60 - 30 - 210 - 90 - 8(19)}$$

$$= \frac{2520 + 1980 + 13500 - 270 - 5020}{800 - 90 - 300 - 150} = \frac{1700 - 528}{80 - 54}$$

$$= \frac{1171}{26} = 45\ V$$

NOTE: This problem, if done properly, takes TIME, but should be straightforward. DO NOT hurry. Use the graph paper provided. You might expect to spend one hour on this problem.

The v-i characteristics for devices A and B are shown on the attached sheets and the diode in the circuit is ideal. Determine v and i if (a) $I_o = +4$ amps, and (b) $I_o = -2$ amps.



~~$V = Ri$~~

~~$i > 0$~~

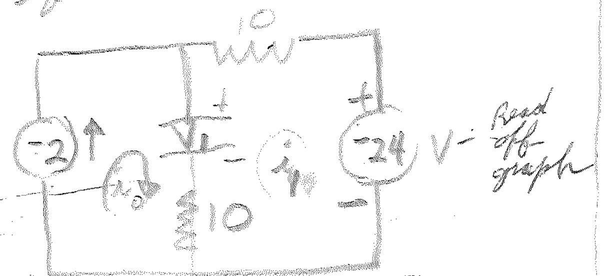
~~$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \begin{bmatrix} 10 & -10 \\ -10 & 20 \end{bmatrix} = \begin{bmatrix} -4 \\ -V \end{bmatrix}$~~

~~$i = \dfrac{\begin{vmatrix} 10 & -I_o \\ -10 & -V \end{vmatrix}}{\begin{vmatrix} 10 & -10 \\ -10 & 20 \end{vmatrix}}$~~

~~$i = \dfrac{-10V - 10I_o}{100}$~~

~~$i = -\frac{1}{10} V - \frac{1}{10} I_o$~~

if $i < 0$



$\begin{bmatrix} i_o \\ i_1 \end{bmatrix} \begin{bmatrix} 10 & -10 \\ -10 & 20 \end{bmatrix} = [\ \ ]$

$-V_1 + 10(i_o - i_1) = 0$

$-V_1 + 10i_1 = 24 + 10(i_1 - i_o)$

$20i_1 - 10i_o - V_1 = 24$

$i_o = -2$

$V_1 - 10i = 20$

$V_1 = 20 + 10i$

$20i - V_1 = 4$

$20i - 20 - 10i = 4$

$10i = 24 \quad i = 2.4A$

$V = 20 + 24$

$= 44$

(a) $v =$ ~~16~~ volts

$i =$ ____ amps

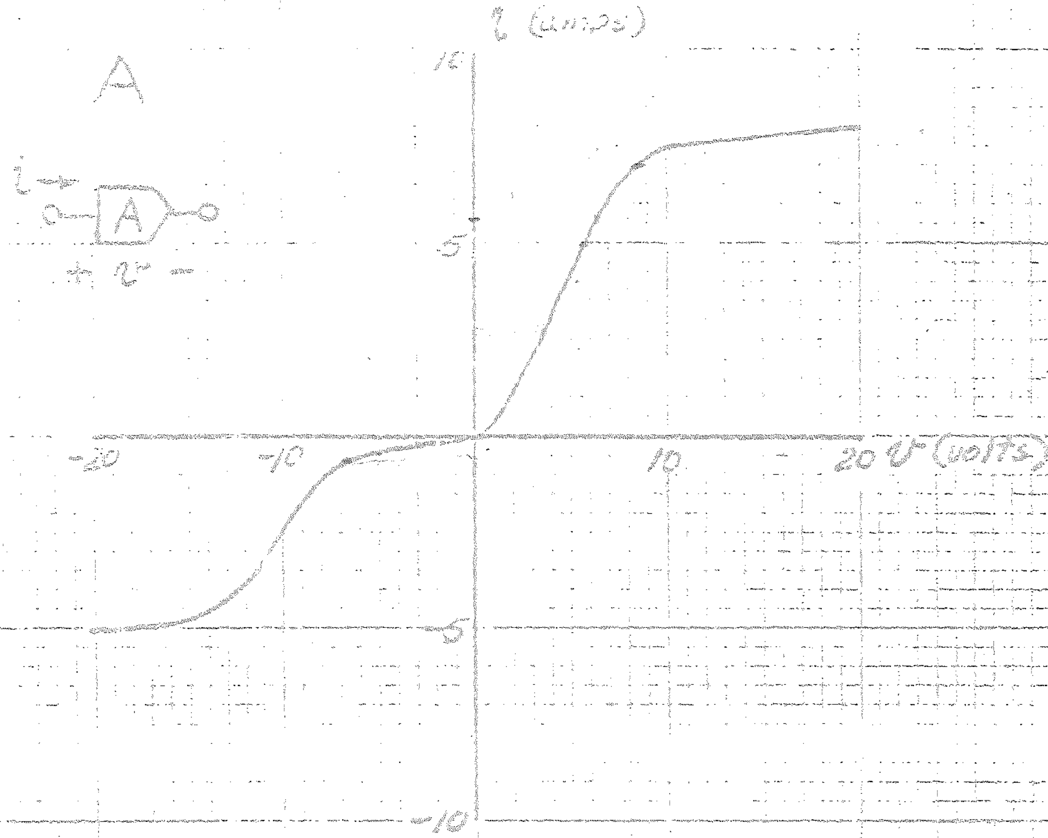
(b) $v =$ ~~44~~ volts

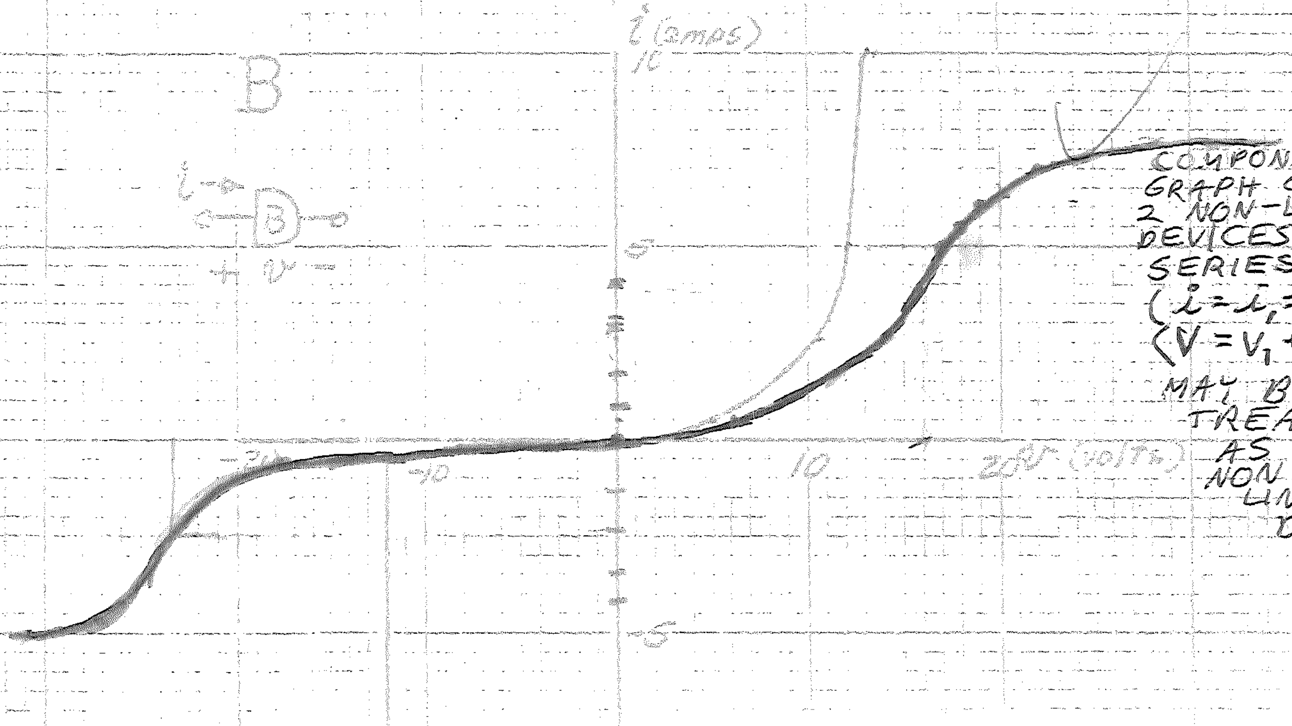
$i =$ ~~2.4~~ amps

(OVER)

10

(Take a break. DO NOT work any Electrical Science problems this weekend!)



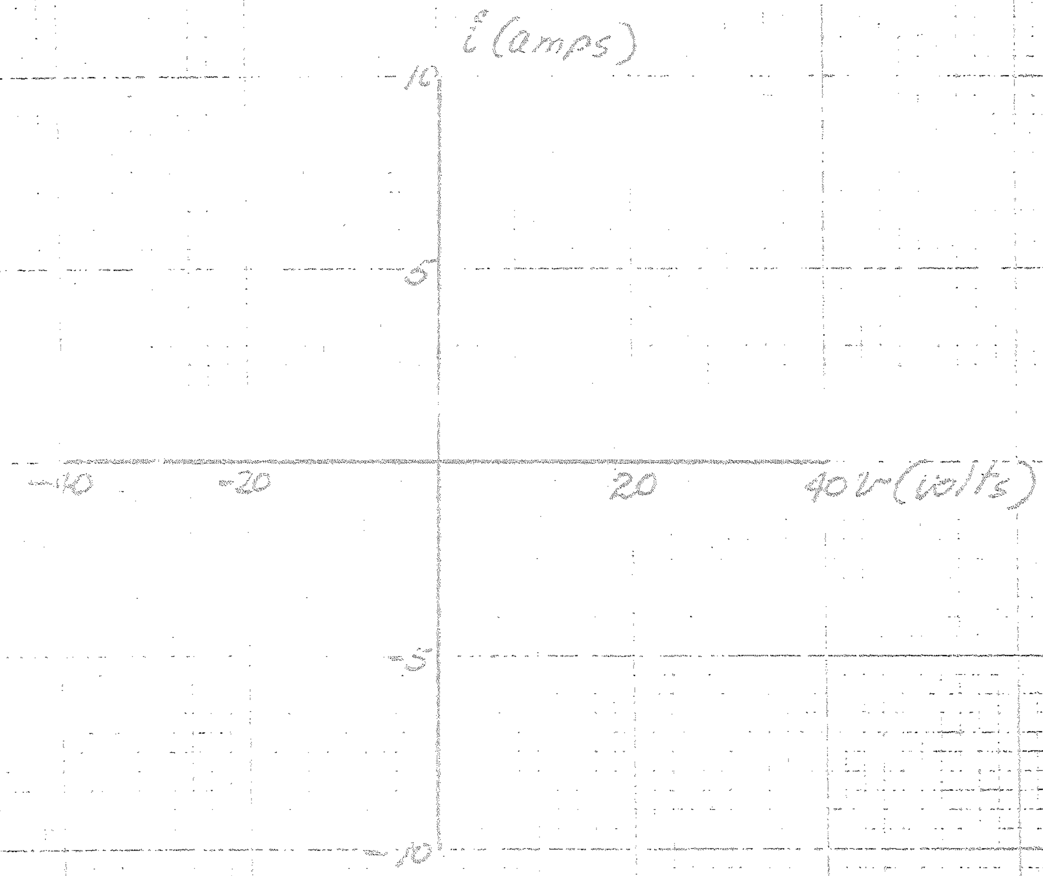


COMPONANT GRAPH OF THE 2 NON-LINEAR DEVICES IN SERIES

$(i = i_1 = i_2)$

$(V = V_1 + V_2)$

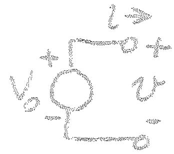
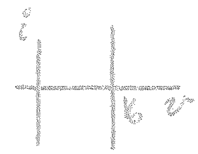
MAY BE TREATED AS ONE NON-LINEAR DEVICE

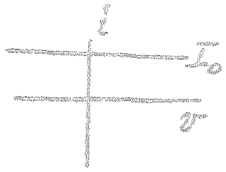
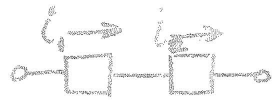


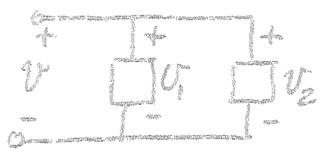
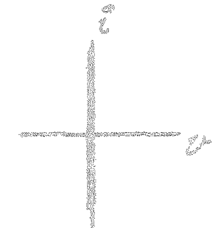
i (amps)
10
5
-40
-20
20
40
v (volts)
-5
-10

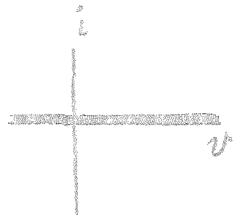
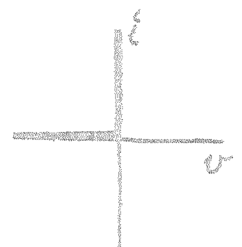
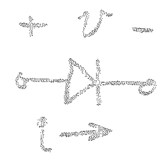
1. (a) $\dfrac{df(x)}{dx} = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$ (b) $\displaystyle\int_a^b f(t)\,dt = \lim_{\Delta t_j \to 0} \sum_{j=1}^{N=\frac{b-a}{\Delta t}} f(t_j)\,\Delta t_j$

(c) $W = \int \vec{F}\cdot d\vec{l}$ (d) $\vec{E} = \lim\limits_{q \to 0} \dfrac{\vec{F}}{q}$ (e) $V_{21} = -\int_1^2 \vec{E}\cdot d\vec{l}$ (f) $i = \dfrac{dq}{dt}$

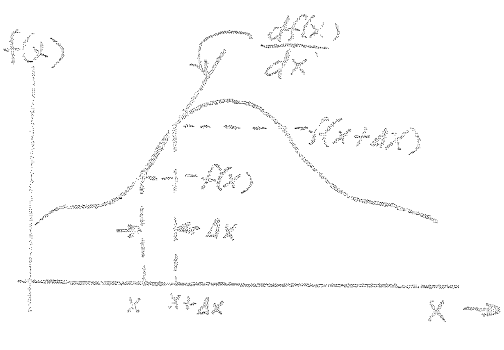
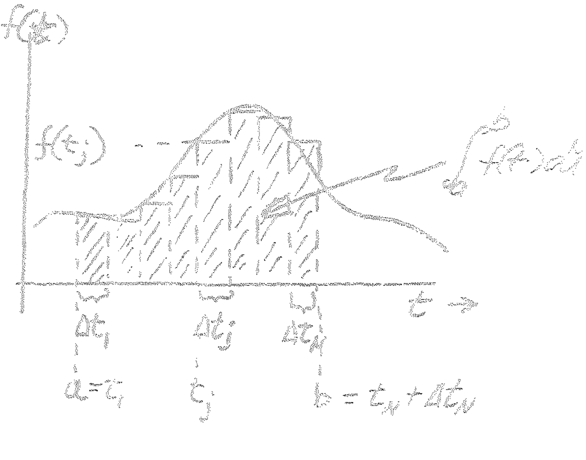
(g) $p = \dfrac{dw}{dt}$ (h)  $v = V_0$ 

(i)  $i = I_0$  (j)  $i = i_1 = i_2$

(k)  $v = v_1 = v_2$ (l)  $v = 0$ 

(m) $i \rightarrow$ $i = 0$  (n)  

2. (a) $\displaystyle\sum_{i=1}^{N} v_i = 0$ (b) $\displaystyle\sum_{i=1}^{N} i_i = 0$ (c) $\displaystyle\oint \vec{J}\cdot d\vec{A} = -\frac{dq}{dt} = -\frac{d}{dt}\int \rho\, dv$

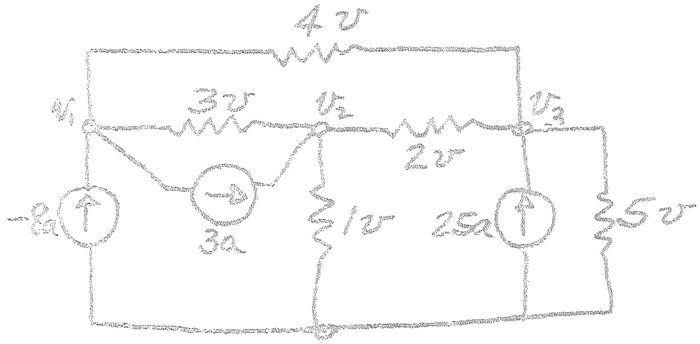
3. (a)  

4. QED

$$\begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 & -4 & -1 \\ -4 & 7 & -1 \\ -1 & -1 & 8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$$i_1 = \frac{\begin{vmatrix} 12 & -4 & -1 \\ 0 & 7 & -1 \\ 0 & -1 & 8 \end{vmatrix}}{\begin{vmatrix} 5 & -4 & -1 \\ -4 & 7 & -1 \\ -1 & -1 & 8 \end{vmatrix}} = \frac{12(56-1)}{5(56-1)+4(-32-1)-1(4+7)} = \frac{12(55)}{11(25-12-1)} = 5a$$

6.



$$\begin{bmatrix} -11 \\ 3 \\ 25 \end{bmatrix} = \begin{bmatrix} 7 & -3 & -4 \\ -3 & 6 & -2 \\ -4 & -2 & 11 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$v_2 = \frac{\begin{vmatrix} 7 & -11 & -4 \\ -3 & 6 & -2 \\ -4 & -2 & 11 \end{vmatrix}}{\begin{vmatrix} 7 & -3 & -4 \\ -3 & 6 & -2 \\ -4 & -2 & 11 \end{vmatrix}} = \frac{7(33+50)+11(-33-8)-4(-75+12)}{7(66-4)+3(-33-8)-4(6+24)} = \frac{382}{191} = 2 \text{ volts}$$

7. (a) 11 volts (b) −22 volts

1.5 amps −2 amps

ELECTRICAL SCIENCE I

Winter Quarter 1970

TEXT: Hayt & Hughes, <u>Introduction to Electrical Engineering</u>, McGraw-Hill, 1968

Period	Week of	Text Pages	Text Problems	Special Problems
1	Jan. 5			
2		1-17	1-5	
3	12	17-21	1-15, 1-16	
4		29-43	2-5	
5		43-55	2-12, 2-18	
6	19		2-19	
7		61-71	3-6, 3-8	
8	EXAM-FRI	71-82	3-10, 3-12	
9	26	82-91	3-13, 3-14	
10			3-28	1
11				2, 3
12	Feb. 2			4
13		105-116	4-4	
14	EXAM-FRI	116-133	4-9	
15	9	133-142	4-12, 4-20	
16				5
17				6
18	16	195-203	6-5	
19		204-217	6-12	
20	EXAM-FRI	217-226		7, 8
21	23	233-247	7-3	
22		247-257	7-7, 7-17	
23			7-24	
24	March 2			9
25		265-274	8-5, 8-10	
26	EXAM-FRI	275-287	8-12, 8-16	
27	9		8-22	10
28		Review		11
29				
30	16			

ROSE POLYTECHNIC INSTITUTE

Department of Chemical Engineering

Moore's Rules

Course Grade

Exams, quizzes	60 (1-y)%
Final	30 (1-y)%
Problems	10 (1-y)%
Instructor's Opinion	100 (1-y)% $\quad 0 \leq y \leq 0.3$

Problems

Problems will be assigned for nearly every class session. The problems are an essential part of the course and each student is expected to understand the correct solution of each assigned problem. No student in the lower half of the class ranking will be permitted to take the final examination until acceptable solutions to at least 90% of the assigned problems have been turned in.

Until this privilege is abused, it will be permissible for students to work together on homework in groups of two or three. However, each student is expected to contribute significantly toward the solution of the problems, to understand the final solution, and to turn in a solution for grading.

Problems are to be turned in at the beginning of the class session on the due date. The paper should be 8 ½" x 11", with holes punched so that the returned problems may be kept in a loose-leaf notebook. Only one side of each page is to be used and the margins should be at least 1 ½" on the left edge and 1" on the right edge. Each problem should be started on a new page. The student's name and the problem number(s) will appear on the reverse side of the last page. In addition, the names of any students who collaborated will be listed.

No numerical grade will be given for problems turned in more than one week after the due date, but such problems will count toward the 90% requirement mentioned above. No late problems will be accepted during the seven days preceding the final exam.

Attendance

No make-up exams are anticipated. It is important that the instructor be notified prior to any absence from an exam. If the instructor can not be contacted, leave a message with the secretary at extension 320.

MOORE'S MYSTICAL GRADING SYSTEM

Purpose: Believe it or not, the system is used in an attempt to be fair. It should help account for variation between tests as well as the usual variation between students.

How Standard Scores Are Calculated: The mean and the standard deviation are computed for each test. [Of course, the mean is the average score and the standard deviation is related to the "spread", or distribution, of scores.] Each raw score is then converted to a standard score by using the following equation.

$$Z_i = \frac{x_i - \bar{x}}{\sigma}$$

where Z_i is the standard score

x_i is the raw score

$\bar{x}$ is the mean

and σ is the standard deviation

Thus, a standard score is the number of standard deviations above or below the mean. A positive score is above the mean and a negative score is below the mean. Standard scores are measures of the performance in comparison with all other scores without regard to the numerical value of the average score or the distribution.

Combining Standard Scores: If all tests were of the same difficulty and importance, an individual's standard scores could simply be added to arrive at a total score. However, not all tests are of equal difficulty and importance. To correct for such differences, weighting factors are used. Every standard score will be multiplied by a weighting factor which is determined by the instructor and is based on the difficulty and importance of the test.

An individual's final cumulative score is the algebraic sum of the products of each standard score multiplied by the appropriate weighting factor. The final cumulative scores are used to rank students from top to bottom. No predetermined distribution of grades, such as a "normal" curve, is assumed.

UNIVERSITY OF KENTUCKY

DEPARTMENT OF CHEMICAL ENGINEERING

Problem Grading Standards

Grade	Criteria
5	Correct method, correct answer, clear presentation, neat paper, on time
4	Correct method but with one of the following: a) minor error in calculation b) unclear presentation c) paper not neat
3	Correct method (or minor error in method) plus more than one of the following: a) error in calculation b) unclear presentation c) paper not neat
2	Major error(s)
0	Paper not turned in

Late papers will receive a maximum grade of 3

FRIDAY 1-5
MONDAY 1-15,16

ENERGY - ability to do work
WORK - Force times distance (MECHANICAL)

RESISTER - DISSIPATES ENERGY (CONSUMES)
INDUCTOR - CONVERTS EL. E. INTO MAGNETIC FLD
CAPACITOR - " " " " ELECTRIC "

DIRECTION OF EL. FLD

$$E = \frac{F}{q}$$

(1-1)

a) 10^{-13} coul $\quad e^-$
$\ominus \quad 10^{-6}$ m $\quad \circ$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \frac{(10^{-13})(1.6\times10^{-19})(9\times10^9)}{(10\cdot10^{-6})^2}$$
$$= 1.44\times10^{-12}\,N$$

b) $E = 3\times10^6 \frac{V}{M}$
$F = (3\times10^6)\,e_q$
$= .481\times10^{-12}\,N$

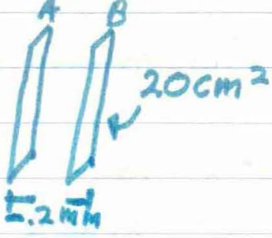
c) $E = \frac{Q}{\varepsilon_0 A}$

$$F = Q_e E = \frac{Q_E Q}{\varepsilon_0 A}$$

$= .481\times10^{-12}$ NT.

X1-3)

a) $10^{-8}c$ $-10^{8}c$



$$E = \frac{Q}{\epsilon_0 A}$$

$$V = Ed$$

$$= \frac{Qd}{\epsilon_0 A}$$

$$= \frac{(10^{-8})(.2\times10^{-3})}{(8.85^{-12})(20\times10^{-4})}$$

$$= 113\ V$$

b) $W = qV$

$$V = \frac{W}{q}$$

$$= \frac{(8\times10^{-18})}{(-1.602\times10^{-19})}$$

$$= {}^{-}50\ V$$

c) $2\times10^{6}\ \frac{V}{M} \rightarrow$

$$V = Ed$$

$$= (2\times10^{6})(.2\times10^{-3})$$

$$= 400\ V$$

X1-4)

a) $2COUL_{12}$

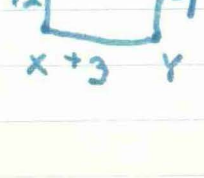
3 JOULES

$V = \frac{.3J}{2C} = {}^{-}1.5$ VOLTS

b) $V = \frac{+9.6\times10^{-19}}{-1.602\times10^{-19}}$

$= {}^{-}6$ VOLTS

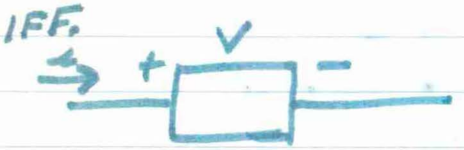
c)



V IS INDEPENDENT OF PATH $\therefore V_{12} = 4$

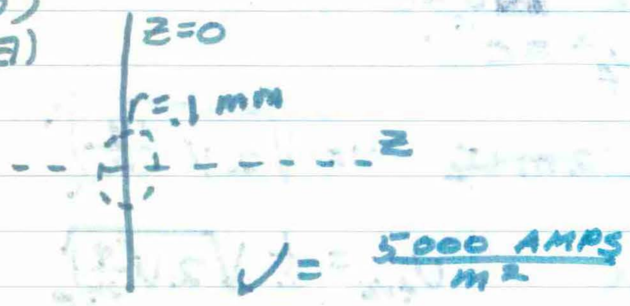
FOR REFERENCE; KEEP CURRENT + GOING IN POSITIVE TERMINAL

$P = vi$



X1-5)

a)



$i = J\pi r^2$

$= -157\ \mu A$

b) $D = 10^8\ e/m$

$u = 10^7\ \frac{m}{s}$

$i = Du = (10^8)(-1.602\times 10^{-19})(10^7 \frac{m}{s})$

$= -160\ \mu A$

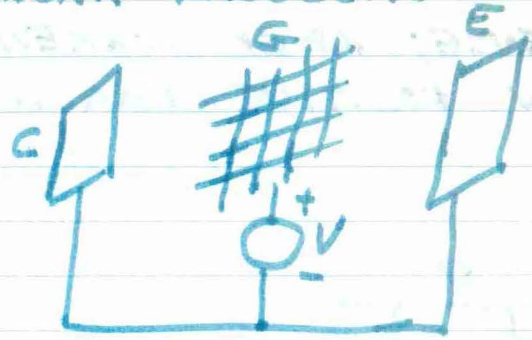
c) $i = \int J\, dA$

$= \int_0^\infty \frac{.2}{r} e^{-10{,}000r}\, \pi 2R\, dR$

$= -126\ \mu A$

HOMEWORK PROBLEMS

1-5)



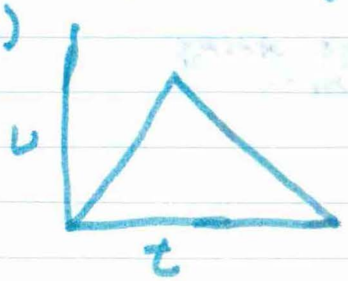
AT GRID, U=0, K.E IS MAX

a) ANSWER [redacted] $U_c = 0$

b) $\frac{1}{2} m u_f^2 = qV$

$\cancel{V = 2m u_f}$ $\quad u = \sqrt{2V \left(\frac{q}{m}\right)_e}$

c)



$u_{AVE} = \frac{1}{2}\sqrt{2V\left(\frac{q}{m}\right)_e}$

1-12-70

1-120-70

THE CATHODE RAY TUBE

+ ION SHIELD: COLLECTS + GAS IONS IN TUBE

FILAMENT: HEATS UP & EMITS ELECTRONS

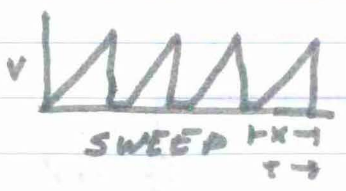
CONTROL GRID: CONTROL'S # e

ANODE - TO ACCELERATS e's

FOCUSING ANODE - NARROWS BEAM DOWN FOR SHARPER PIC.

SCREEN - COVERED WITH PHOSPHER

CATHODE RAY OSCIL.



SWEEP CONTROLS at (HORIZONAL DEFLECTION PLATES

IMPUT CONTROLS VERTICAL D.P.

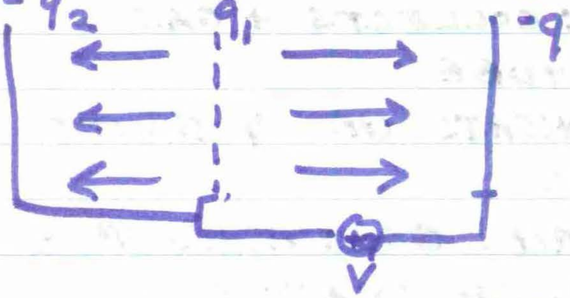
PROBLEM #16

a) WOULD DOUBLE

b)

1-14-70

1-5) $E = \frac{Q}{\varepsilon_0 A}$ $E = \frac{V}{d}$

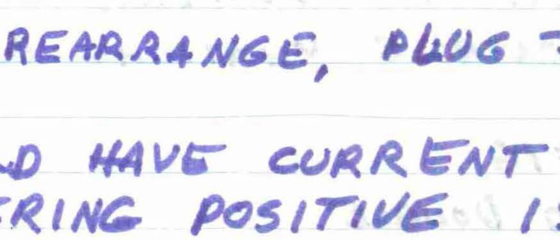


$E_1 \neq -E_2$

$\therefore$ Q MUST BE KNOWN IN THIS CASE.

WOULD USE $E = V/d$

2-24)

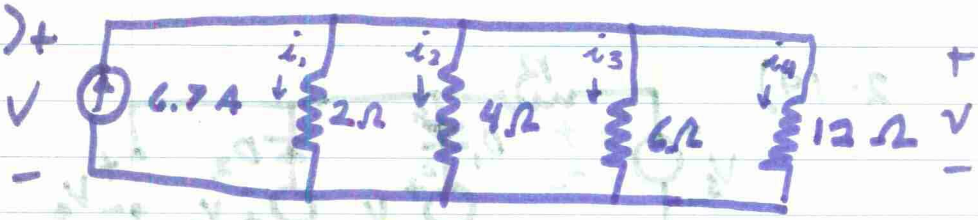


REARRANGE, PLUG & CHUG.

SHOULD HAVE CURRENT ENTERING POSITIVE IS CHOOSING TERMINALS

$P_{cs} = -741$ WATTS (GIVING OFF ENER)

b)



$V = i Z$ (Z = IMPEDENCE)

$6.7\ A = i_1 + i_2 + i_3 + i_4$

$V = 2i_1 = 4i_2 = 6i_3 = 12i_4$

$i_1 = 6i_4$ ETC.

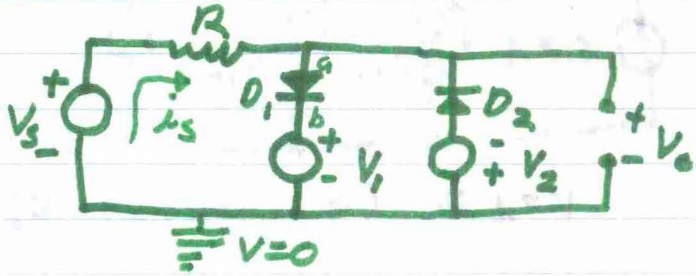
$P_{cs} = -Vi \cong -45$ WATTS

(USE $-Vi$ IN THAT THERE IS INCONSISTANCE IN REFERENCES)

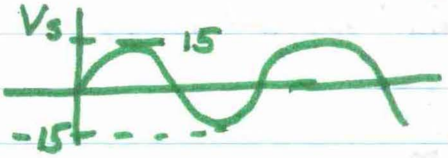
IF CURRENT ENTERS A NEGATIVE TERMINAL, $P = -Vi$

1-16-70

2-19)



$V_1 = 5V \quad V_2 = 2V \quad V_s = 15 \sin \omega t$



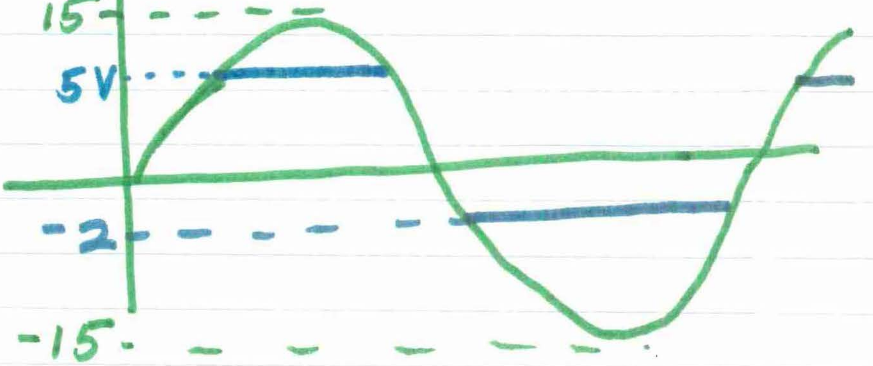
V_s IN NOT IN // WITH V_1 CAUSE OF RESISTOR

D_1 WILL CONDUCT FOR $V_s > 5$

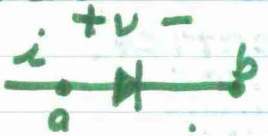
(NO V DROP OVER R ⇒ $i = 0$)

$V_{a_{MAX}} = 5V$

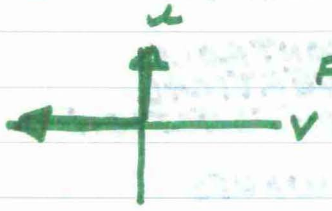
∴



DIODE:

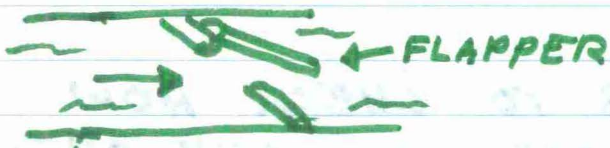


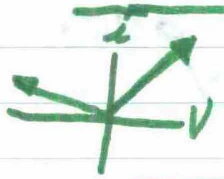
$V_b \times V_a$



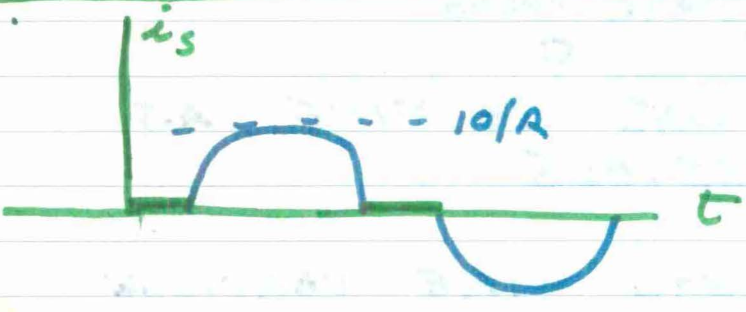
FOR IDEAL DIODE

FLUID ANALOGY:





FOR REAL DIODE



1-16-70 - LAB

THE OSCILLOSCOPE

A B CONTROL VERTICAL CALIBRATION (SENSITIVITY)

VERTICLE ← CONTROLS VERTICAL DEFLECTION

AC
DC
GROUND

VOLTAGE INPUTS

GROUND

A & B TO CHECK HOW GOOD OSS. IS. ANY OUT-SIDE INTER FERANCE

INTENSITY FOCUS

MAKE LINE AS FINE AS POSSIBLE

HORIZONAL LIKE VERTICLE BLOCK. WHEN IN USE, PUT TIME SWEEP = 0

FREQ

TIME SWEEP

IF ON PLUSS, OSS. WOULD START ON FIRST POSITIVE SLOPE

500 MV 5 MV
O O -TO TEST

CALIBRATION

1-19-70

1) FIELD EMISSION OF ELECTRON
PULL OF ELECTRONS

2) PHOTOELECTRIC EMISSION
(BY MEANS OF RADIATION, SUCH AS LIGHT)
a) PHOTOTUBE-MUCH LIGHT YIELDS MINIMUM CURRENT
b) PHOTOMULTIPLIER-LIGHT EMITS ELECTRONS, WHOSE NUMBERS ARE MULTIPLIED BY SEC. EMISSION (Pg. 31)

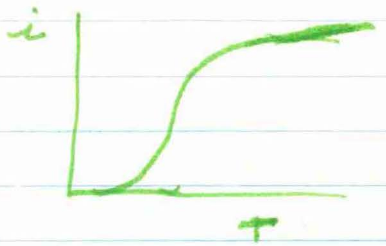
3) SECONDARY EMISSION-LIKE POOL.
~~HEAT~~ ELECTRON EMITS ELECTRONS FROM IT'S ~~POTENTIAL~~ KINETIC ENERGY

4) THERMIONIC EMISSION - BY THERMO ENERY (BY HEATING CATHODE)
a) MOST COMMON TYPE
b) USED IN VACUUM TUBES

WHEN ELECTRIC FIELD INTENSITY IS GREAT ENOUGH TO REMOVE ALL THE EMITTED ELECTRONS:

$$i(T) = a A T^2 e^{-b/T}$$

FOR A GIVEN VOLTAGE.



TAPERS OFF BECAUSE TRANSPORTATION CAN'T MEET EMISSION

CHILD-LANGMUIR LAW

$$i(V) = 2.33 \times 10^{-6} \frac{A}{d^2} V^{3/2}$$

AT A GIVEN TEMP.

HERE, EMISSION CAN'T MEET TRANSPORTATION.

X 2-3) $i(\pi) = 43.7$ mA - CAPACITY FROM CATH. (MAX.)

$i(1) = 4.58$ mA

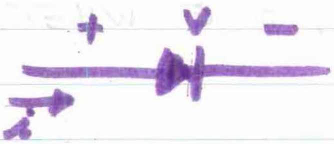
$i(2) = 12.95$ mA

$i(10) = 150$ mA

1-21-69

DEFINITIONS:

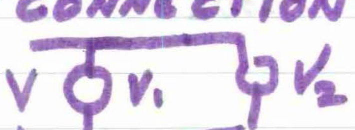
1) VOLTAGE SOURCE HAS SAME POTENTIAL FOR ALL CURRENTS
2) CURRENT SOURCE HAS SAME CHARGE DISPLACEMENT FOR ALL VOLTAGE
3) OHM'S LAW: $V = iR$
   (i MUST GO INTO + TERMINAL)
   INCREMENTAL: $r = \frac{dv}{di}$
4) DIODE · NO REVERSE CURRENT & NO FORWARD VOLTAGE.



$v < 0$
$i > 0$

5) OPEN CIRCUIT: $i = 0$
   (A CURRENT SOURCE)
6) SHORT CIRCUIT: $V = 0$
   (A VOLTAGE SOURCE)
7) SERIES CONNECTION
   $i = i_1 = i_2$

8) // CONNECTION

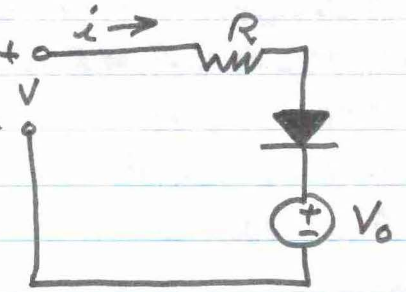


$V = V_1 = V_2$

$$v_{AVE} = \bar{V} = \frac{1}{T}\int_{t_0}^{t_0+T} V\,dt$$

1-23-70 TEST UP TO PAGE 63

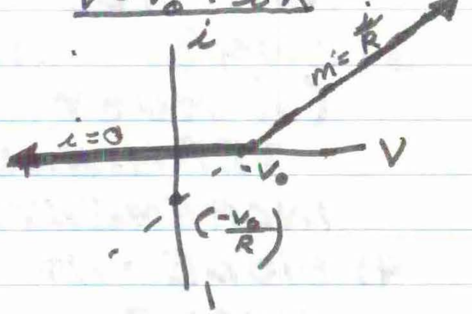
2-16) c)



non-conduct $i=0$

conduct

$\underline{v = V_0 + iR}$



THE DIODE

GRAPH TWO PARTS. ONE PART WHERE IT CONDUCTS, ONE WHERE IT DOESN'T

KIRCHHOFF'S LAWS

1) $\sum_{i=0}^{N} V_i = 0$ (Σ VOLAGES AROUND A CLOSED PATH)

A CLOSED PATH! NOT NESISARILY A MESH!

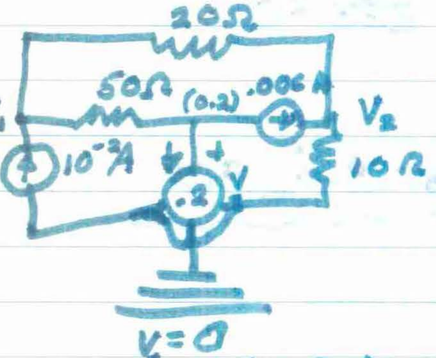
2) $\sum_{j=1}^{M} i_j = 0$ (Σ CURRENT GOING IN AND OUT OF NODE)

1-26-70

NTD TO FIND VOLTAGES BY KIRCHHOFF'S LAW

1) DEFINE i's THROUGH V SOURCE
2) Σ CURRENTS AT NODE
3) Z NODES $\rightarrow$ Z-1 INDEPENDENT EQUATIONS

3-5)



DEFINE i THRU VS TO ENTER + TERMINNAL

1) $-10^{-2} + \left(\frac{V_1 - .2}{50}\right) + \left(\frac{V_1 - V_2}{20}\right) = 0$

2) $\frac{V_2 - V_1}{20} + (-.006) + \left(\frac{V_2 - 0}{10}\right) = 0$

3) $\frac{V_1 - .3}{50} + (-i_S) - .006 = 0$

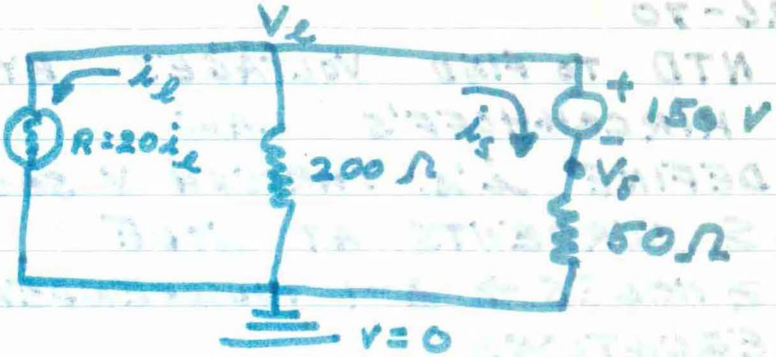
4) $10^{-2} - i_s + \frac{0 - V_2}{10} = 0$

$V_1 = +0.3$

$.2 - V_1 = -0.1$

$V_2 - V_1 = ?$

3-10)



A) ℓ) $i_s + i_\ell + \frac{V_\ell}{200} = 0$

5) $i_s - \frac{V_5}{50} = 0$

(GRD) $\frac{V_5 - 0}{50} + \frac{V_\ell}{200} + i_\ell = 0$

$V_\ell - V_5 = 150$

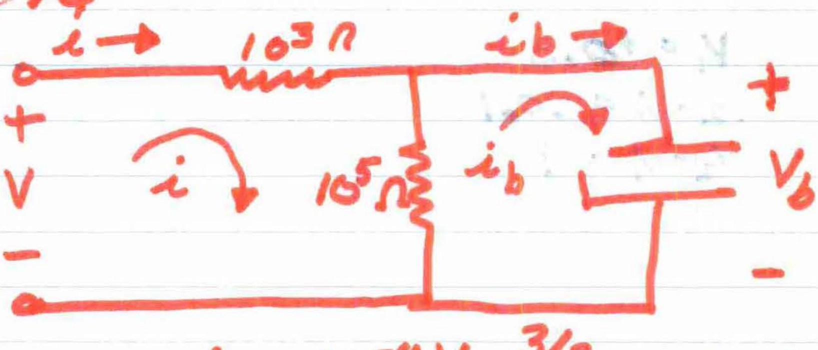
$\frac{V_\ell - 0}{20 i_\ell} = i_\ell$

2 VALUES OF i_ℓ, use $i_\ell > 0$

B) THEN $V_\ell - V_5 = 300$? $R \stackrel{?}{=} \frac{8}{3}\,\Omega$

1-28-79

3-28 EX)



$i_b = 10^{-4} V_b^{3/2}$ AMPS

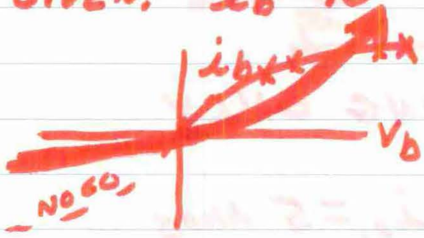
$i_b > 0$

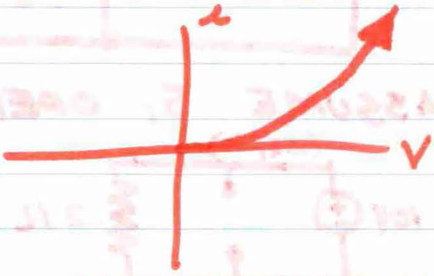
(FOR MESH CURRENT, PUT VOLTAGE ACROSS i SOURCE)

$i)\quad -V + 10^3 i + 10^5(i - i_b) = 0$

$i_b)\quad 10^5(i_b - i) + V_b = 0$

GIVEN: $i_b = 10^{-4}\, V_b^{3/2}$



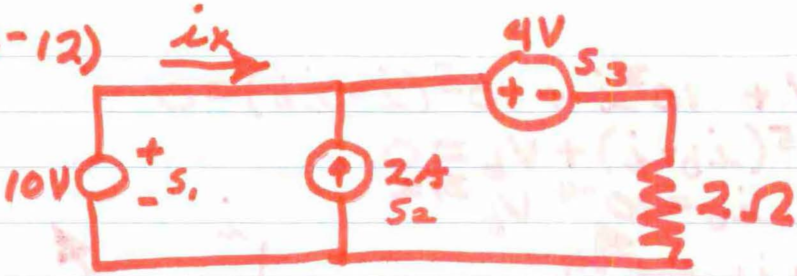


MESH CURRENT IS HYPOTHETICAL
DEFINE VOLTAGE ACROSS
CURRENT SOURSES
USE KIRCHHOFF'S V LAW
EQUATIONS = # MESHES

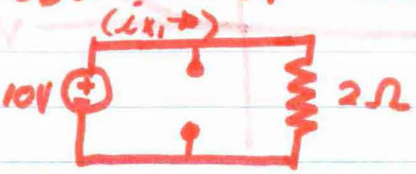
NON-LINEAR ELEMENTS
KIRCCHOFF'S LAW STILL
APLY.

LOAD LINE - GRAPHICAL
SOLUTION OF NON-LINEAR
DEVICE
SUPERPOSITION CAN ONLY BE
USED ON LINEAR SYSTEM
REPLACE V.S. WITH SHT. CIRCUIT
" i.S " OPEN "

X3-12)

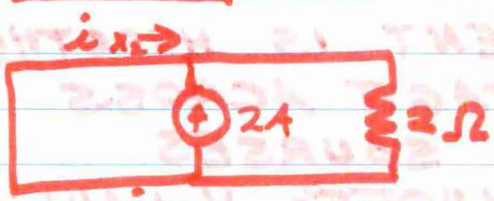


① ASSUME S_1 OPERATING ONLY



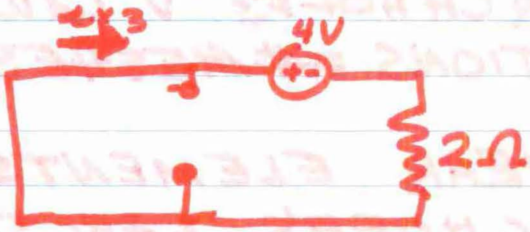
$i_{x_1} = 5$ AMPS

②



$i_{x_2} = -2$ AMPS

③

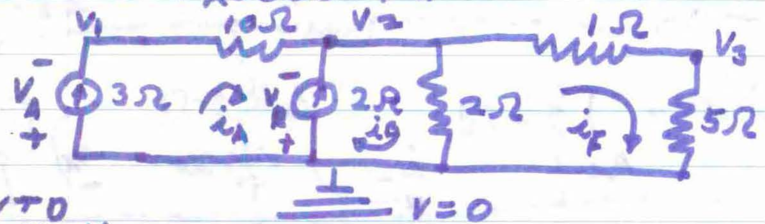


$i_{x_3} = -2$ AMPS

TOTAL
$i_x = i_{x_1} + i_{x_2} + i_{x_3} = +1$ AMP

1-30-70

SPECIAL PROBLEM # 2



by NTD

1) $\frac{V_1 - V_2}{10} - 3 = 0$

2) $\frac{V_2 - V_1}{10} - 2 + \frac{V_2 - V_3}{1} + \frac{V_2}{2} = 0$

3) $\frac{V_3 - V_2}{1} + \frac{V_3}{5} = 0$

GR) $3 + 2 - \frac{V_2}{2} - \frac{V_3}{5} = 0$

by MESH

A) $V_A + 10 i_a - V_B = 0$

B) $2(i_b - i_f) + V_B = 0$

F) $2(i_f - i_b) + 6 i_F = 0$

$i_a = 3$

$i_b - i_a = 2 \Rightarrow i_b = 5$

~~$V_1 = -V_a$~~

$V_a = V_0 - V_1$

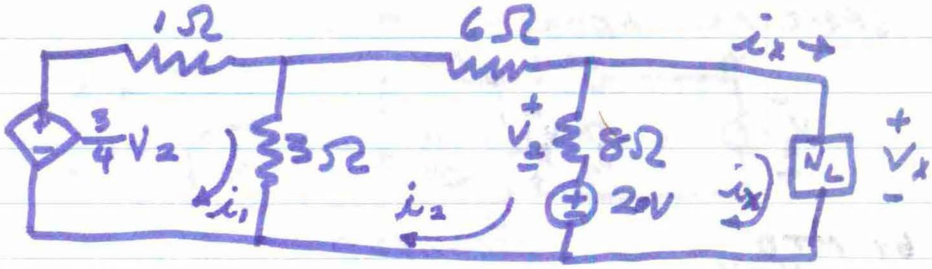
$V_2 - V_0 = -V_B = V_1 - 30 = 2(i_B - i_f)$

$V_3 - V_0 = 5 i_F$

$V_3 - V_1 = -10 i_a - i_F 1$

FOR FINDING V BETWEEN 2 PTS. IN CIRCUIT, EXTEND OPEN CIRCUIT & SUM UP VOLTAGES AROUND LOOP

SPECIAL PROBLEM # 4 HELPFUL HINTS ?



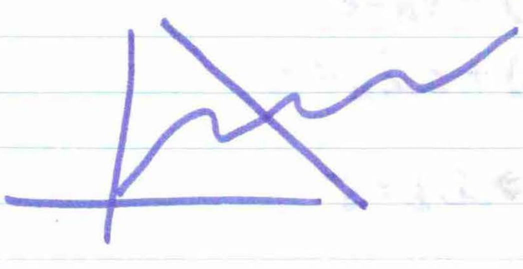
$-\frac{3}{4}V_2 + i_1 + 3(i_1 - i_2)$

$3(i_2 - i) + 6i_2 + 8(-i_2 - i_x) + 20 = 0$

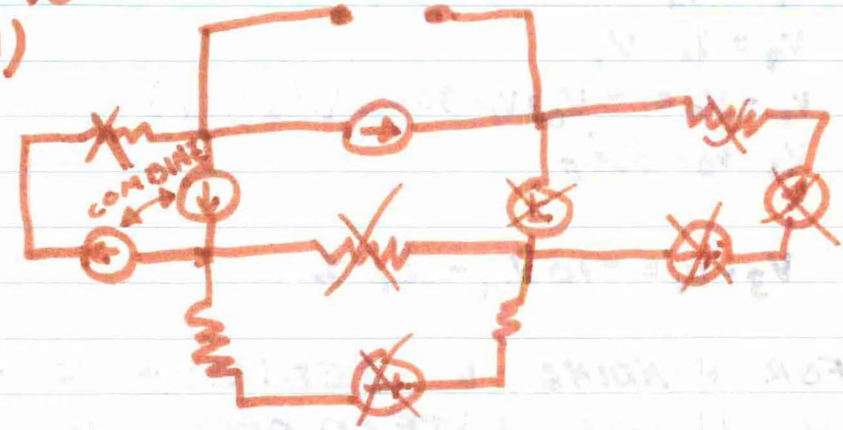
$-20 + 8(i_x - i_2) + V_x = 0$

$V_2 = 8(i_2 - i_x)$

FIFTH RELATION IS GRAPH.
REDUCE ABOVE TO $i_x = f(V_x)$



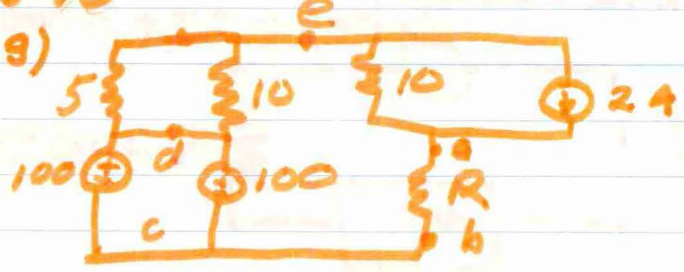
2-2-70
4-4)



$i = 0$

2-6-70

4-9)

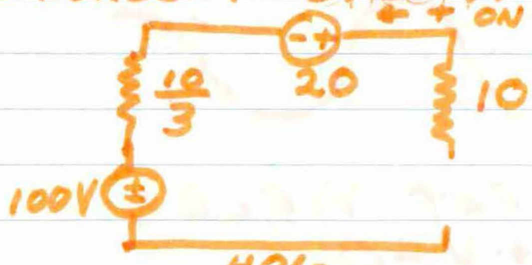


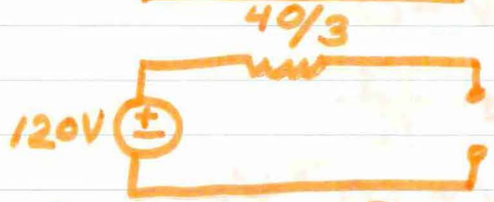
SIMPLIFY:

MAX POWER WHEN $R = R_{eq}$

EQUIVALENT CIRCUIT:

+ ON SAME END AS ARROW HEAD





PUT IN R IN CIRCUIT:

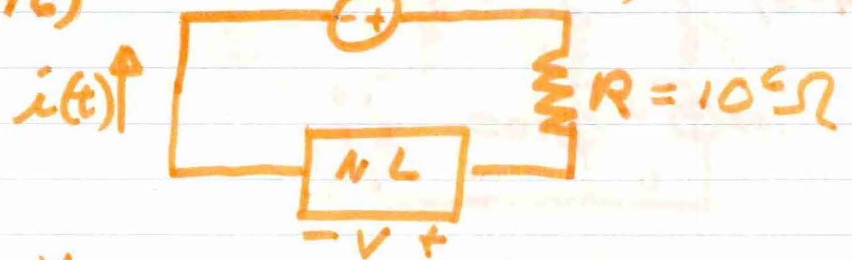
$$R = 13\tfrac{1}{3}\,\Omega$$

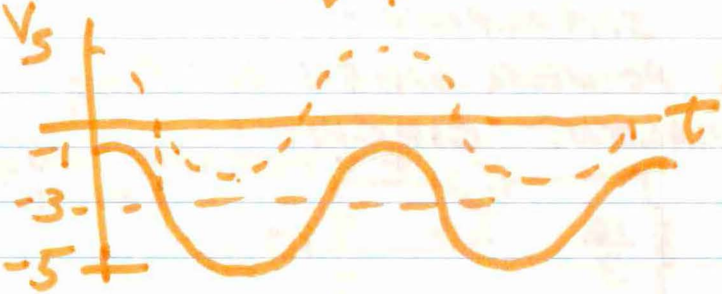
$$P = i^2R = \left(\frac{-120}{R + 40/3}\right)R$$

$$\frac{dP}{dR} = 0 \Rightarrow R = 13.33$$

NON-LINEAR JUNK:

3-96) $V_s(t) = -3 + 2\cos 2\pi t$





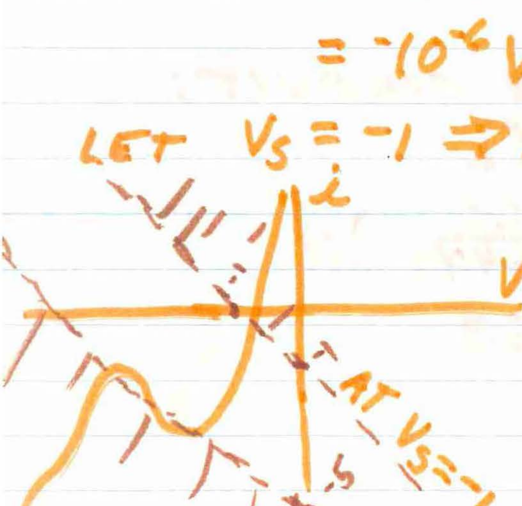
MESH CURRENT YIELDS

$$-V_s + Ri + V = 0$$

$$\Rightarrow i = -\frac{1}{R}V + \frac{V_s}{R}$$

$$= -10^{-6}V + 10^{-6}V_s$$

LET $V_s = -1 \Rightarrow i = -10^{-6}V - 10^{-6}$

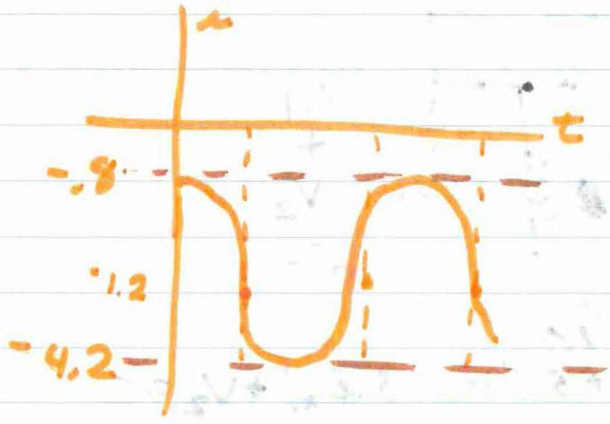


$i \simeq .8\,\mu A$

LET $V_s = -5$

$\Rightarrow i = -10^{-6}V + 10^{-6}(-5)$

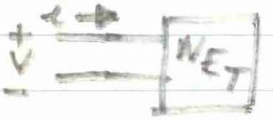
$i \simeq 4.3\,\mu A$



SAME T
SCALE AS
V_S

2-9-70

TWO TERMINAL:



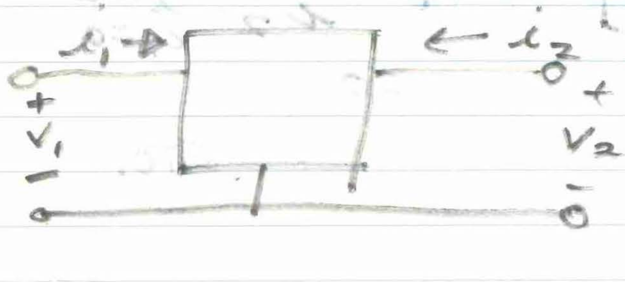
i SHOULD ENTER POSITIVE TERMINAL

$$i = \frac{1}{R_{eq}} V + I_{eq}$$
$$V = R_{eq}\, i + V_{eq}$$

THREE TERMINAL:

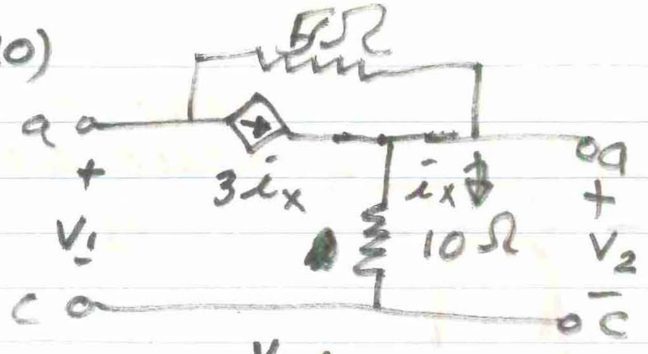
T FORM
$$\begin{cases} V_1 = A i_1 + B i_2 + C \\ V_2 = D i_1 + E i_2 + F \end{cases}$$

Π FORM
$$\begin{cases} i_1 = G V_1 + H V_2 + J \\ i_2 = K V_1 + L V_2 + M \end{cases}$$
CAPITOLS HAVE UNITS OF CONDUCTANCE ($= 1/\Omega$)



MUST SOLVE FOR 6 PARAMETERS

4-20)

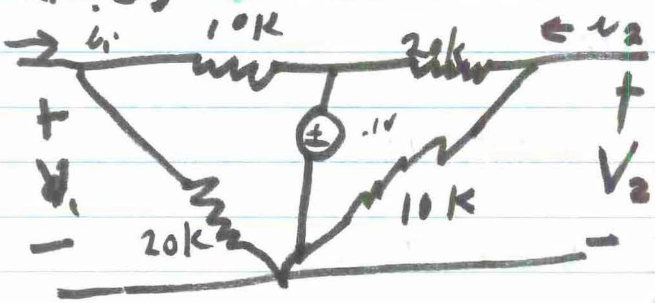


$i_x = V_2/10$

$\Sigma V = 0 = -V_1 + 5(-i_1 - 3i_x) + V_2 = 0$

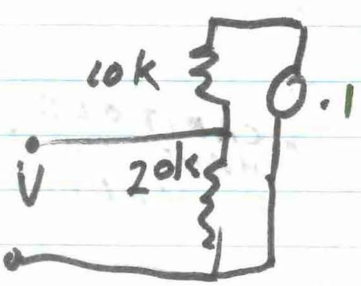
ETC.

4-21-a)



$C = V_1 \quad i_1 = i_2 = 0$

$= (2/3)(.1)\ V$



$A = \dfrac{V_1}{i_1} \quad i_2 = 0,\ s = 0$

$= \dfrac{(10k)(20k)}{(10+20)k}$

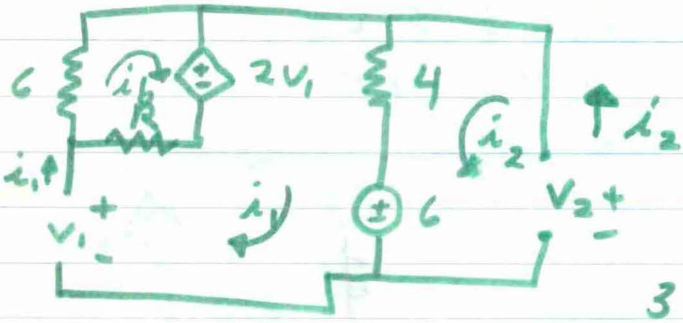
$B = \dfrac{V_1}{i_2} \quad s = 0,\ i_1 = 0$

$= 0$

ETC.

2-11-70

5)



$3V_1 = V_2$

WITH $R \neq 0$ WHEN $R = 0$

B) $6 i_b + 2V_1 + R(i_b - i_1) = 0$

1) $-V_1 + R(i_1 - i_b) - 2V_1 + 4(i_1 + i_2) + 6 = 0$

2) $-V_2 + 4(i_2 + i_1) + 6 = 0$

SOLVE FOR i_1 AND i_2 IN TERMS OF V_1 & V_2

$i_b = \frac{1}{6+R}(-2V_1 + R i_1)$

$0 = -3V_1 + i_1(4+R) + 4i_2 + 6 - R\left[\frac{1}{6+R}(Ri_1 - 2V_1)\right]$

$V_1\left[-3 + \frac{2R}{6+R}\right] + i\left[4 + R - \frac{R^2}{R+6}\right] + i_2 4 + 6 = 0$

$i_2 = \frac{V_2}{4} - i_1 - 1.5$

$$i_1\left[\frac{6R}{6+R}\right] - V_1\left[\frac{R+18}{6+R}\right] + V_2 = 0$$

$$\longrightarrow i_1 = V_1\left[\frac{R+18}{6R}\right] + {}^{-}V_2\left[\frac{6-R}{6R}\right]$$

$$\longrightarrow i_2 = -V_1\left[\frac{R+18}{6R}\right] + V_2\left[\frac{1}{4} + \frac{6+R}{6R}\right] - \frac{3}{2}$$

BY INSPECTION $(R=0)$,

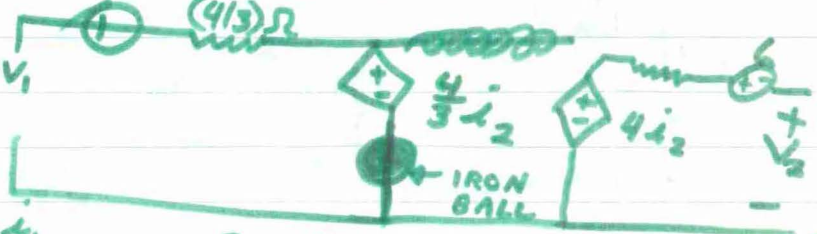
π EQUIVALENT IS

$V_2 = 3V_1$

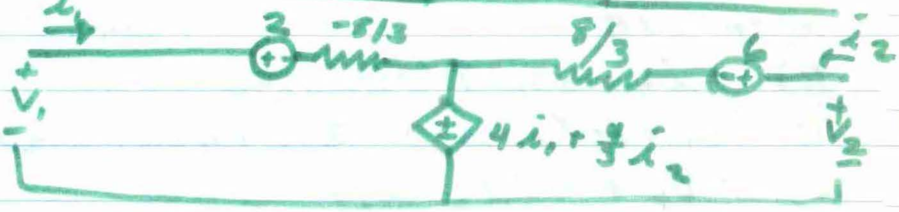
USING T EQ. YIELDS: (R=0)

$V_1 = \frac{4}{3} i_1 + \frac{4}{3} i_2 + 2$

$V_2 = 4 i_1 + 4 i_2 + 6$



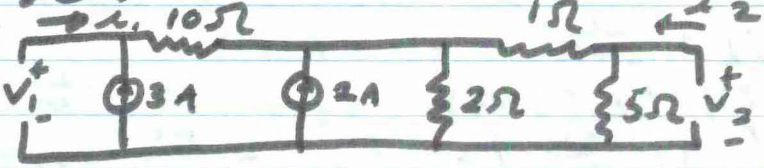
OR:



?

2-13-70.

S.P. #6)



$V_1 = A i_1 + B i_2 + C$

$V_2 = D i_1 + E i_2 + F$

SOLVING TERM BY TERM

$C = V_1 \big|_{i_1 = i_2 = 0}$

$= 3(10) + 5\left(\frac{(2 \times 6)}{2+6}\right)$ ← BY DIVIDING UP CURRENTS

$= 3(10) + \frac{3}{4}\frac{3}{3}(5)$

$= 37\frac{1}{2}$

$F = V_2 \big|_{i_1 = i_2 = 0}$

$= (5\Omega)\ \frac{5}{4}$ AMPS

$= 6.25$

$A = \frac{V_1}{i_1}\Big|_{\substack{c=0 \\ i_2=0}}$ (C=0 WHEN ALL SOURCES ARE IDLE)

OPEN CIRCUIT (0 ε) FOR i_s

$A = \left(\frac{2\cdot 6}{2+6}\right) + 10$

$= 11.5$

$E = \frac{V_2}{i_2}\Big|_{\substack{i_1=0 \\ F=0}}$

$\boxed{\cancel{= \frac{5\left(1+\frac{2\cdot 10}{2+10}\right)}{5+\left(1+\frac{2\cdot 10}{2+10}\right)}} = \cancel{\frac{40}{23}\ (?) = 1.74}}$

$E = \frac{(3)(5)}{8} = 1.875$

$B = \frac{V_1}{i_2}\Big|_{\substack{i_1=0 \\ c=0}}$

ASSUME $i_2 = 1$

$B = \frac{\left(\frac{5}{8}\right)(2)}{1} = 1.25\Omega$

$D = \frac{V_2}{i_1}\Big|_{\substack{i_2=0 \\ F=0}}$

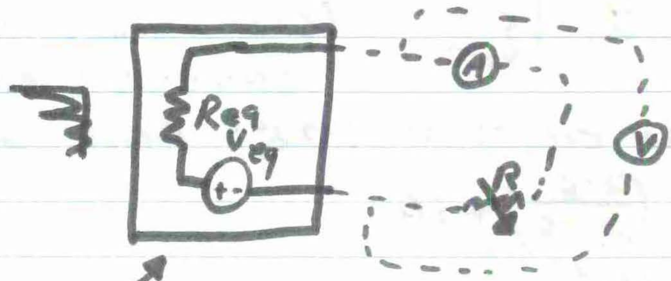
let $i_2 = 8$

$= \frac{8\left(\frac{2}{8}\right)5}{8} = 1.25\Omega$

IFF $\frac{V_1}{i_2} = \frac{V_2}{i_1}$, CIRCUIT

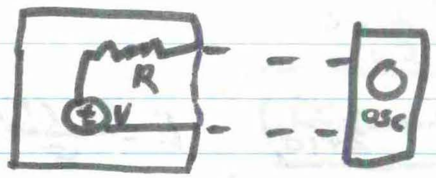
DISPLAYS RECIPROCITY

BACK TO 2 TERMINAL:

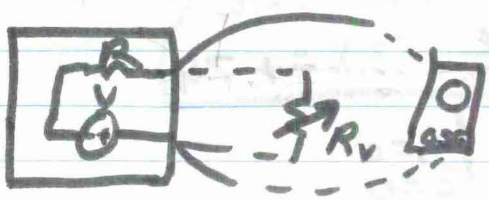


ONE WAY

BLACK BOX

① 

MEASURING V_{oc} ($=V_{eq}$)

② 

$R_v >> 1$

③ REDUCE R_{EXT}

IF $V = V_{oc}/2$

THEN $R_{EQ} = R_v$

2-17-70 LAB

PULSE GENERATOR

PRF

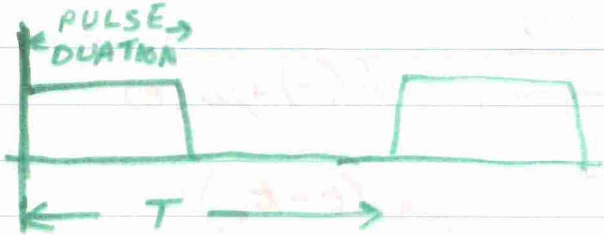
PULSE REPITITION FREQ.

PULSE DURATION

RANGE OF P.D

AMPLITUDE (V)

FOR INBETWEEN VALUES OF PRF



$\frac{1}{T}$ = P.R.F.

P.D. < T

2-16-70

ENERGY STORAGE THINGIES
(ALL TIME DEPENDENT)

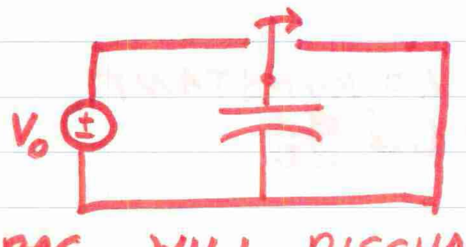
CAPACITOR - STORES EL. ENERGY ~~IN~~ BY WAY OF AN ELECTRIC FIELD

CAPACITANCE FUNCTION OF ϵ

$$i = \frac{d(CV)}{dt} = \frac{C\,dv}{dt} + V\frac{dC}{dt}$$

$$V_c(t) = \frac{1}{C}\int_0^t i_c(x)\,dt + K$$

K = V AT TIME = 0

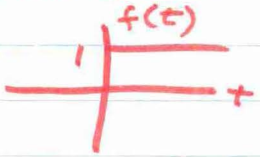


CAPAC. WILL DISCHARE SPARKWISE

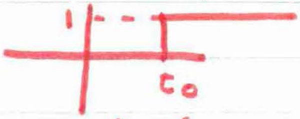
1) CAPAC. ACTS LIKE OPEN CIRCUIT IN D.C.

2) WILL NOT LET VOLTAGE ACROSS ITSELF CHANGE ABRUPTLY

STEP FUNCTION



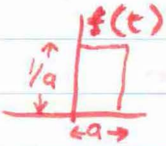
$f(t) = \mu(t)$



$\mu(t - t_0)$

$\mathcal{L}\{\mu(t)\} = \frac{1}{s}$

IMPULSE



→ IMPULSE = 

$a \cdot \infty = 1$

$\lim_{a \to 0} f(t) = \delta(t)$

$\mathcal{L}\{\delta(t)\} = 1$

MAGNETISM

$$V = \frac{d(N\Phi)}{dt} = \frac{d(Li)}{dt}$$

INDUC

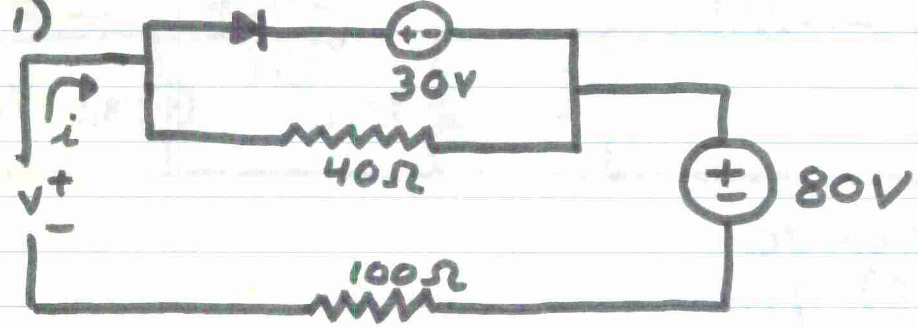
$\cancel{P = \frac{L i di}{dt}}$

$P = \frac{d\lambda}{dt} i$ | FOR L = KONSTANT $p = L i \frac{di}{dt}$

2-18-70

EXAM # 2, PROB # 3

1)



a) CONDUCT~~OR~~S:

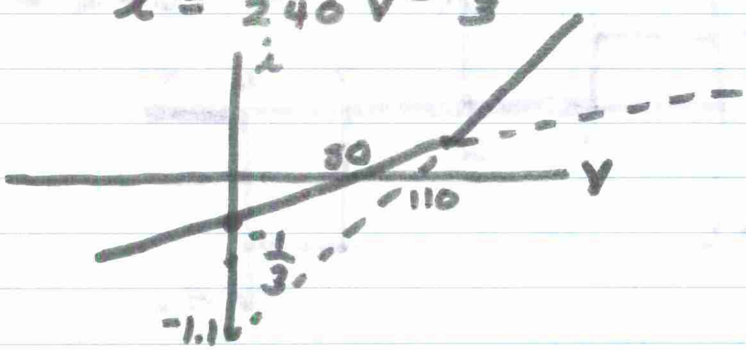
$-V + 30 + 30 + 100i = 0$

$i = \frac{1}{100} V - 1.1$

b) DOESN'T CONDUCT:

$-V + 140i + 80 + 100i = 0$

$i = \frac{1}{240} V - \frac{1}{3}$



2) REVERSING POLARITY ON 30V SOURSE

a) NON CONDUCTS (SAME AS 1b)

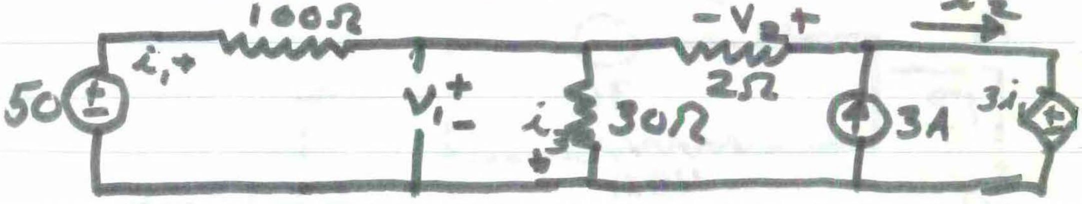
b) CONDUCTS: $i = \frac{V}{100} - \frac{1}{2}$

3) REVERSE POLARITY ON 80V SOURSE

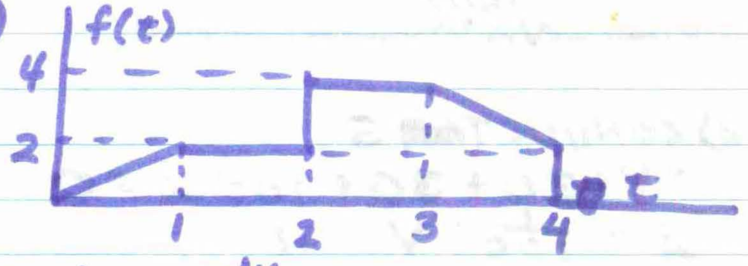
a) CONDUCTS - $i = \frac{V}{120} + \frac{1}{3}$

b) NON-CONDUCTS - $i = \frac{V}{100} + \frac{1}{2}$

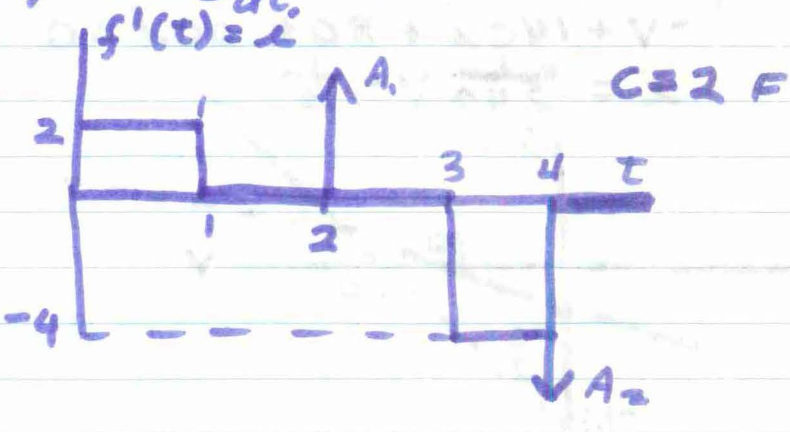
EXAM #2 PROB #3
MAY REDRAW CIRCUIT:



2-20-70
6-12)



a) $i = C\frac{dv}{dt}$



MUST FIND MAGNETUDE OF IMPULSE THINGIES

$$v = \frac{1}{C}\int i\,dt + k$$

SO CHECK BACKWARDS TO f(t)

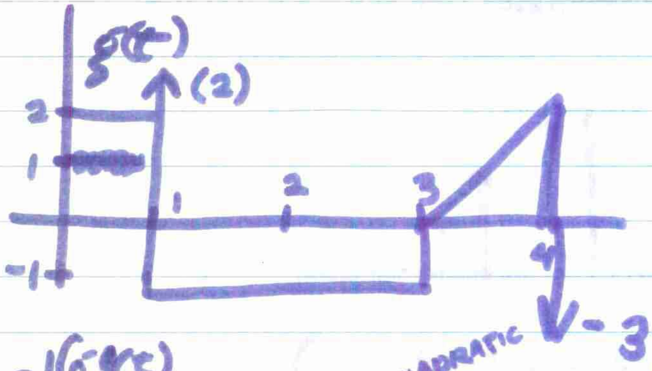
$\therefore A_1 = 4$, FOR $f(t)$ JUMPS TO FOUR. $A_2 = -4$

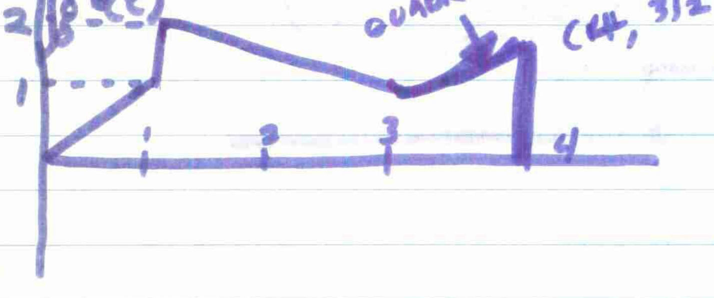
- $i = C \frac{dv}{dt}$

$v = \frac{1}{C} \int i\,dt + K$

$v = L \frac{di}{dt}$

$i = \frac{1}{L} \int v\,dt + K$

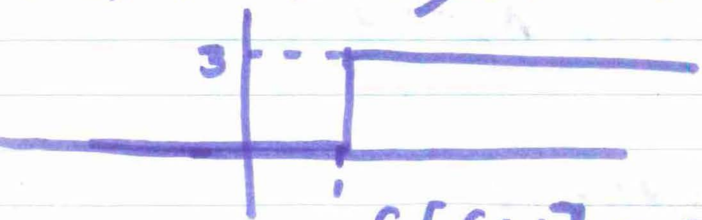




$C = 2F$

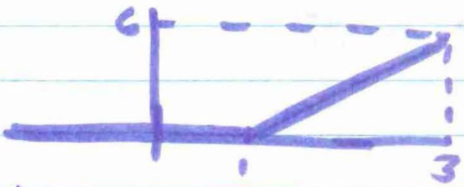
ON BOTH GRAPHS, MULTIPLYING FUNCTION SCALE BY TWO YIELDS ANSWERS TO INDUCTOR PARTS OF THIS NEAT PROBLEM.

6-5) $f(t) = 3u(t-1)$

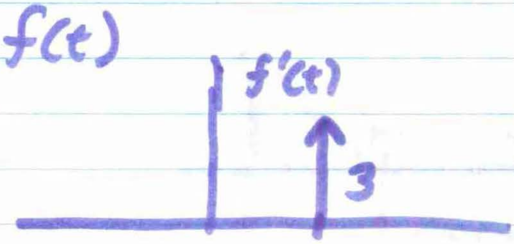


$\mathcal{L}[f(t)] = \frac{3e^{-s}}{s}$

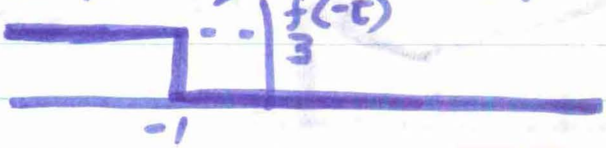
b) $\int_{-\infty}^{t} f(t)\,dt$



c) $\frac{d}{dt} f(t)$



d) $f(-t) = 3u(-t-1)$



2-20-70

$Y'' - 4Y = 0 \qquad Y(0) = a \quad Y'(0) = b$

COFEFFICIENTS $F(x)$

EX) $(X^2+1)Y' - 2XY' + 2Y = 0$

BACK TO $Y'' - 4Y = 0$

$\Rightarrow Y'' = 4Y \Rightarrow Y''(0) = 4a$

$Y''' = 4Y' \Rightarrow Y'''(0) = 4b$

$Y^{IV} = 4Y'' \Rightarrow Y^{IV}(0) = 4^2 a$

$\vdots \qquad \vdots \qquad \vdots$

LOOKS A HELL OF A LOT LIKE A TAYLOR SERIES:

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2}f''(0) \dots \frac{x^n}{n!}f^n(0)$$

$$Y(x) = a + b(x) + \frac{1}{2}x^2 4a + \frac{1}{6}x^3 + \frac{1}{24}x^4 16a + \frac{1}{120}x^5 16b + \dots\dots$$

$$= a\left(1 + \frac{4x^2}{2} + \frac{16x^4}{4!} + \dots\right) + b\left(x + \frac{4x^3}{3!} + \frac{16x^5}{5!} + \dots\right)$$

$$= a\cosh 2x + \frac{1}{2}b\sinh 2x$$

BY $\mathcal{L}$

$s^2\bar{Y} - as - b - 4\bar{Y} = 0$

$(s^2-4)\bar{Y} = as + b$

$\bar{Y} = \dfrac{as+b}{s^2-4}$

$Y = a\cosh 2x + \frac{1}{2}b\sinh 2x$

$Y'' - 4Y = 0$

$Y = a_0 + a_1X + a_2X^2 + a_3X^3 \dots + a_KX^K \dots$

$= \sum_{n=0}^{\infty} a_n X^n$

$Y' = a_1 + 2a_2X + 3a_3X^2 \dots + k\,a_KX^{K-1}$

$= \sum_{n=1}^{\infty} n a_n X^{n-1} = \sum_{n=0}^{\infty} n a_n X^{n-1}$

$Y'' = \sum_{n=0,1,2}^{\infty} n(n-1) a_n X^{n-2}$

TAKE $(X^2+1)Y'' - 2XY' + 2Y = 0$

$(X^2+1)(2a_2 + 6a_3X + 12a_4X^2)$
$\quad - 2X(a_1 + 2a_2X + 3a_3X^2 + 4a_4X^3)$
$\quad + 2(a_0 + a_1X + a_2X^2 + a_3X^3 + a_4X^4) = 0$

$(2a_2 + 2a_0) + (6a_3 + \cancel{2a_1} - \cancel{2a_1})X$
$\quad + (12a_4 + \cancel{2a_2} - \cancel{4a_2} + \cancel{2a_2})X^2$
$\quad$ + (HOFFY DIDN'T DO IT) $= 0$

THEREFORE: $2a_2 + 2a_0 = 0$

$6a_3 = 0$

$12a_4 = 0$

IF CONTINUED, ALL $a_K = 0$

$\therefore Y = a_0 + a_1X + a_2X^2 + a_3X^3 \dots\dots$

EVERY TERM$(n < 3) = 0$

FURTHERMORE, $a_2 = {}^{-}a_2$
OR
$$Y = a_0(1-X^2) + a_1 X$$

GOING BACK TO Y, Y, ETC.
AS INFINITE SERIES;
$\underline{Y}(0) = a_0$; $Y'(0) = a_1$

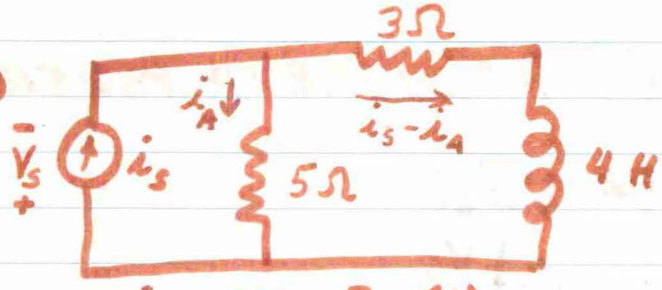
COEFFICIENTS MAY BE EXPRESSED
IN TERMS OF OTHER CO-EFFICIENT,
AND MADE INTO FINITE FORMULA

A) THE GENERAL SOLUTION OF
$PY'' + QY' + RY = 0$
OBTAINS TWO ARBITRARY
CONSTANTS; $Y = aY_1 + BY_2$
WHERE X_0 & Y_0 ARE BOTH
SOLUTIONS.

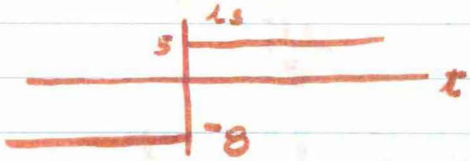
B) A PARTICULAR SOLUTION MAY
BE OBTAINED BY PRESCRIPTION
OF TWO CONDITIONS
e.g. $Y(0) = a$; $Y'(0) = b$
OR $Y(0) = a$; $Y'(2) = b$

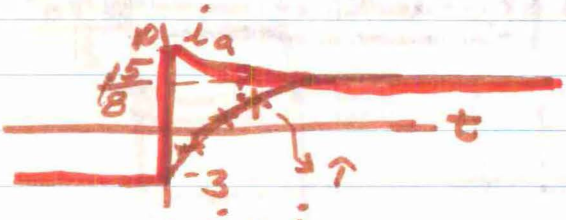
2-23-70

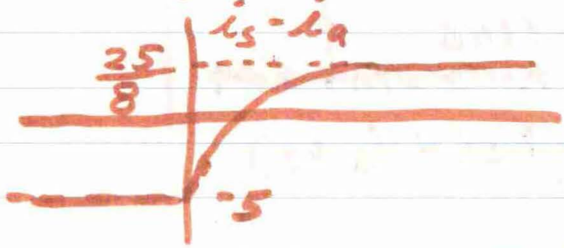
7-3)



$i_s = {}^-8 + 3u(t)$



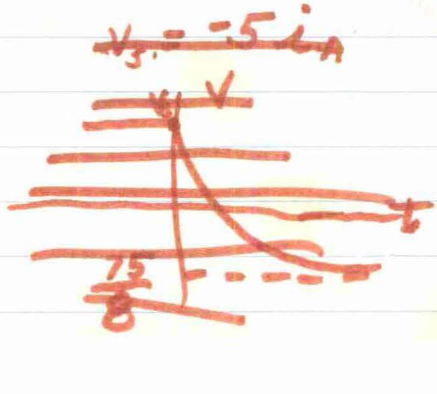




T = TIME CONSTANT = $\frac{L}{R} = \frac{4}{8} = \frac{1}{2}$

(R IS R_{EQ} FROM INDUCTOR'S TERMINALS)

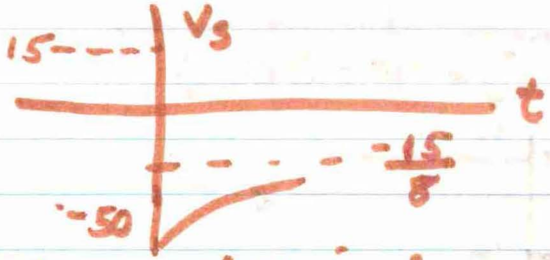
~~$V_s = {}^-5\, i_A$~~

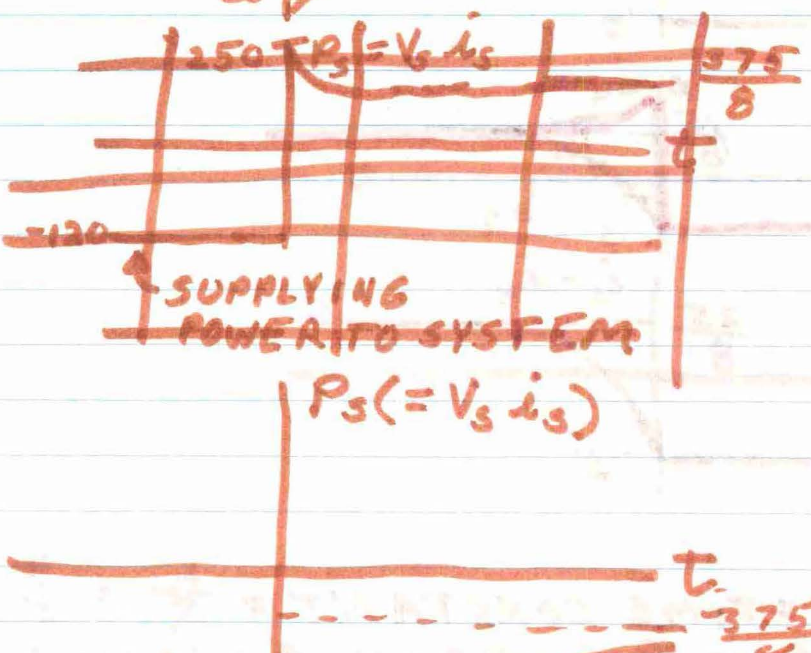


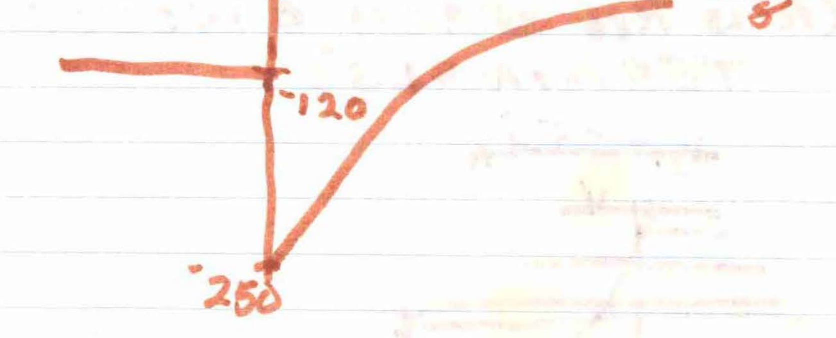
$P_s = V_s\, i_s$

(POWER DELIVERED = $^-$POWER OF SCE)

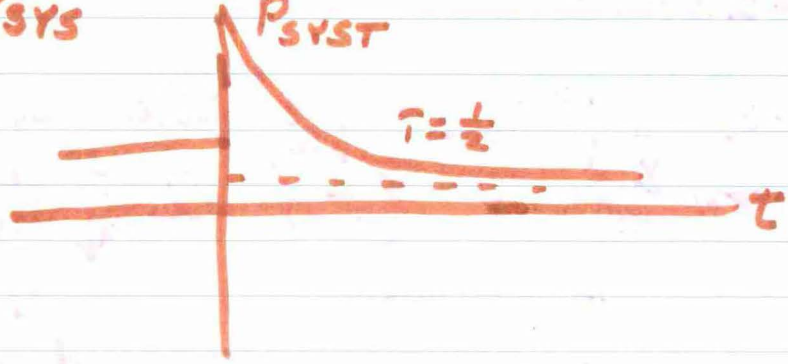
$V_s\ (= {}^-5\, i_A)$







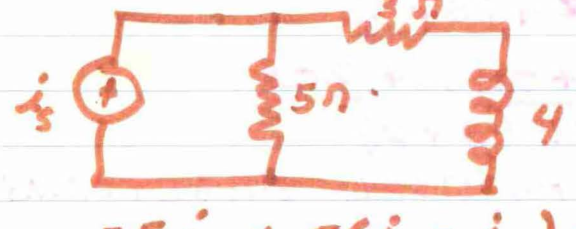
$\therefore P_{SYS}$



1) MUST HAVE $t = 0-$
2) $t = 0+$
3) $t = \infty$
4) $\tau = ?$

IN SYSTEM WITH ONE TIME ~~IND~~DEPENDENT ~~TIME~~ THINGIE, THESE FOR CONDITIONS DEFINE A CURVE.

7-3) BY ALGEBRA



$$-5 i_a + 5(i_s - i_a) + 4\,\frac{d(i_s - i_A)}{dt} = 0$$

$$5(-I_A(s)) + 3\,[I_s(s) - I_A(s)] + 4\,s\,[I_s(s) - i(0+) - I_A(s)] = 0$$

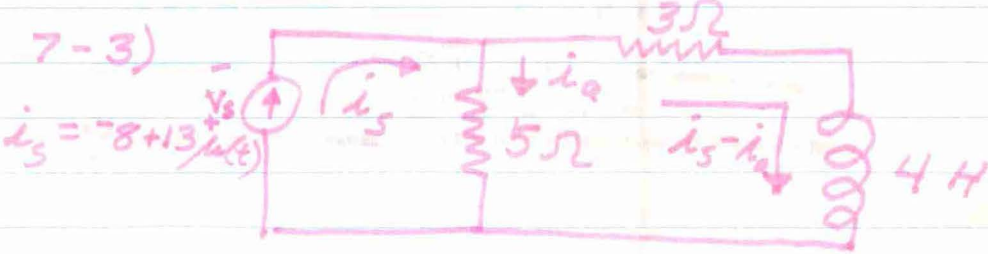
$t > 0$; $i_s = \cancel{15} 5 \Rightarrow I_s = \frac{5}{s}$

$$I_A(s)\{-4s - 8\} = \frac{-15}{s} - 20 + 20s$$

AT THIS POINT, MOORE BLEW IT, SO NOTES AT THIS POINT SHALL BE TERMINATED.

2-25-70

7-3)



$$0 = 5(-i_a) + 3(i_s - i_a) + 4\frac{d(i_s - i_a)}{dt}$$

$$-5I_A + 3I_s - 3I_A + 4[(sI_s - 5)]$$
$$-4[sI_A - 10] = 0$$

$$I_A(-8-4s) + I_s(3+4s) + 20 = 0$$

$$I_A = \frac{I_s[3+4s]+20}{4s+8}$$

$$I_s = \frac{5}{s} \quad (\mathcal{L} \text{ OF STEP})$$

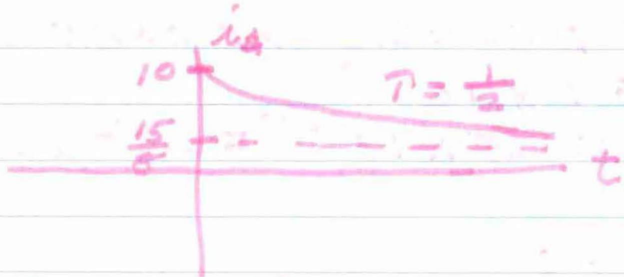
$$I_A = \frac{\frac{5}{s}[3+4s]+20}{4(s+2)}$$

$$= \frac{15+20s+20s}{4s(s+2)}$$

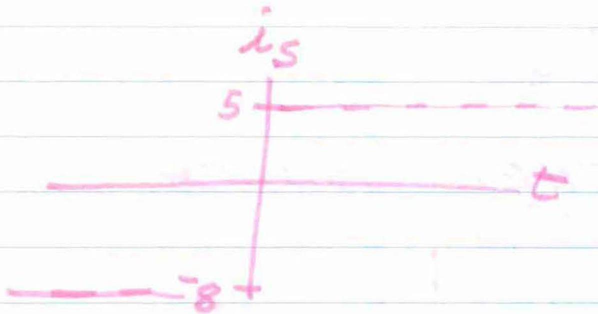
EXPAND BY PARTIAL FRACTIONS

$$\frac{15+40s}{4s(s+2)} = \frac{15}{8s} + \frac{65}{8(s+2)}$$

$$\therefore i_a = \frac{15}{8} + (e^{-2t})\frac{65}{8}$$

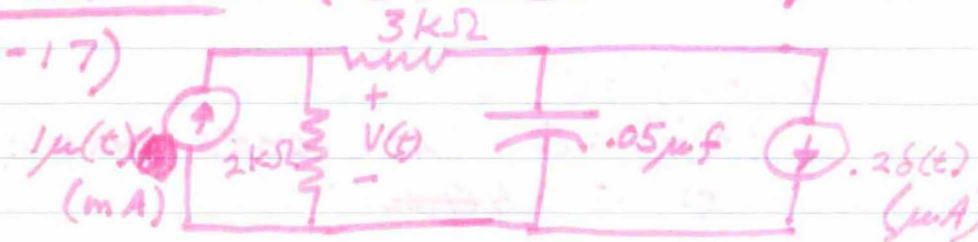


$$P = i_s V_s = (5)(-5 i_A) = -25 i_A$$

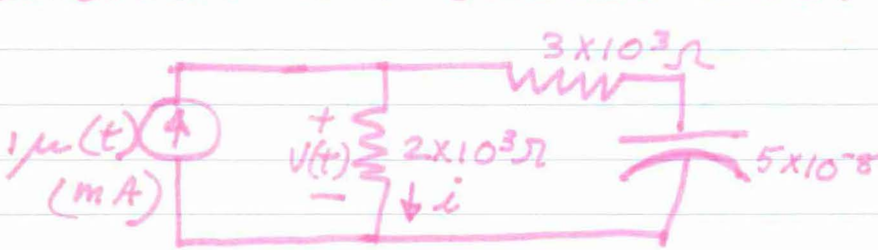


$$\underline{P(t) = -25\left(\frac{15}{8} + \frac{65}{8} e^{-2t}\right)}$$

7-17)



MAY DO BY SUPERPOSITION:
CONSIDER LEFT SOURCE ONLY:



$$i(0-) = 0$$

$$i(0+) = 0.6\times10^{-3} \text{ CONT.} \rightarrow$$

$$= \left(\frac{3000}{5000}\right)(1\times10^{-3})$$

CAP. ON D.C. ACTS AS OPEN CIRCUIT

$i(\infty) = 1 \times 10^{-3}$

$\tau = RC = 25 \times 10^{-5} = 250\ \mu\text{SEC}$

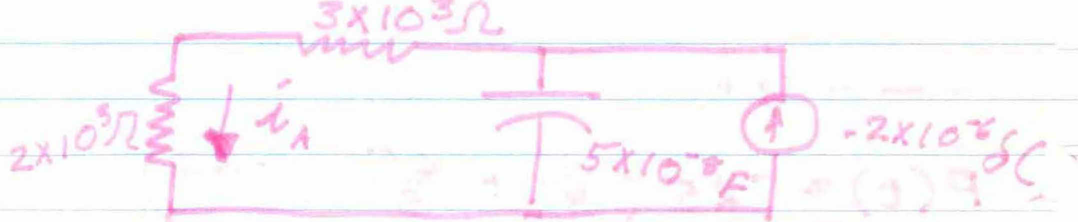
$V(0-) = 0$

$V(0+) = 1.2$

$V(\infty) = {}^{+}2$

$\tau = 250\ \mu\text{SEC}$

CONSIDER OTHER SOURCE

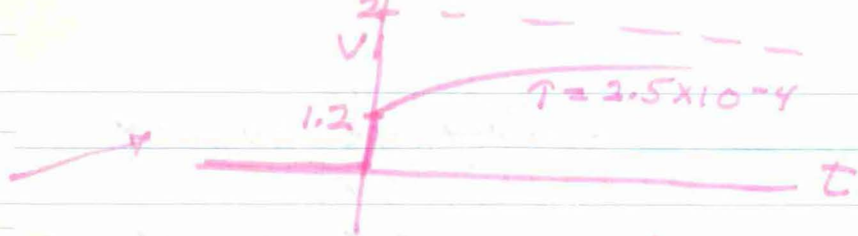


$i(0-) = 0$

$i(0+) = 0$ (ALL GOES THRU C)

$i(\infty) = \cancel{250\ \mu}\ 0$

$\tau = 250\ \mu\text{SEC}$ (CHACTERISTIC OF SYSTEM)

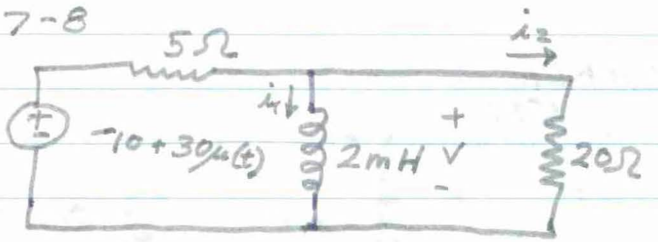


$$V(t) = \left[2 - .8\, e^{\frac{-t}{250\times10^{-6}}}\right] u(t)$$

3-2-70

Pg. 252-3

X 7-8



1) LAPLACE TRANSFORMS (MESH CURR.)
2) 4 NUGGETS OF KNOWLEGE IF ONE TIME DEPEND.

4 NUGGETS OF KNOWLEGE

1) $V(0-)$

INDUCTOR ACTS AS SHORT CIRCUIT

$\therefore V_1(0-) = 0$ $\quad (i_1(0-) = 2 \text{ AMPS})$

2) $V(0+)$ $\quad (i_1(0+) = -2 \text{ AMPS})$

SUMMING V $\quad 20 + 5(i_1 + i_2) + 20 i_2 = 0 \quad i_1 = -2$

$\therefore i_2 = \frac{30}{25} = \frac{6}{5}$

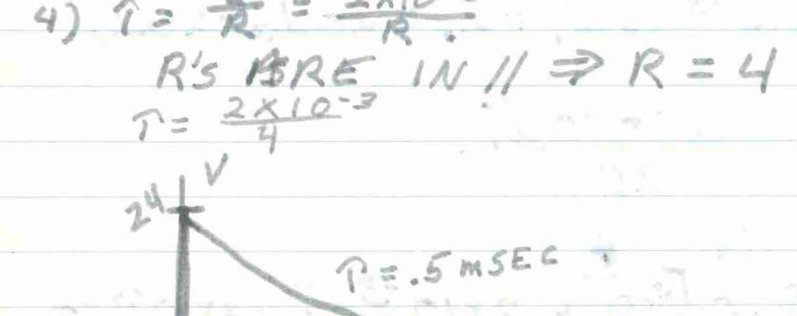
$\Rightarrow V(0+) = 20 i_2 = 24V$

3) $V(\infty) = 0$ $\quad$ (IND. ACTS AS SHT. CIR)

4) $\tau = \frac{L}{R} = \frac{2\times10^{-3}}{R}$

R'S ~~IS~~ ARE IN // $\Rightarrow R = 4$

$\tau = \frac{2\times10^{-3}}{4}$



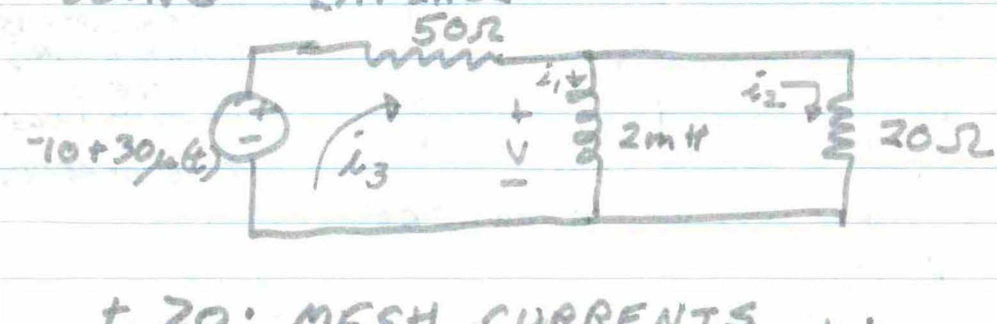
(CONT.)

AT 0+, I_1 IS ~~CONSUMING~~ SUPPLYING ENERGY

$$P_I(0+) = V(0+) \cdot i_1(0+)$$
$$= (24)(-2) = -48 \text{ WATTS}$$

MINUS SIGN IMPLIES THE INDUCTOR IS SUPPLYING ENERY (OPPOSED TO i^2R (>0) WHICH ALWAYS CONSUMES ENERGY)

USING LAPLACE



t > 0: MESH CURRENTS

① $-20\mu(t) + 5i_3 + 2\times10^{-3}\dfrac{di_1}{dt} = 0$

② $-20\mu(t) + 5i_3 + 20(i_3 - i_1) = 0$

③ $V = 2\times10^{-3}\dfrac{di_1}{dt} = 20(i_3 - i_1)$

1a) $\dfrac{-20}{s} + 5\bar{i}_3 + 2\times10^{-3}[s\bar{i}_1 + 2] = 0$

2a) $\dfrac{-20}{s} + 5\bar{i}_3 + 20\bar{i}_3 - 20\bar{i}_1 = 0$

3a) $\bar{V} = 2\times10^{-3}(sI_1 + 2) = 20I_3 - 20I_1$

SOLVING:

(2a) $\bar{i}_3 = \dfrac{\frac{20}{s} + 20\bar{i}_1}{25}$

SUBING INTO (1a)

$$\frac{-20}{s} + 5\left[\frac{\frac{20}{s} + 20\bar{i}_1}{25}\right] + 2\times10^{-3}s\bar{i}_1 + 4\times10^{-3} = 0$$

$$\bar{i}_1 \left[4 + 2\times 10^{-3} s \right] = \frac{20}{s} - \frac{4}{s} - 4\times 10^{-3}$$

$$\bar{i}_1 = \frac{\frac{16}{s} - 4\times 10^{-3}}{(2\times 10^{-3} s + 4)} = \frac{16 - 4\times 10^{-3} s}{s(s+2000)(2\times 10^{-3})}$$

$$= \frac{\frac{16}{4}}{s} + \frac{\frac{16+8}{-4}}{s+2000}$$

$$\therefore \bar{i}_1 = \frac{4}{s} - 6 \frac{1}{s+2000}$$

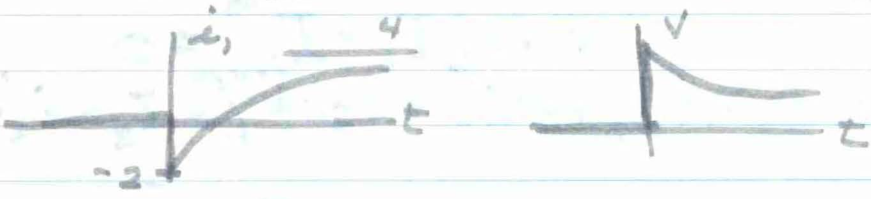
$$\underline{i(t) = \left[4 - 6 e^{-2000t} \right] u(t)}$$

$$\bar{V} = 2\times 10^{-3} \left[s I_1 + 2 \right]$$

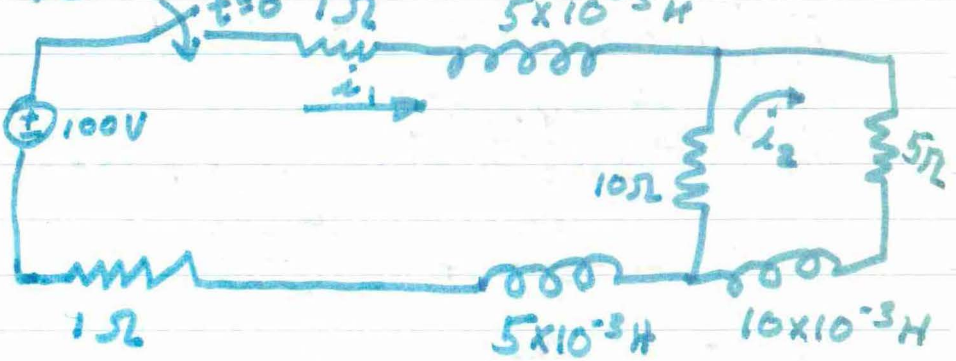
$$= 2\times 10^{-3} \left[\frac{16 - 4\times 10^{-3}}{(s+2000)(2\times 10^{-3})} + 2 \right]$$

$$= \frac{16 - 4\times 10^{-3} s + 4\times 10^{-3} s + 8}{s+2000}$$

$$= \frac{24}{s+2000} \Rightarrow V(t) = 24 e^{-2000t} u(t)$$



3-5-70



FIND $i_2(t)$

$t > 0$

① $-100 + i_1 + 5\times10^{-3}\frac{di_1}{dt} + 10i_1 - 10i_2 + 5\times10^{-3}\frac{di_1}{dt} + i_1 = 0$

② $10i_2 - 10i_1 + 5i_2 + 10^{-2}\frac{di_2}{dt} = 0$

---

① $-\frac{100}{s} + 2I_1 + 10^{-2}[sI_1 - i_1(0+)] + 10[I_1 - I_2] = 0$

② $10I_2 - 10I_1 + 5I_2 + 10^{-2}[sI_2 - i_2(0+)] = 0$

SOLVING FOR I_2

① $I_1(2 + 10^{-2}s + 10) = \frac{100}{s} + 10I_2$

$I_1 = \frac{\frac{100}{s} + 10I_2}{10^{-2}s + 12} = \frac{10^4 + 10^3 s I_2}{s(s+1200)}$

SUBSTITUTE INTO 2

$I_2(15 + 10^{-2}s) - 10\left\{\frac{10^4 + 10^3 s I_2}{s(s+1200)}\right\}$

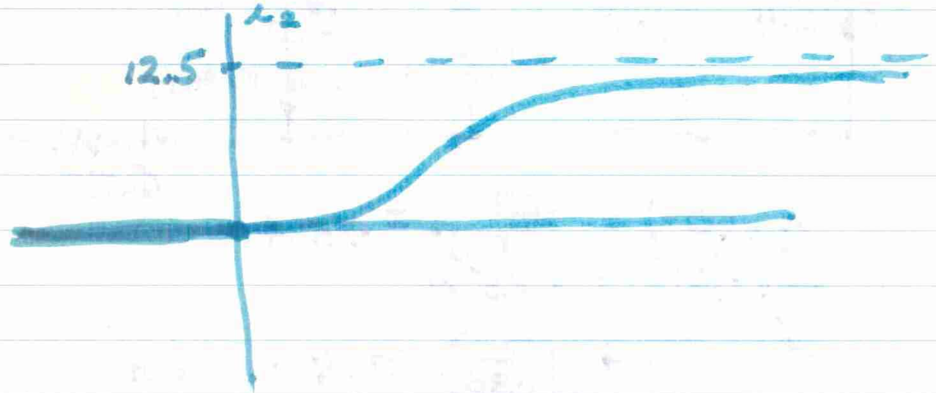
$$I_2\left\{15+10^{-2}s-\frac{10^4 s}{s(s+1200)}\right\}=\frac{10^5}{s+1200}$$

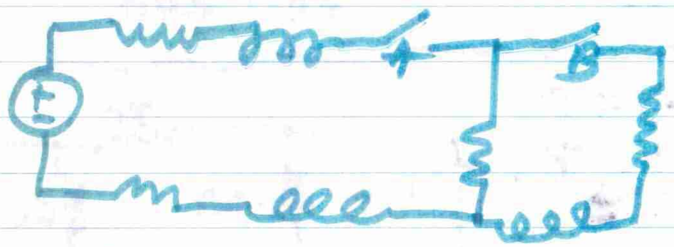
$$I_2=\frac{10^5}{s(s+1200)(15+10^{-2}s)-10^4 s}$$

$$=\frac{10^5}{s\{10^{-2}s^2+27s+8000\}}$$

$$=\frac{12.5}{s}+\frac{2.09}{s+2360}+\frac{-14.59}{s+339}$$

$$i_2(t)=\left(12.5+2.09e^{-2360t}-14.59e^{-339t}\right)u(t)$$



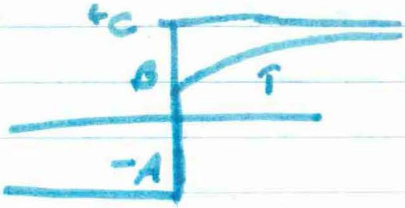


A CLOSED
B OPEN $\quad T_1=\frac{1}{1200}$

A OPEN
B CLOSED $\quad T_2=\frac{1}{1500}$

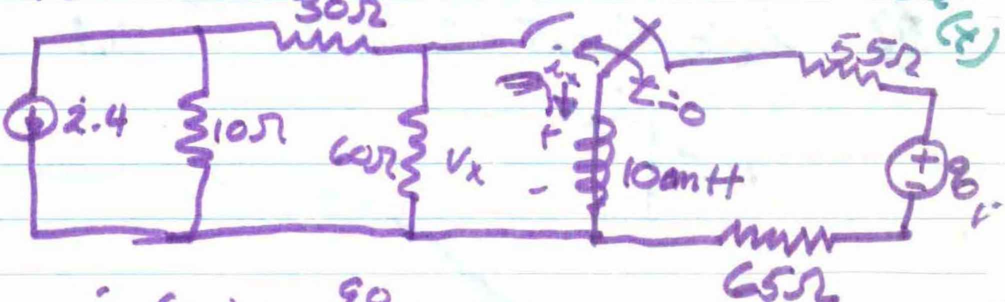
∴ UNRELATED TO INITIAL PROBLEM

FOR CIRCUIT WITH 1 E.S.C



$$Y(t) = {}^{-}A\,\mu(t) + \left\{ (C-B)\left[1-e^{-\frac{t}{T}}\right] + B \right\} \mu(t)$$

3-7-70



$i_x(0-) = \frac{90}{55+65} = .75\,A$

$i_x(0+) = .75\,A$

$i_x(\infty) = \left(\frac{10}{10+30}\right) {}^{-}2.4 = {}^{-}.6\,A$

$T = \frac{L}{R} = \cancel{100 \times 10^{-3}}\ \frac{1}{24} = \frac{1}{240}$

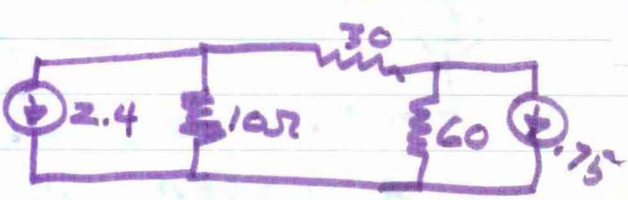
1) $V_x(0-) = 0$

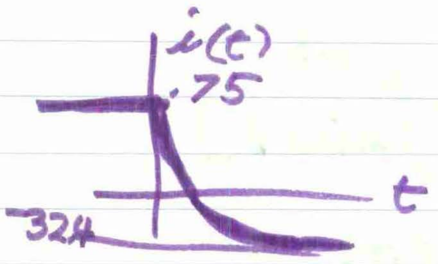
2) $V_x(0+) = ?$

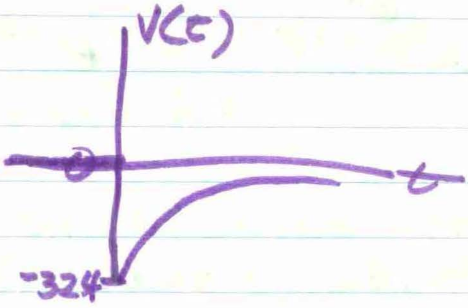
$V_x(0+) = {}^{-}32.5\,V$

3) $V_x(\infty) = 0$

4) T

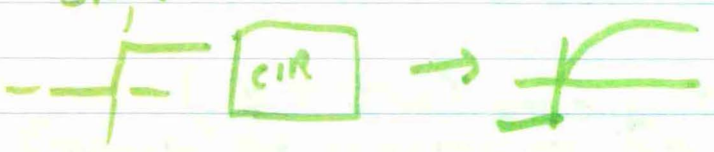






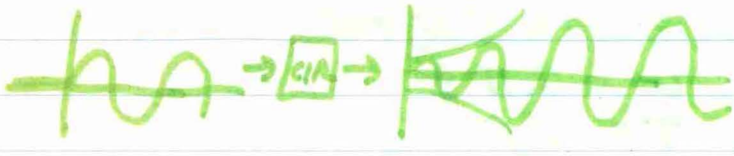
3-15-70

CH 7



TRANSIENT RESPONSE

CH 8



FREQ. RESPONSE

DIFFERS IN AMPLI.
& PHASE ∠

INPUT: OUTPUT

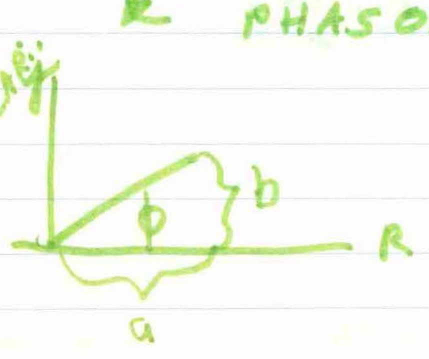
$A_1 \cos(\omega t + \phi_1)$; $A_2 \cos(\omega t + \phi_2)$

cos ωt APPEARS
IN BOTH EXPRESSIONS.
REDUCED TO

POLAR: $A_1 \angle \phi_1$; $A_2 \angle \phi_2$

PHASOR

RECTANGULAR:



$a + jb$

EULER (EXP) $e^{a+jb} = e^{a} e^{jb} = A_1 e^{jb}$

$= A_1 (\cos b + j \sin b]$

MAY EXPRESS IN TERMS OF POLAR, COOROINITE (RECTANGULAR), OR EULER (EXPODENTIAL)

PHASOR

$A_1 \angle \phi_1 \leftrightarrow A_1 \cos(\omega t + \phi_1)$

(MUST BE FUNCTION OF TIME)

WILL REPRESENT VOLTAGE AND CURRENTS.

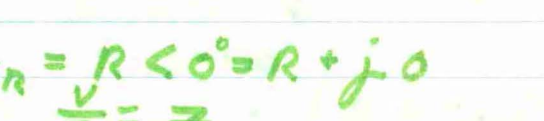
$\omega = 2\pi f$

$T = 1/f$

SHOULD KNOW EULER'S EQUA.

$Z_R = R \angle 0^\circ = R + j0$

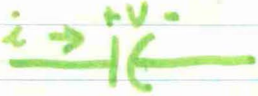
$\frac{V}{I} = Z$



$R = \frac{v(t)}{i(t)}$

$V = RI \Rightarrow R = \frac{V(s)}{I(s)}$

Z_C



$v = \frac{1}{C}\int i \, d\tau$

$V_s = \frac{1}{C}\frac{I}{s}$

$\frac{V_s}{I_s} \Rightarrow \frac{V_s}{I_s} = \frac{1}{Cs}$

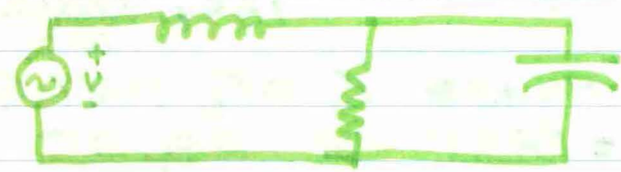
$Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

$= 0 - j\left(\frac{1}{\omega C}\right)$

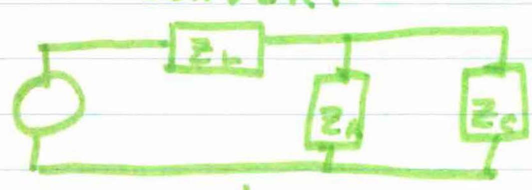
$= \frac{1}{\omega C} \angle -90^\circ$

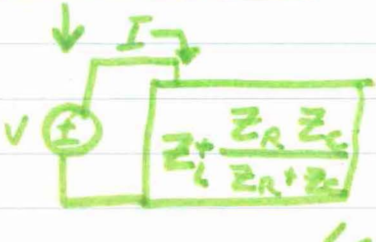
$Z_L = j\omega L$

EX



CONVERT



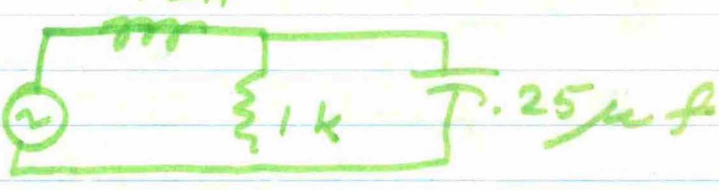


(CONT →)

$$I = \frac{V}{Z_{EQ}} = \frac{V}{Z_L + \frac{Z_R Z_C}{Z_R + Z_C}}$$

$$I\angle\phi_3 = \frac{V\angle\phi_1}{|Z_{eq}|\angle\phi_2} \quad \text{IF } \phi_2 = 0$$

(SUBTRACT $\angle$'S WHEN DIVIDING)



$$Z_{eq} = Z_L + \frac{Z_R Z_C}{Z_C + Z_R}$$

$$= j\omega(.2) + \frac{(1000)\left(\frac{-j}{.25\times10^{-6}\omega}\right)}{1000 + \left(\frac{-j}{.25\times10^{-6}\omega}\right)}$$

$$= j\omega(.2) + \frac{\frac{16\times10^{4}}{\omega^2} - \frac{4\times10^{12}}{\omega^2} j}{10^6 + \frac{16\times10^{12}}{\omega^2}}$$

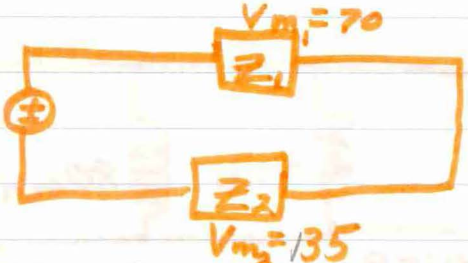
IF PHASE $\angle = 0$, UNREAL PART OF EQ. $= 0$

$\omega = 2000$

3-13-70

8-10)

$V_s = 100\angle 0°$



FIND 2 PHASOR VOLTAGES

$V_1 = \overset{70}{V_m}(\cos\phi_1 + j\sin\phi_1)$

$V_2 = 135(\cos\phi_2 + j\sin\phi_2)$

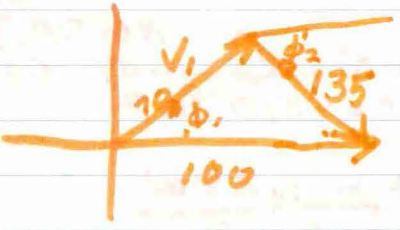
$V_s = V_1 + V_2$

$100 + j0 = 70\cos\phi_1 + j70\sin\phi_1$
$+ 135\cos\phi_2 + j135\sin\phi_2$

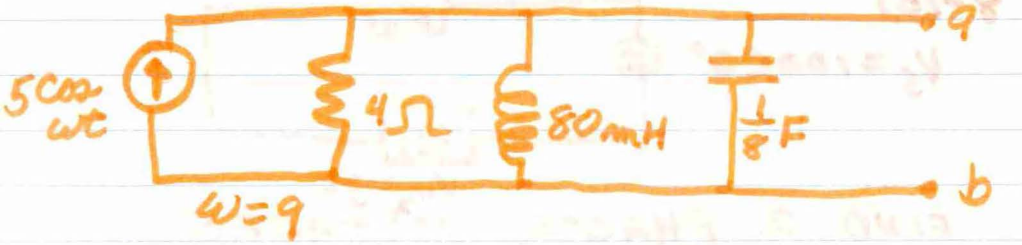
$100 = 70\cos\phi + 135\cos\phi_2$

$0 = 70\sin\phi_1 + 135\sin\phi_2$

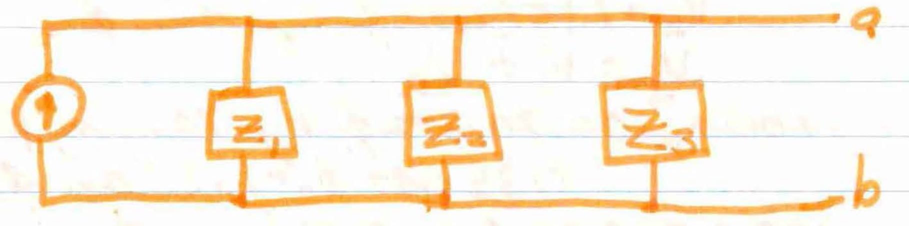
ϕ_1	ϕ_2
$-76°$	$+30.3°$
$76°$	$-30.3°$



8-16)



FIND THEV. EQ. IN PHASOR FORM



$$Z_1 = 4\angle 0^\circ$$
$$Z_2 = (80\times 10^{-3})\, j\omega$$
$$Z_3 = -\frac{8j}{\omega}$$

LET $Z_4 = \frac{Z_1 Z_2}{Z_1+Z_2} = \frac{4(.08)j\omega}{4+.08j\omega}$

$$Z_4 = \frac{j.32\,\omega(4-j.08\omega)}{16+64\times10^{-4}\omega^2}$$ MULTIPLY BY CONJUGATE

$$= \frac{.0256\,\omega^2 + j\,1.28\,\omega}{16+.0064\,\omega^2}$$

$$Z_{eq} = \frac{10.24\,\omega - j(2.048)\omega^2}{.0256\,\omega^2 + j\,1.28\omega^2 - 128j - .0512\,\omega^2 j}$$

ETC.

CONT →

$$\frac{a+jb}{c+jd} = \frac{N\angle\phi_1}{D\angle\phi_2}$$

IF $\phi_1 = \phi_2$, THEN $Z = R$

a) $Z = 2.75\angle 46.6°$ $\qquad \omega = 9$

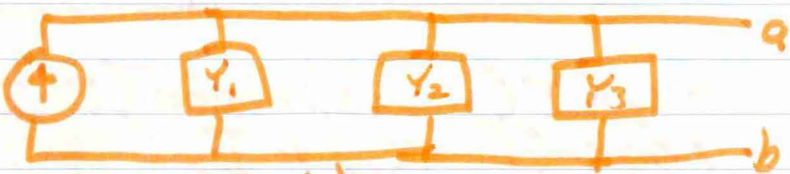
$V_{eq} = 13.75\angle 46.6°$

b) $Z_{eq} = 2.89\angle -43.7$ $\qquad \omega = 11$

$V_{eq} = 14.45\angle -43.7°$

c) $\omega = 10$

8-16)



$\left(Y = \frac{1}{Z}\right)$ (EMITTENCE)

$Y_1 = .25$

$Y_2 = \frac{1}{j.080\omega} = -\frac{j}{.080\omega}$

$Y_3 = j\omega/8$

$Y_{eq} = Y_1 + Y_2 + Y_3$

$= (.25) + j8^{-1}\omega - \frac{j}{.080\omega}$

$= .25 + j\left(\frac{\omega}{8} - \frac{1}{.080\omega}\right)$

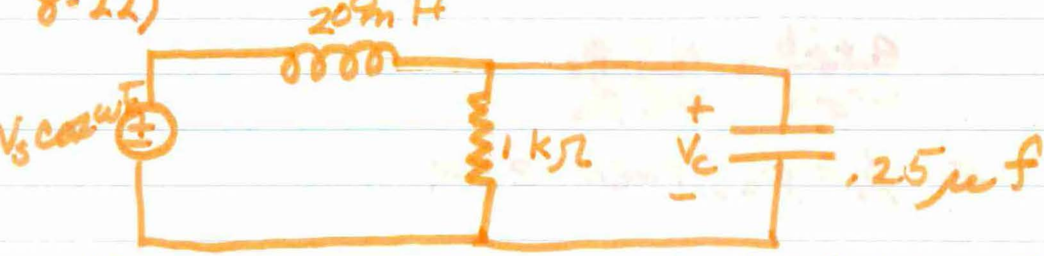
$Z_{eq} = \frac{1}{Y_{eq}}$

$= \frac{1}{.25 + j\left(\frac{\omega}{8} + \frac{1}{.080\omega}\right)}$

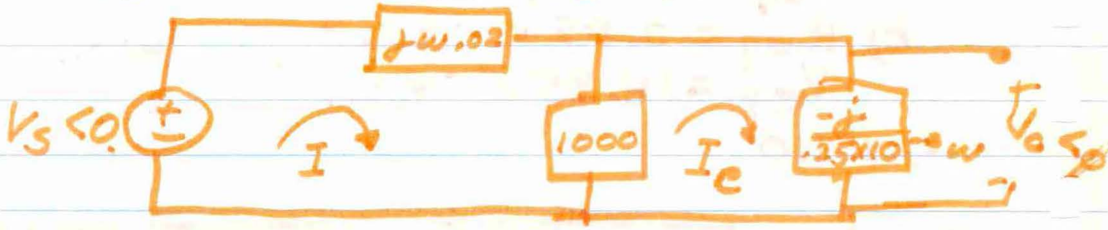
$\frac{\omega}{8} - \frac{1}{.080\omega} = 0 \Rightarrow \omega = 10$

FOR $\angle Z = 0°$

8-22)



b) FIND ω FOR WHICH AMPLITUDE IS A MAXIMUM



$$-V_S + I(j\omega .2) + 1000(I - I_c) = 0$$

$$1000(I_c - I_A) + I_c\left(\frac{-4j}{10^{6}\omega}\right) = 0$$

$$V_c = -I_c \frac{4j}{10^{-6}}$$

$$\frac{dV_c}{d\omega} = 0$$

$$\frac{dV_c}{d\omega} = 0 = 2(4\times10^{-9} - 200\omega^2)(-400\omega) + .64\times10^{12}(2\omega) = 0$$

$$\omega = 3460$$

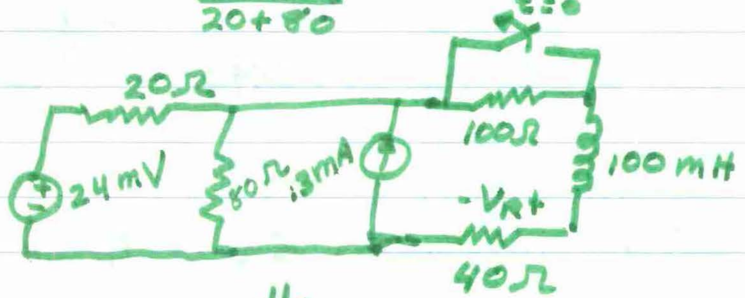
3-16-70

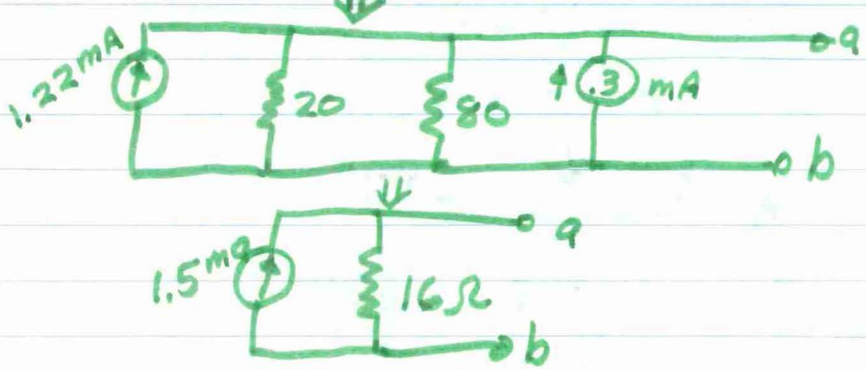
ANSWERS TO LAST TEST

$\boxed{1}$ $A = -30$; $B = 40$

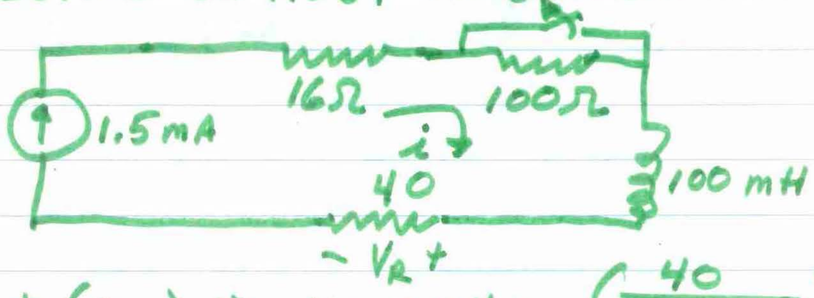
$\boxed{2}$ $T = \frac{L}{R} = \frac{100\times10^{-3}}{\frac{(20)(80)}{20+80}} = 6.25$ m SEC

$\boxed{3}$





CONNECTING ON REST OF CIRCUIT



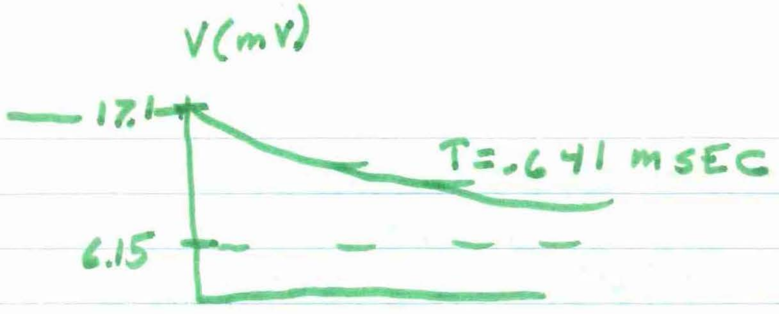
$t(0-) \Rightarrow V_R = 24mv\left(\frac{40}{40+16}\right)$
$= 17.1$ mV

$t(0+) \Rightarrow V_{R(0+)} = 17.1$mV $\quad i = \frac{24\times10^{-3}}{56}$

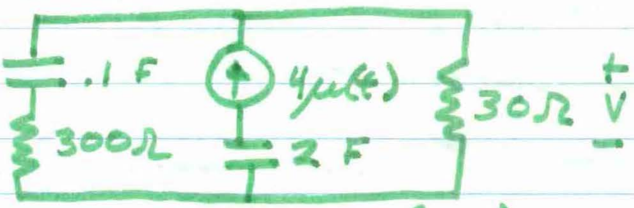
$t(\infty) \Rightarrow V_R(\infty)$
$= 24\left(\frac{40}{156}\right) = 6.15$ mV

$T = \frac{L}{R} = \frac{.1}{156} = 0.641$ mSEC

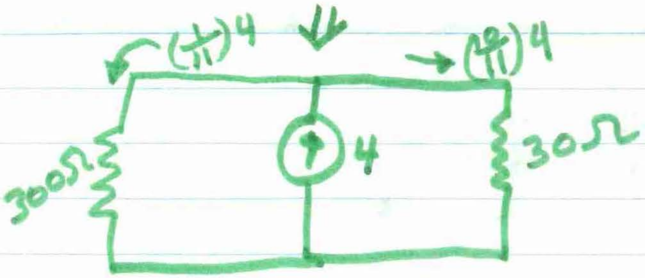
CONT →



$$\underset{(mV)}{V_R(t)} = 17.1\,u(-t) + \left[6.15 + 10.95\,e^{-\frac{t}{\tau}}\right]u(t)$$

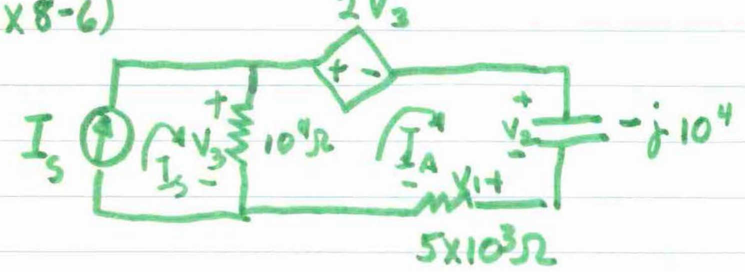


FIND V(0+)



$\therefore$ $v = \frac{10}{11}(4)(30) = 109$ V

X 8-6)



$I_s = 2\times10^{-3}\angle 90^\circ$

~~$10^4 I_A - 2\times10^{-3}\angle -90$~~

$10^4(I_A + j\,2\times10^{-3}) + 2V_3 - I_A 10^4 j + 5\times10^3 I_A = 0$

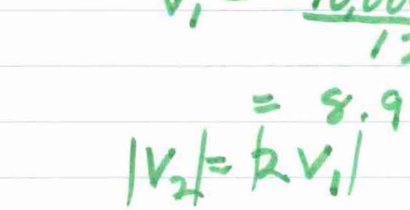
$V_3 = 10^4\left[-j\,2\times10^{-3} - I_A\right]$

$10^4(-j\,2\times10^{-3} - I_A) + I_A(-j\,10^4) + I_A(5\times10^3) = 0$

$I_A = \dfrac{j\,20}{-10^4 - j10^4 + 5\times10^3}$

$= \dfrac{20j}{-5\times10^3 - j\,10^4} = \dfrac{j\,2\times10^{-2}}{-5 - j\,10}\,\dfrac{(-5+j10)}{(-5+j10)}$

$= \dfrac{-.2 - 0.1j}{125} = \dfrac{-2}{1250} - j\dfrac{1}{1250}$



$\Rightarrow I_A = \dfrac{2(1.118)}{1250}\angle -153^\circ$

$V_1 = \dfrac{10{,}000(1.118)}{1250}\angle -153^\circ$

$= 8.94\angle -153^\circ$

$|V_2| = |2V_1|$

I) INTRODUCTION TO VOLTAGE AND CURRENT

A) DEFINITION (WORD) - 3-4

1) ~~ME~~ VOLTAGE - MEASURE OF ENERGY NESSESARY TO MOVE AN ELECTRIC CHARGE FROM ONE TERMINAL TO ANOTHER

2) CURRENT - RATE AT WHICH CHARGE MOVES THROUGH ELEMENT

3) POWER = (CURRENT)(VOLTAGE)

WATT = (AMP)(VOLT)

B) THE ELECTRIC FIELD - 4

1) COULOMB'S LAW - $F = \frac{q_1 q_1}{4\pi\varepsilon_0 r^2}$

a) $\varepsilon_0 = 8.85 \times 10^{-12} \frac{\text{FARADAY}}{\text{METER}}$

b) F IS IN NEWTONS

2) ELECTRON

a) MASS $= 9.109 \times 10^{-31}$ kg

b) $q = -1.602 \times 10^{-19}$ COUL

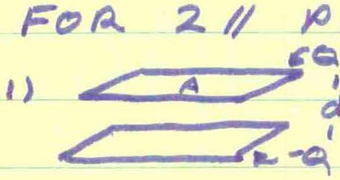
3) ELECTRIC FIELD INTENSITY

a) $E = \frac{dF}{dQ}$

b) MAGNITUDE OF A FIELD: $E = \frac{q}{4\pi\varepsilon_0 r^2}$ (POINT)

e) $E = \frac{N}{C} = \frac{V}{m}$

f) FOR 2 ∥ PLATES

1) 

$E = \frac{Q}{\varepsilon_0 A}$

2) ACCEL OF AN ELECTRON

a) $E = Fd$

b) $F = ma$ ETC

C) VOLTAGE-$\mathcal{E}$ (POTENTIAL DIFFERENCE)

1) PLACE TEST CHARGE Q IN A FIELD E. Q DOESN'T EFFECT E

a) $F_{FIELD} = QE$

b) MOVE CHARGE AGAINST FIELD -L (RIGHT)

1) $W = QEL$ (W IN JOULES)

2) $W_f - W_0 = QEL$

c) TAKE AWAY FORCE THAT MOVED Q

1) WOULD GO TO THE RIGHT

2) $W = K.E. = \frac{1}{2}mu^2 = QEL$

d) W INDEPENDENT OF PATH

e) $W = W_f - W_0 = -Q\int_1^2 E_L\,dL$

FOR NON-UNIFORM FIELDS

2) $V = \frac{\text{JOULE}}{\text{COULOMB}}$

3) $V = Ed$

D) CURRENT (i)

1) a) LINEAR, $i = \frac{q}{t}$

b) NON-LINEAR, $i = \frac{dq}{dt}$

2) $\text{AMP} = \frac{\text{COULOMB}}{\text{SEC}}$

3) CURRENT DENSITY a) $J = \frac{di}{dA}$

b) UNITS OF $\frac{\text{AMP}}{\text{METER}^2}$

c) CURRENT $\perp$ AREA

4) a) LINEARLY, $i = JA$

b) NON-LINEAR, $i = \iint J\,dA$

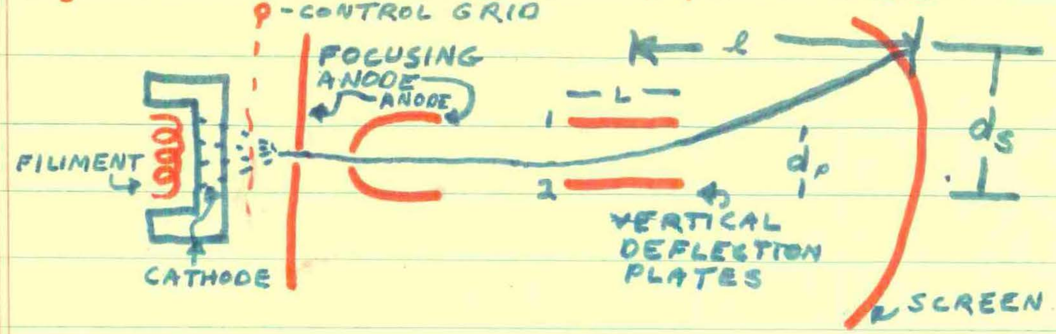
5) CHARGE DENSITY (ρ)

a) $\rho = \frac{dq}{dV}$

b) UNITS - $\frac{\text{COULOMBS}}{m^3}$

6) $J_L = \rho\,.\,U_L$ (U = CHARGE VELOCITY)

E) THE CATHODE RAY TUBE



(OF AN ELECTRON)

1) $\frac{1}{2} m_e U_{fa}^2 = eV_0$

$\therefore U_{fa} = \sqrt{\frac{2eV_0}{m_e}} = 5.93 \times 10^5 \sqrt{V_0}$

2) PASSING THRU V.D.P.

a) $V_{V.D.P._{21}} = E L_{21}$

b) $E_L = \frac{V_d}{d_p}$

3) AFTER V.D.P.

a) UPWARD $a = \frac{eE_L}{m_e}$ (DURING. V.D.P.)
(TAKES $\cancel{U_{fa}}\ L/U_{fa} = t$) TO GO THRU V.D.P.

~~b) HITS SCREEN.~~

b) $U_Y = \frac{eE_L L}{m_e U_{fa}}$

c) $d_s = U_Y \frac{l}{U_{fa}} = \frac{V_{vdp} L l}{2 V_0 d_p}$

4) OTHER INTERESTING JUNK

a) AT CATHODE

1) $i = ne$ (n = # ELECTRONS/SEC)

2) i IS FROM R TO L

b) CURRENT DENSITY

$J = \frac{I}{A} = \frac{ne}{\pi r^2}$ (r = RADIUS OF e BEAM

c) CHARGE DENSITY

$\rho = \cancel{A}.\ \frac{J}{U_0} = \frac{I}{\pi r^2}\sqrt{\frac{m}{2eV_0}}$

II) VOLTAGE-CURRENT RELATIONSHIPS - 29

A) TIME-INDEPENDENT V-i RELATIONSHIPS (LINEAR & NONLINEAR)

B) FAMILY OF V-i CURVES

1) OCCURS WHEN OTHER VARIABLE IS PRESENT.

2) EX) $i = 10^{-6}V + 10^{-3}L$

3) SET 3RD VARIABLE TO CONVENIENT CONSTANTS TO PLOT FAMILY OF CURVES

4) A CURVE IN WHICH $i = 0$ FOR $V<0$ AND LINEAR FOR $V>0$ IS CALLED "PIECEWISELINEAR"

C) THE V-AMP CHARACTERISTIC OF A PHYSICAL DEVICE

1) a) ELECTRON EMISSION OCCURS AT CATHODE

b) ENERGY NEEDED FOR EMISSION IS CALLED "POTENTIAL BARRIER"

2) PHOTOEMISSION (LIGHT ENERGY WHICH CAUSES EMISSION)

W (ENERGY OF A PHOTON) $= \frac{hc}{\lambda}$

a) h = PLANCK'S CONSTANT

b) c = VEL. OF LIGHT

c) λ = WAVELENGTH OF LIGHT

d) $\therefore W = \frac{1.242 \times 10^{-6}}{\lambda}$ eV

3) TYPES OF EMISION

a) PHOTOEMISSION - BY LIGHT

b) SECONDARY EMM. - BY E. OF OTHER e'S

c) FIELD EMISSION - BY ELECTRIC FIELD

d) THERMIONIC EMISSION - BY HEAT

4) CURRENT AS FUCTION OF TEMP

$$i(T) = aAT^2 e^{-b/T}$$

a) $T = °K$ b) a, b = CONSTANT CHARAC. OF MATERIAL

c) A = SURFACE AREA OF MATERIAL

d) ELECTRIC FIELD STRENGTH AT CATH. MUST BE GREAT ENOUGH TO REMOVE ALL THE EMITTED ELEC. ie, IFF ANODE-CATHODE V IS SUFFIENTLY HIGH, i VARIES WITH T

5) CHILD - LANGMUIR LAW

$$i(V) = 2.33 \times 10^{-6} \frac{A}{d^2} V^{3/2}$$

FOR // PLANE CAPACITOR

a) d = DISTANCE BETWEEN THE TWO

b) A = AREA (SURFACE)

c) HOLD TO POINT WHERE EVERY ELECTRON HAS BEEN USED

D) METHODS OF REPRESENTING V-i RELATIONSHIPS - RARELY DO TRUE V-i RELATION. HAVE SIMPLE FUNCTIONS

E) DEFINITIONS - IDEAL

1) LUMPED CIRCUIT ELEMENT - REGION OF SPACE, WITH AT LEAST 2 TERMINALS

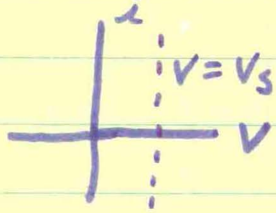
2) LUMPED CIRCUIT - A COLLECTION OF CONNECTED LUMPED CIRCUIT ELĒ.

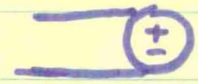
3) CIRCUIT ANALYSIS - STUDY OF V-i RELATIONSHIP IN LUMPED CIRCUIT

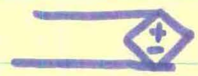
4) VOLTAGE SOURCE

a) A LUMPED 2 TERMINAL CIR. ELE.

b) $V =$ CONSTANT $= V_S$ FOR ALL i



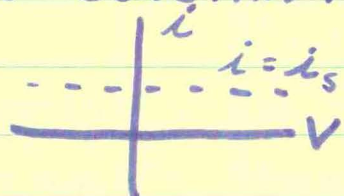
c) INDEPENDENT:  V_S

d) DEPENDENT:  V_S

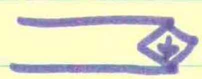
5) CURRENT SOURCE

a) A LUMPED 2 TERMINAL CIR. ELE.

b) $i =$ CONSTANT $= i_s$



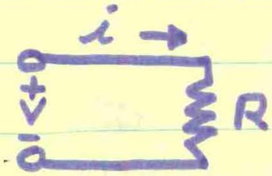
c)  i_s - INDEP.

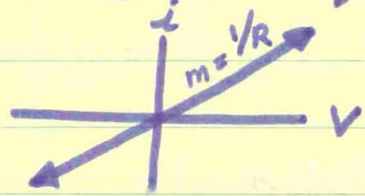
d)  i_s - DEPEN.

6) LINEAR RESISTOR

a) A LUMPED 2 TERMINAL CIR. ELE

b) $V = iR$ (OHM'S LAW)





c) MEASURED IN OHMS (Ω) ($\frac{\text{VOLT}}{\text{AMP}} = \Omega$)

d) POWER INTO A RESISTOR ALWAYS HAS THE SAME SIGN AS R

e) FOR NONLINEAR RESISTORS:

1) $R = dV/di$

2) R IS RECIPRICAL OF V-i SLOPE

7) IDEAL DIODE - 2 TERMINAL L.C.E.

a) CONDITIONS

1) $V=0 \quad i>0$

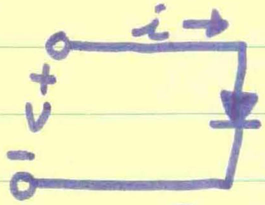
2) $i=0 \quad V<0$

3) $Vi=0$

b)

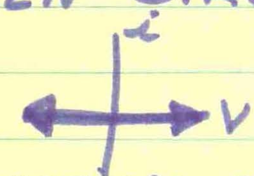
(PIECEWISE LINEAR)

c)



8) OPEN CIRCUIT - $i=0$ FOR ALL V

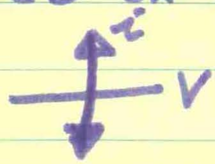
a)



b) SPECIAL CASE OF CURRENT SOURCE

9) CLOSED OR SHORT CIRCUIT - $V=0$ FOR ALL i

a)

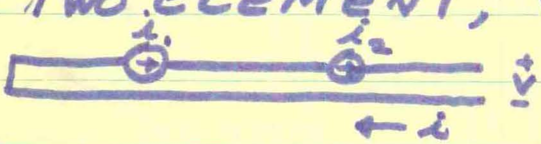


b) SPECIAL CASE OF VOLTAGE SOURCE

10) SERIES CONNECTION

a) TWO ELEMENT, $\forall \; i=i_1=i_2$

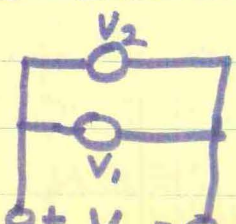
b)



11) PARALLEL CONNECTION

a) TWO ELEMENTS $\forall \; V=V_1=V_2$

b)



III) NETWORK SOLUTIONS

A) USEFUL DEFINITIONS

1) NODE - POINT AT WHICH 2 OR MORE CIRCUIT ELEMENTS MEET

2) BRANCH - PART OF NETWORK FROM ONE NODE TO ANOTHER (LUMPED CIR. ELE.)

3) LOOP - A SEQUENCE OF CIRCUIT ELEMENTS FORMING A CLOSED PATH

4) MESH - A LOOP WITH NO CIR. ELE. INSIDE

5) PLANAR NETWORK - ANY NETWORK THAT CAN BE DRAWN ON PAPER WITHOUT CROSSING LINES

6) ONE PORT NETWORK - 2 TERMINALS

B) KIRCHHOFF'S LAWS & SUCH

1) IN A CIRCUIT, ONE NEEDS 2 EQUATIONS FOR EACH ELEMENT TO SOLVE FOR V & i

2) SUM OF V AROUND CLOSED PATH = 0

$$\sum_{i=1}^{N} V_i = 0$$

3) SUM OF CURRENT ENTERING NODE = 0

$$\sum_{j=1}^{M} i_j = 0$$

4) MAY APPLY TOWARD SOLVING V & i

C) MESH & NODE TO DATUM

1) MESH

a) SOLVE FOR V USING CURRENT LAW ($V = iR$)

b) MUST DEFINE VOLTAGES ACROSS ALL CURRENT SOURSES

2) NODE TO DATUM

a) DEFINE DATUM ($V=0$)

b) PUT CURRENTS THRU ALL VOL. SOURSES

c) USE ~~VOL~~ CURRENT LAW TO FIND VOLTAGES ($i = V/R$)

D) NON-LINEAR ELEMENTS

1) KIRCHHOFF'S LAWS ~~DON'T~~ ARE EFFECTIVE.

2) SOLVE GRAHPHICALLY, INSTEAD OF ALGEBRAICALLY.

E) CIRCUITS CONTAINING 2 OR MORE NON-LINEAR ELEMENTS MAY BE ADDED GRAPHICALLY AS ONE

F) LINEARITY AND SUPERPOSITION

1) LINEARITY - PROPERTY OF HAVING A v-i RELATIONSHIP IN FORM $i = aV + b$ OR $V = ci + d$. ⇒ GRAPHICALLY, A STRAIT LINE (IF EVERY SOURCE IN CIRCUIT IS DOUBLED, V & i ACROSS EVERY BRANCH WILL DOUBLE

2) SUPERPOSITION PRINCIPLE - THE RESPONSE OF ANY <u>LINEAR</u> NETWORK TO MORE THAN ONE INDEPENDENT EXCITATION IS GIVEN BY THE SUM OF THE RESPONSES TO EACH EXICATION ACTING ALONE

IV) NETWORK EQUIVALENCE

A) DEFINITION - 2 NETWORKS ARE EQUIVALENT IFF THEY HAVE SAME V-i RELATIONSHIP

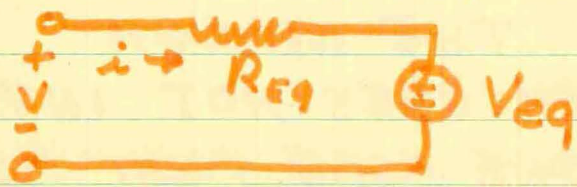
B) TABLE OF EQUIVALENCE OF LUMPED CIRCUIT ELEMENTS

ELEMENTS	SERIES	PARALLEL
V_{s1}, V_{s2}	$V_s = V_{s1} + V_{s2}$	IMPOSSIBLE
I_{s1}, I_{s2}	IMPOSSIBLE	$I_s = I_{s1} + I_{s2}$
R_1, R_2	$R_{EQ} = R_1 + R_2$	$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2}$
V_{s1}, R_2	UNCHANGED	$V_s = V_{s1}$
V_{s1}, I_{s2}	$I_s = I_{s2}$	$V_s = V_{s1}$
I_{s1}, R_2	$I_s = I_{s2}$	UNCHANGED

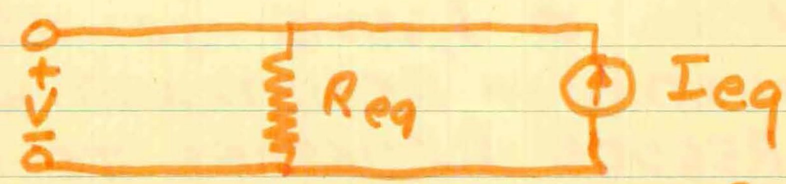
C) THE ULTIMATE REDUCTION: THÉVENIN & NORTON EQUIVALENTS

1) ANY LINEAR NETWORK MAY BE REDUCED TO THESE EQUIVALENTS

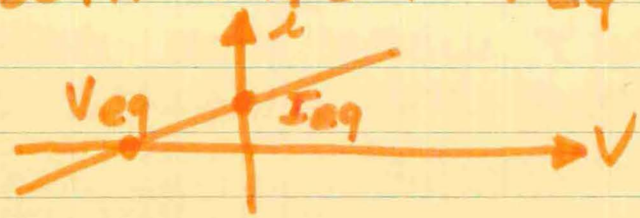
2) THÉVENIN EQUIVALENT:



3) NORTON EQUIVALENT:



4) IN BOTH CASES: $V_{eq} = R_{Eq} I_{eq}$



D) METHODS OF REDUCTION TO EQUIVALENTS

1) USING DIRECT DETERMINATION OF a V-i EQUATION
   a) THÉVENIN EQ → $V = ai + b$
      NORTON EQ → $i = cV + d$
   b) $R_{EQ} = a = 1/c$
   c) $V_{eq} = b$
   d) $I_{eq} = -d$
2) DETERMINATION OF V-i CURVE BY 2 PTS.
   a) $R_{EQ} = (V_2 - V_1)/(i_2 - i_1)$
   b) $V_{EQ} = V_1 - R_{eq} i_1 = V_2 - R_{EQ} i_2$
   c) $I_{EQ} = V_1/R_{EQ} - i_1 = V_2/R_{EQ} - i_2$
3) USE OF OPEN CIRCUIT V & SH. CIR. CURRENT
   a) ~~YIELD INTERCEPTS OF V-i GRAPH~~
   b) USING $(V_{oc}, 0)$ AND $(0, i_{sc})$
      1) $R_{eq} = -V_{oc}/i_{sc}$
      2) $V_{eq} = V_{oc}$
      3) $I_{eq} = \cancel{-}i_{sc}$
   c) DOESN'T WORK IF INDEPENDENT SOURSE IS IN THE CIRCUIT
   d) IF GRAPH DOES NOT INTERSECT ORIGIN, ONE MORE POINT MUST BE DETERMND. MAY, IF CONVENIENT, SET i OR V TO 1, AND SOLVE
4) REDUCTION OF RESISTANCE WITH V_{oc} OR i_{sc}
   a) REDUCE RESISTORS TO $R_{EQ} = 1/\text{SLOPE}$
   b) USE V_{oc} OR i_{sc} TO DETERMINE POINT
   c) DOESN'T WORK FOR DEP. SOURSES
5) USE TABLE TO REDUCE SUPERFLOUS COMBINATIONS

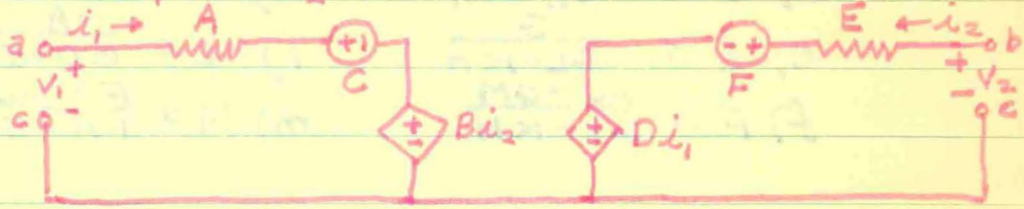
E) NETWORKS WITH THREE NODE SPECIFIED

1) IN T FORM

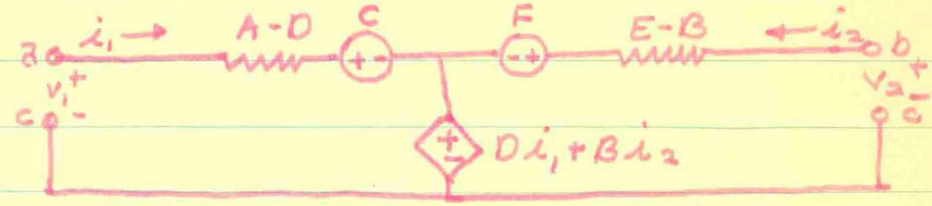
a) $V_1 = Ai_1 + Bi_2 + C$

$V_2 = Di_1 + Ei_2 + F$

b) 1)

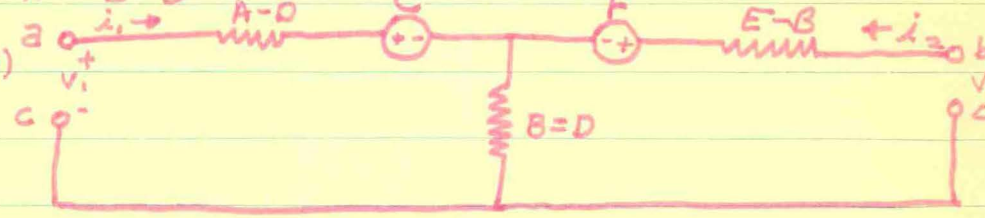


2)



c) IF $B=D$ (KNOWN AS RECIPROCY)

1)



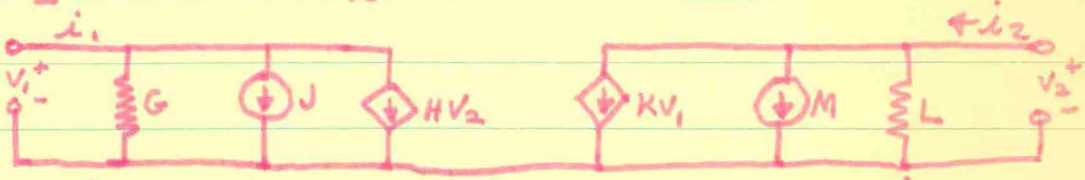
2) OCCURS WHEN INITIAL CIRCUIT IS COMPOSED OF LINEAR AND INDEPENDENT CIRCUIT ELEMENTS.

2) IN π FORM

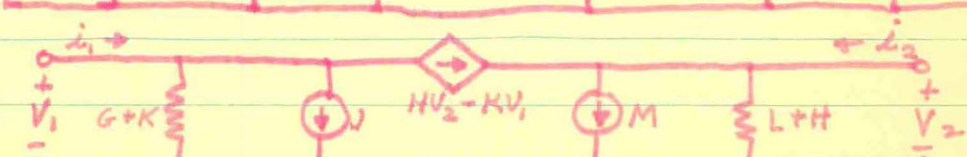
a) $i_1 = GV_1 + HV_2 + J$

$i_2 = KV_1 + LV_2 + M$

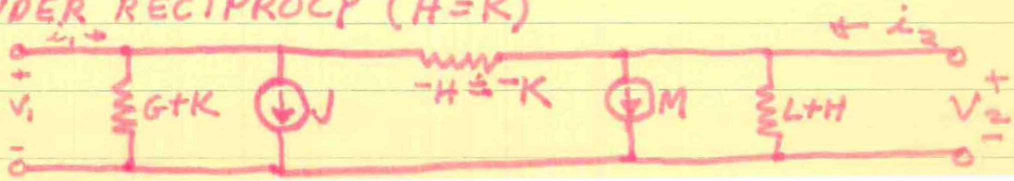
b) 1)



2)



c) UNDER RECIPROCY ($H=K$)



3) SOLVING ALGEBRICALLY FOR COEFF. OF T AND π EQUIVALENTS

a) $A = \frac{L}{LG - KH}$

b) $B = \frac{-H}{LG - KH}$

c) $C = \frac{MH - JL}{LG - KH}$

d) $D = \frac{-K}{GL - KH}$

e) $E = \frac{G}{GL - KH}$

f) $F = \frac{KJ - GM}{GL - KH}$

g) $G = \frac{E}{EA - BD}$

h) $H = \frac{-B}{EA - BD}$

j) $J = \frac{BF - CE}{EA - BD}$

k) $K = \frac{D}{BD - EA}$

l) $L = \frac{-A}{BD - EA}$

m) $M = \frac{FA - CD}{BD - EA}$

V) ENERGY STORAGE ELEMENTS

A) THE CAPACITOR

1) GAUSS' LAW & SUCH

   a) $\oint_s D_n \, ds = \int \rho \, dv = q$

      D_n = COMPONENT OF THE ELECTRICAL DISPLACEMENT NORMAL TO THE ENCLOSING CIRCUIT

   b) IF SYSTEM IS LINEAR:

      $D = \epsilon E$

      (ϵ = PERMITTIVITY OF THE MEDIUM

   c) $q = Cv$ (C = CAPACITANCE)

      MKS FOR C IS FARAD = $\frac{\text{COUL}}{\text{VOLT}}$

   d) CAPACITANCE

      1) IN LINEAR SYSTEM, ϵ WILL APPEAR IN THE EXPRESSION ON C

      2) C DEPENDS OF SYSTEM'S GEOM.

      3) IN A LINEAR MEDIUM, C IS INDEPENDENT OF q, V, AND i

      4) IF NON-LINEAR, $C = dq/dv$

   e) PERMITIVITY CONSTANT & SUCH

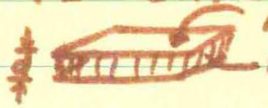
      $\varepsilon = \varepsilon_R \varepsilon_0$

      1) ε = PERMITIVITY OF MATERIAL

      2) ε_0 = " CONSTANT ($= 8.85 \times 10^{-12} \frac{F}{m}$)

      3) ε_R = RELATIVE PERMITIVITY OF MATERIAL (> 1)

2) FOR A // PLATE CAPACITOR



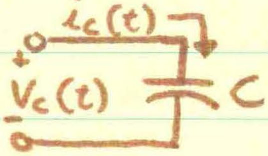
a) $E = \frac{q}{\epsilon A}$

b) $V = Ed = \frac{qd}{\epsilon A}$

c) $C = \frac{q}{V} = \frac{\epsilon A}{d}$

3) CAPACITOR - DEFINITION & SUCH

a) V-i RELATIONSHIP → $i_c(t) = C \frac{d V_c(t)}{dt}$



OR $V_c(t) = \frac{1}{C} q(t)$

→ b) $V_c(t) = \frac{1}{C} \int i_c(t)\,dt + k$

(K SUMMERIZES EFFECT OF V_C BEFORE $i_c(t)$ WAS KNOWN)

c) POWER IN A CAPACITOR

$P(t) = V_c(t)\, i_c(t) = V_c(t)\, C \frac{dV_c(t)}{dt}$

d) WORK (ENERGY STORED)

$$W_C = \int_0^{V_c(t)} C V_c(t)\, dV_c(t)$$

$$= \frac{1}{2} C V_c^2(t) = \frac{1}{2} q V_c(t) = \frac{1}{2} \frac{q^2(t)}{C}$$

e) WHEN V=0, CAP. ACTS AS OPEN CIRCUIT

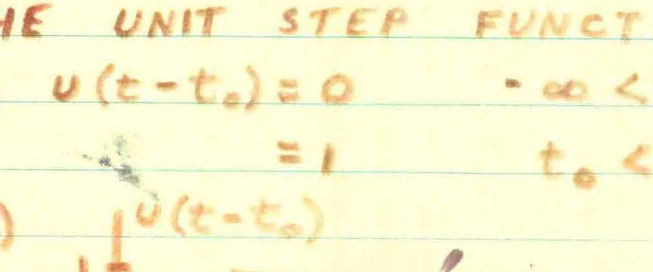
f) THERE ARE NO DISCONTINUITIES IN A CAPAC UNLESS $i = \infty$

B) THE UNIT STEP FUNCTION

1) $u(t - t_0) = 0 \quad -\infty < t < t_0$

$= 1 \quad t_0 < t < \infty$

2)

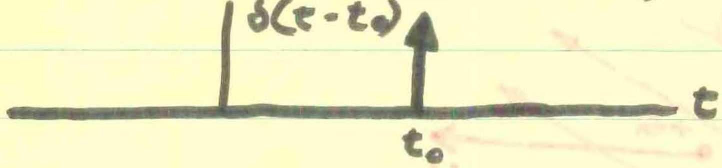


C) UNIT IMPULSE FUNCTION δ

1) $u(t-t_0) = \int_{-\infty}^{t} \delta(t-t_0)\,dt$

2) a) $\delta(t-t_0) = 0$ FOR $t < t_0$; $t > t_0$

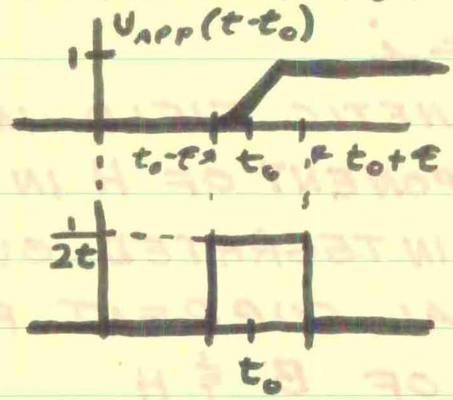
b)



3) $\int_{-\infty}^{\infty} \delta(t-t_0)\,dt = 1 \Rightarrow$ AREA OF UNIT IMPUSE $(0 \cdot \infty)$ IS UNITY

4) BY APPROXIMATION

a)



b) $u(t_0+t) = \lim_{\tau \to 0} U_{APP}(t-t_0)$

c) $\delta(t-t_0) = \lim_{\tau \to 0} \frac{d}{dt}\left[U_{APP}(t-t_0)\right]$

D) THE MAGNETIC ~~FLUX~~ FIELD

1) THE MAGNETUDE OF FORCE ON A POINT CHARGE MOVING IN A MAGNETIC AT A VELOCITY

$$F = QUB\sin\theta \;(= Q\vec{U} \times \vec{B})$$

a) B = MAGNETIC FLUX DENSITY $\left(\frac{\text{WEBER}}{m^2} = \text{TELSA}\right)$

b) Q = POINT CHARGE

c) U = VELOCITY OF PARTICLE

d) θ = ANGLE BETWEEN U AND B

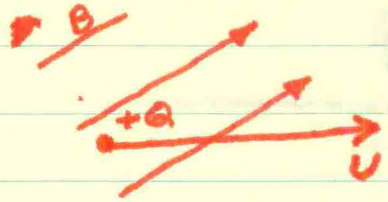
e) DIRECTION OF F DETERMINED BY RIGHT HAND SCREW RULE.

2) IF MAGNETIC FLUX DENSITY ($\vec{B}$) IS PRESENT:

a) $\Phi = \int_S B_n \, dS$ (= FLUX)

1) B_n = COMPONENT OF B NORMAL TO SURFACE ELEMENT

2) S = SURFACE

3) 

b) WHEN COMPUTED AROUND A CLOSED SURFACE, $\Phi = 0$

3) AMPERE'S LAW:

$$\oint H_l \, dl = i$$

a) H = MAGNETIC FIELD INTENSTITY $\left(\frac{AMP}{m}\right)$

b) H_l = COMPONENT OF H IN THE DIRECTION OF dl, INTEGRATED OVER A CLOSED PATH

c) $\hat{i}$ = TOTAL CURRENT PASSING THRU

4) RELATION OF B & H

a) IN FREE SPACE; $B = \mu_0 H$

$\left(\mu_0 = 4\pi \times 10^{-7} \frac{HENRIES}{METER}\right)$

b) IF THE MATERIAL IS LINEAR, HOMOGENEOUS, AND ISOTROPIC:

1) $B = \mu H$

2) $\mu = \mu_R \mu_0$

(a) μ_R = DIMINSIONLESS RELATIVE PERMEABILITY)

E) FARADAY'S LAW

1) $\oint E_\ell \, d\ell = -\frac{d}{dt}\int_S B_n \, ds = -\frac{d\Phi}{dt}$

2) FOR A LOOP OF WIRE:

a) $\oint E_\ell \, d\ell = \frac{2\pi r_0^2 B_0}{T}$

b) OR $E_{\ell_{AVE}} = \frac{r_0 B_0}{T}$

3) $\int B_n \, dS = N\Phi$

F) THE INDUCTOR

1) IN A COIL: a) $L = \frac{N\Phi}{I}$

b) NON-LINEAR: $L = \frac{d(N\Phi)}{di}$

2) FOR A SOLENOID: 

$L = \mu n^2 d \pi a^2$

3) $V = \frac{d(Li)}{dt} = \frac{i\,dL}{dt} + \frac{L\,di}{dt}$

4) DEFINITION: $N\Phi = Li$; A LUMPED TWO TERMINAL CIRCUIT ELEMENT. V-i RELATIONSHIP $V_L(t) = L\frac{di_L(t)}{dt}$

5) $i(t) = \frac{1}{L}\int_0^t V_L(t)\,dt + k$

$(K = \frac{1}{L}\int_{-\infty}^{t_0} V_L(t)\,dt$

F) REDUCTION OF SERIES AND PARALLEL COMBINATIONS OF CAPACITORS AND INDUCTORS

1) CAPACITORS

a) IN PARRALLEL: $C_{eq} = C_1 + C_2$

b) IN SERIES: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

2) INDUCTORS

a) IN PARRALLEL: $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$

b) IN SERIES: $L_{eq} = L_1 + L_2$

6) MOST IMPORTANT PLUGS AND CONCEPTS

1) $i_c(t) = C \frac{d V_c(t)}{dt}$

2) $V_c(t) = \frac{1}{C} \int_0^{t_0} i_c(t) dt + k$

3) $p_c(t) = V_c(t) \, C \frac{d V_c(t)}{dt}$

4) CAPACITOR ACTS AS AN OPEN CIRCUIT WHEN THE VOLTAGE ACROSS IT IS NOT CHANGING

5) A CAPACITOR CAN NOT HAVE AN IMEDIATE VOLTAGE CHANGE ACROSS IT

6) $V_L = L \frac{di}{dt} + i \frac{dL}{dt}$

7) $i_L = \frac{1}{L} \int_0^t V_L(t) dt + k$

8) AN INDUCTOR BEHAVES AS A SHORT CIRCUIT WHEN THE CURRENT THRU IT IS NOT CHANGING

9) THERE CAN BE NO INSTANTANEOS CURRENT CHANGE THRU AN INDUCTOR

VI) INTRODUCTION TO RL & RC CIRCUITS

A) WAVEFORM TERMINOLOGY

1) V(t) IS PERIODIC IFF:

a) $v(t) = v(t+T)$ FOR ALL t

b) T = PERIOD (OF REPITITION)

2) $\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)\,dt < \infty > \frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\,dt$

3) ANY WAVEFORM THAT IS NOT PERIODIC IS CALLED APERIODIC

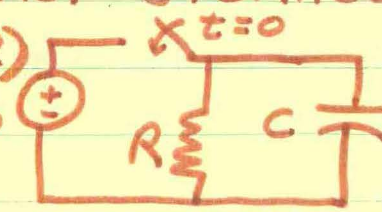
4) SOME ARE "ALMOST PERIODIC"

EX: $\sin t + \sin \pi t$

5) RANDOM (ALL MIXED UP)

B) SUDDEN CHANGES IN NETWORKS CONTAINING A DC. SOURCE, A RESISTOR, AND AN ENERGY STORAGE ELEMENT.

1) a)



NTD YIELDS:

$\frac{v}{R} + C\frac{dv}{dt} = 0$; $t > 0$

OR $v(t) = e^{K} e^{-t/RC} \mu(t)$

AT $t(0-)$; $v(0^-) = V_0$

AT $t(0+)$; $v(0^+) = V_0$

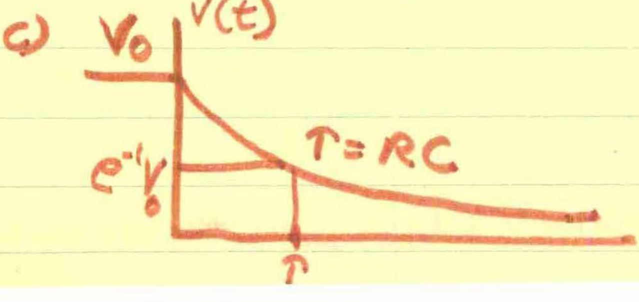
AT $t(\infty)$; $v(\infty) = V_0$

$\therefore v(t) = V_0\,\mu(-t) + V_0\, e^{\frac{-t}{RC}} \mu(t)$

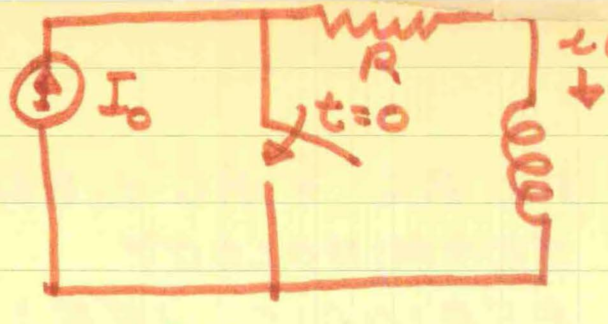
b) τ = (TIME CONSTANT =) RC

AT τ, FUNCTION DROPS ONE "eTH," OR ABOUT 36.8%

c)



2) a)



BY MESH:

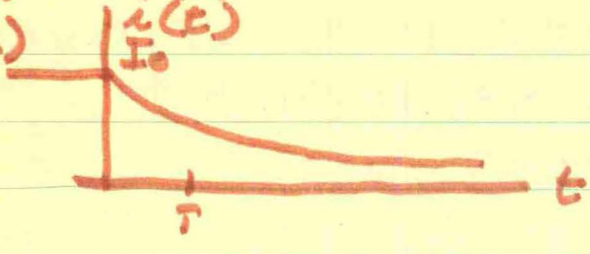
$$R i(t) + L\frac{d i(t)}{dt} = 0$$

OR

$$i(t) = I_0 e^{-\frac{Rt}{L}} \mu(t)$$

b) $\tau = \frac{L}{R}$

c)



c) MORE GENERAL CIRCUITS CONTAINING ONE ENERGY STORAGE ELEMENT

1) ALL CIRCUITS CONTAINING ONE E.S.E. CAN HAVE AN EASILY DERIVED τ

   a) $\tau = R_{eq}C$ OR $\tau = \frac{L}{R_{eq}}$

   b) R_{eq} AS SEEN FROM ELEMENT'S TERMINALS

2) DETERMINE "4 NUGGETS OF KNOWLEDGE"

   a) $Y(0-)$

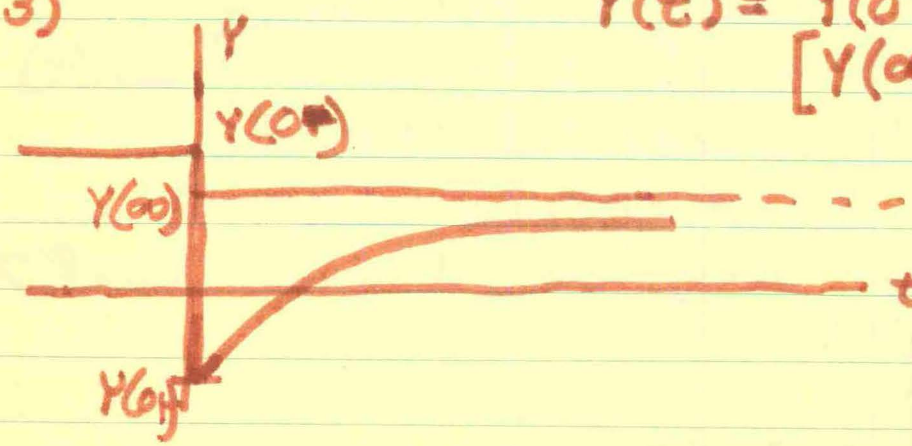
   b) $Y(0+)$

   c) $Y(\infty)$

   d) τ

3)

$$Y(t) = Y(0^-)\mu(-t) + \left[Y(\infty) + \{Y(0) - Y(\infty)\}e^{-\frac{t}{\tau}}\right]\mu(t)$$



D) GENERAL OUT LINE FOR FINDING FOUR NUGGETS OF KNOWLEDGE

1) $Y(0-)$ - FIND THE INITIAL STEADY STATE VALUE OF V OR i FOR THE NETWORK IN ITS CONDITION PRIOR TO CHARGE
   $C \cdot i$ AND $L \cdot V = 0$
2) $Y(0+)$ - REPEAT ABOVE FOR $t > 0$, AGAIN. ~~SWITCH~~ + AGAIN
   $C - i$ AND $L - V = 0$
3) DETERMINE Y JUMP AT INSTANT OF CIRCUIT CHANGE, CAPACITOR VOLTAGE OR INDUCTOR CURRENT MUST REMAIN CONSTANT OVER THIS SHORT INTERVAL OF TIME
4) COMPUTE τ (= RC OR $\frac{L}{R}$)
5) $Y(\infty)$ AGAIN CAP. i AND $L \cdot V$ IS ZERO

$$Y(t) = Y(0^-)\mu(t) + \left[Y(0) + \left\{Y(\infty) + Y(0)e^{\frac{t}{\tau}}\right\}\right]\mu(t)$$

V FOR INDUCTORS
i FOR CAPACITOR

VII) SINUSOIDAL STEADY STATE ANALYSIS

A) THE PHASOR REPRESENTATION OF A SINUSOID

1) CONSIDER $i = I_m \cos(\omega t + \phi)$
   a) PERIOD $T = \frac{2\pi}{\omega}$
   b) FREQUENCY $f = \frac{1}{T}$
   c) ANGULAR OR RADIAN FREQUENCY ω
   d) ϕ IS THE PHASE ANGLE
2) SOME IDENTITIES
   a) $e^{j\theta} = \cos\theta + j\sin\theta$
   b) $\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$
   c) $\sin\theta = \frac{-j}{2}(e^{j\theta} - e^{-j\theta})$
   d) $\therefore I_m \cos(\omega t + \phi) = i = Re\left[I_m e^{j(\omega t + \phi)}\right]$
   (Re DENOTES REAL PART OF)
3) PHASOR - FOR ANY SINUSOIDAL FUNCTION OF TIME, $a(t) = A_m \cos(\omega t + \phi)$, THE COMPLEX QUANTITY, $A = A_m \angle\phi = A_m e^{j\theta} = A_m \cos\phi + j A_m \sin\phi$ IS THE PHASOR REPRESENTATION OF $a(t)$, HENCE
   $a(t) = A_m \cos(\omega t + \phi) = Re(A e^{j\omega t})$
   WHERE
   $A = A_m e^{j\phi} = A_m \angle\phi = A_m \cos\phi + j A_m \sin\phi$

B) PHASOR v-i RELATIONSHIPS

1) IF $i = i_1 \pm i_2 \Rightarrow I = I_1 \pm I_2$
2) IF $V = Ri \Rightarrow V = RI$
3) IF $V = L\frac{di}{dt} \Rightarrow V = Lj\omega I$
4) IF $V = \frac{1}{C}\int i\,dt \Rightarrow V = \frac{-j}{C\omega} I$

C) IMPEDANCE

1) FOR R, L, C

a) $V = RI$

b) $V = j\omega L I$

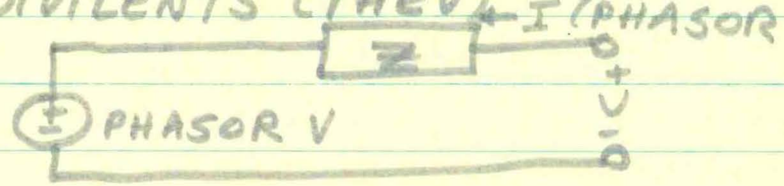
c) $V = I / j\omega C$

2) DEFINITION: FOR 2 TERMINAL NETWORK COMPOSED OF IDEAL CIRCUIT ELEMENTS, OPERATING IN SINOSOIDAL STEADY STATE, THE RATIO OF PHASOR VOLTAGE V ACROSS THE TERMINALS IT PHASOR CURRENT I:

$$V = ZI$$

(Z MEASURED IN OHMS)

D) EQUIVILENTS (THEV)



E) REACTENCE: $Z = R + jX$ (X = REACT.)

F) ADMITTANCE $Y = 1/Z$

G) SUBCEPTANCE B: $Y = G + jB$

H) POWER - RMS VALUE

1) FOR ANY PERIODIC WAVE FORM

$$P_{AV} = \overline{p(t)} = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)\, i(t)\, dt$$

2) FOR SINUSOIDAL WAVE

$$P_{AV} = \frac{1}{2} V_m I_m \cos(\beta - \alpha)$$

3) FOR A SOURCELESS 2 TER. CIR.

$$P_{AVE} = \frac{1}{2} I_m^2 R_{eq} = \frac{1}{2} V_m^2 G_{EQ}$$

I) RMS VALUE (TIME INDEPENDENT)

a) $P_{AV} = I_{RMS}^2 R = \frac{R}{T} \int_{t_0}^{t_0+T} i^2(t)\,dt$

b) $I_{RMS} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} i^2(t)\,dt}$

c) $V_{RMS} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} v^2(t)\,dt}$

d) IF $v(t)$ OR $i(t)$ IS SINUSOIDAL:

1. $V_{RMS} = V_m / \sqrt{2}$ (SIN WAVE)
2. $I_{RMS} = I_m / \sqrt{2}$

COMPLEX NUMBERS
POLAR FORM CORRESPONDING TO RECTANGULAR FORM 1 + J Y

Y	MAGNITUDE	PHASE ANGLE		Y	MAGNITUDE	PHASE ANGLE		Y	MAGNITUDE	PHASE ANGLE	
		RADIANS	DEGREES			RADIANS	DEGREES			RADIANS	DEGREES
0.025	1.000	0.0249	1.432	1.274	1.620	0.9056	51.892	2.599	2.785	1.2036	68.962
0.050	1.001	0.0499	2.862	1.299	1.640	0.9150	52.431	2.699	2.879	1.2160	69.676
0.075	1.002	0.0748	4.289	1.324	1.660	0.9242	52.957	2.799	2.973	1.2277	70.346
0.099	1.004	0.0996	5.710	1.349	1.680	0.9332	53.471	2.899	3.067	1.2387	70.974
0.124	1.007	0.1243	7.125	1.374	1.700	0.9419	53.972	2.999	3.162	1.2490	71.564
0.149	1.011	0.1488	8.530	1.399	1.720	0.9505	54.462	3.099	3.257	1.2587	72.121
0.174	1.015	0.1732	9.926	1.424	1.740	0.9588	54.940	3.199	3.352	1.2679	72.645
0.199	1.019	0.1973	11.309	1.449	1.761	0.9670	55.407	3.299	3.448	1.2765	73.141
0.224	1.025	0.2213	12.680	1.474	1.782	0.9750	55.863	3.399	3.543	1.2847	73.610
0.249	1.030	0.2449	14.036	1.499	1.802	0.9827	56.309	3.499	3.640	1.2924	74.054
0.274	1.037	0.2683	15.376	1.524	1.823	0.9903	56.745	3.599	3.736	1.2998	74.475
0.299	1.044	0.2914	16.699	1.549	1.844	0.9978	57.171	3.699	3.832	1.3068	74.875
0.324	1.051	0.3142	18.004	1.574	1.865	1.0050	57.587	3.799	3.929	1.3134	75.256
0.349	1.059	0.3366	19.290	1.599	1.886	1.0121	57.994	3.899	4.026	1.3197	75.618
0.374	1.068	0.3587	20.556	1.624	1.908	1.0191	58.392	3.999	4.123	1.3258	75.963
0.399	1.077	0.3805	21.801	1.649	1.929	1.0259	58.781	4.099	4.220	1.3315	76.292
0.424	1.086	0.4018	23.025	1.674	1.950	1.0325	59.162	4.199	4.317	1.3370	76.607
0.449	1.096	0.4228	24.227	1.699	1.972	1.0390	59.534	4.299	4.414	1.3422	76.907
0.474	1.107	0.4434	25.407	1.724	1.993	1.0454	59.898	4.399	4.512	1.3473	77.195
0.499	1.118	0.4636	26.565	1.749	2.015	1.0516	60.255	4.499	4.609	1.3521	77.471
0.524	1.129	0.4834	27.699	1.774	2.037	1.0577	60.603	4.599	4.707	1.3567	77.735
0.549	1.141	0.5028	28.810	1.799	2.059	1.0636	60.945	4.699	4.805	1.3611	77.988
0.574	1.153	0.5218	29.898	1.824	2.081	1.0695	61.279	4.799	4.903	1.3653	78.231
0.599	1.166	0.5404	30.963	1.849	2.102	1.0752	61.606	4.899	5.000	1.3694	78.465
0.624	1.179	0.5585	32.005	1.874	2.124	1.0808	61.927	4.999	5.098	1.3733	78.689
0.649	1.192	0.5763	33.023	1.899	2.147	1.0863	62.241	5.199	5.295	1.3808	79.114
0.674	1.206	0.5937	34.019	1.924	2.169	1.0916	62.548	5.399	5.491	1.3876	79.508
0.699	1.220	0.6107	34.991	1.949	2.191	1.0969	62.850	5.599	5.688	1.3940	79.875
0.724	1.235	0.6273	35.942	1.974	2.213	1.1020	63.145	5.799	5.885	1.4000	80.217
0.749	1.249	0.6435	36.869	1.999	2.236	1.1071	63.434	5.999	6.082	1.4056	80.537
0.774	1.265	0.6593	37.775	2.024	2.258	1.1120	63.718	6.199	6.280	1.4108	80.837
0.799	1.280	0.6747	38.659	2.049	2.280	1.1169	63.996	6.399	6.477	1.4157	81.119
0.824	1.296	0.6897	39.522	2.074	2.303	1.1217	64.269	6.599	6.675	1.4204	81.384
0.849	1.312	0.7044	40.364	2.099	2.325	1.1263	64.536	6.799	6.873	1.4247	81.634
0.874	1.328	0.7188	41.185	2.124	2.348	1.1309	64.798	6.999	7.070	1.4288	81.869
0.899	1.345	0.7328	41.987	2.149	2.371	1.1354	65.055	7.199	7.269	1.4327	82.092
0.924	1.362	0.7464	42.768	2.174	2.393	1.1398	65.308	7.399	7.467	1.4364	82.303
0.949	1.379	0.7597	43.531	2.199	2.416	1.1441	65.555	7.599	7.665	1.4399	82.504
0.974	1.396	0.7727	44.274	2.224	2.439	1.1484	65.798	7.799	7.863	1.4432	82.694
0.999	1.414	0.7853	44.999	2.249	2.462	1.1525	66.037	7.999	8.062	1.4464	82.874
1.024	1.431	0.7977	45.707	2.274	2.485	1.1566	66.271	8.199	8.260	1.4494	83.046
1.049	1.449	0.8097	46.397	2.299	2.507	1.1606	66.501	8.399	8.459	1.4523	83.210
1.074	1.468	0.8215	47.069	2.324	2.530	1.1646	66.726	8.599	8.657	1.4550	83.367
1.099	1.486	0.8329	47.726	2.349	2.553	1.1684	66.948	8.799	8.856	1.4576	83.516
1.124	1.505	0.8441	48.366	2.374	2.576	1.1722	67.166	8.999	9.055	1.4601	83.659
1.149	1.523	0.8550	48.990	2.399	2.599	1.1760	67.380	9.199	9.254	1.4625	83.796
1.174	1.542	0.8656	49.600	2.424	2.623	1.1796	67.590	9.399	9.452	1.4648	83.927
1.199	1.562	0.8760	50.194	2.449	2.646	1.1832	67.796	9.599	9.651	1.4670	84.053
1.224	1.581	0.8861	50.774	2.474	2.669	1.1868	67.999	9.799	9.850	1.4691	84.173
1.249	1.600	0.8960	51.340	2.499	2.692	1.1902	68.198	9.999	10.049	1.4711	84.289

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CHAPTER 1

15) a) $i = \frac{q}{t}$

$= \frac{10^{11}\,e}{m} \frac{(3 \times 10^{7}\,m)}{s} \frac{(1.06 \times 10^{-19}\,\text{COUL})}{e}$

$= 3.18 \times 10^{-1}\ \frac{\text{COUL}}{\text{SEC}}$

$= .318$ AMPS

.48

b) $V = \frac{W_{P2} - W_{P1}}{Q}$

$W_{P1} = 0$

$V = \frac{W_{P2}}{Q} = \frac{\Delta K.E.}{Q}$

$= \frac{mu^2}{2Q}$

$$V = \frac{(9.11 \times 10^{-31}\,kg)(3 \times 10^{7}\,m)^2}{2\,(1.06 \times 10^{-19}\,\text{COUL})\,\text{SEC}^2}$$

$= 38.7 \times 10^{2}\ \frac{\text{JOULE}}{\text{COUL}}$

$= 3.87 \times 10^{3}$ VOLTS

2.156

c) $P = iV$

$= (.318\ \text{AMPS})(3.87 \times 10^{3}\ \text{VOLTS})$

$= 1.23 \times 10^{4}\ \text{WATTS} \left(\frac{1\ \text{KILOWATT}}{10^{3}\ \text{WATTS}}\right)$

$= 12.3$ KILOWATTS

$E = (12.3\ \text{KILOWATTS})(2\ \text{HOURS})$

$= 24.6$ KILOWATT-HOURS

16) a) $E_L = neV_0$ $\quad E_S = (\frac{1}{2}n)eV_0$
(ASSUMING $\frac{1}{2}$ THE ELECTRONS EMERGE FROM THE SMALLER FILAMENT)

$$\frac{E_S}{E_L} = \frac{\frac{1}{2}neV_0}{neV_0} = \frac{1}{2}$$

$$\therefore E_S = \frac{1}{2}E_L$$

b) $U_L = \sqrt{2eV_0/m_e}$ $\quad U_S = \sqrt{2eV_0/m_e}$

$$U_L = U_L$$

$$\frac{d}{U} = t$$

$$t_L = \frac{d\sqrt{m_e}}{\sqrt{2eV_0}} \qquad t_S = \frac{\frac{1}{2}d\sqrt{m_e}}{\sqrt{2eV_0}}$$

$$\frac{t_S}{t_L} = \frac{1}{2}$$

$$\therefore t_S = \frac{1}{2}t_L$$

c) FROM (b), $U_{Lx} = U_{Sx}$

$$U_{YL} = \frac{d_s U_0}{l_{ps}} \qquad U_{YS} = \frac{\frac{1}{2}d_s U_0}{\frac{1}{2}l_s}$$

$$U_{YL} = U_{YS}$$

$$\therefore U_{fL} = U_{fS}$$

d) $\cancel{V_{dL}}$

$$V_{d_L} = \frac{d_s\, 2V_0\, d_p}{l_p\, l_{ps}} \qquad V_{d_S} = \frac{(\frac{1}{2}d_s)2V_0(\frac{1}{2}d_p)}{(\frac{1}{2}l_p)(\frac{1}{2}l_{ps})}$$

$$\therefore V_{d_L} = V_{d_S}$$

$$d_{s_L} = U_{Y_L}\frac{l_{ps}}{U_{0_L}} \qquad d_{s_S} = U_{Y_S}\frac{(\frac{1}{2}l_{ps})}{U_{0_S}}$$

FROM C) $U_{YL} = U_{YS}$ $\quad U_{0_L} = U_{0_S} \Rightarrow \frac{d_{sS}}{d_{sL}} = \frac{1}{2}$

$$\therefore \left(\frac{d_s}{V_d}\right)_S = \frac{1}{2}\left(\frac{d_s}{V_d}\right)_L$$

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5) a) $i(V) = 2.33 \times 10^{-6} \frac{A}{d^2} V^{\frac{3}{2}}$

$\frac{A}{d^2} = \frac{i(V)}{(2.33 \times 10^{-6})\, V^{3/2}}$

$= \frac{.1 \times 10^6}{(2.33)(10^2)^{3/2}} = \frac{10^5}{2.33 \times 10^3} = \frac{10^2}{2.33} = .429 \times 10^2 = 42.9$

$i(V) = (2.33 \times 10^{-6})(42.9)(400)^{3/2}$
$= (2.33 \times 10^{-6})(42.9)(8. \times 10^3)$
$= .800$ AMPS

$i = \frac{\Delta q}{\Delta t}$

$\frac{\Delta q}{\Delta t} = .800$ AMPS

$\Delta t = 1$ SEC

$\Delta q = .800$ COULOMBS

$\# e = \Delta q \left(\frac{\# e}{q}\right) = (.800 \text{ COUL})\left(\frac{1\ e}{1.602 \times 10^{-19} C}\right)$
$= .753 \times 10^{+19}$ ELECTRONS
$= 7.53 \times 10^{18}$ ELECTRONS 5×10^{8}

b) $J = \frac{-4 \quad V^{3/2}}{9 x^2 (1.90 \times 10^5)}$

$J = \rho u$

$\rho = \frac{-4\, V^{3/2}}{9x^2 (1.90 \times 10^5)\, u}$

$\rho = \frac{-4\, V^{3/2}}{9x^2 (1.90 \times 10^5)(5.93 \times 10^5)\sqrt{V}}$

$\rho = \frac{q}{Vol}$

$q = \frac{-4\, V^{\frac{1}{2}}}{9\, d^2 (1.90 \times 10^5)(5.93 \times 10^5)} \left(\frac{d^2}{A}\right)\left(\frac{1}{d^4}\right)$

$= \frac{-4\sqrt{V}}{9(1.90 \times 10^{10})(5.93)\, d^6} \left(\frac{d^2}{A}\right)$

$= \frac{-4\,(20)}{9(1.90 \times 10^{10})(5.93)(10^{-18})(42.9)}$

$= .0184 \times 10^8$ COUL

$= 1.84 \times 10^6$ COUL

2.02×10^{-10}

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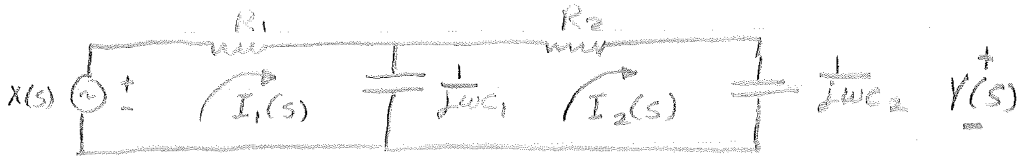
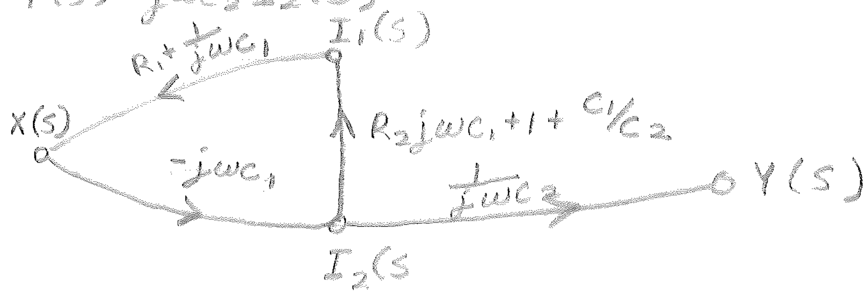
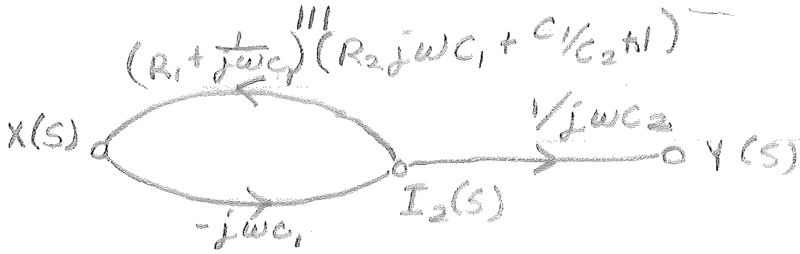
11-10-70



$$X(s) = I_1(s)\left[R_1 + \frac{1}{j\omega C_1}\right] + I_2(s)\left[\frac{-1}{j\omega C_1}\right]$$
$$I_2(s)\left[R_2 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}\right] = \frac{1}{j\omega C_1} I_1(s)$$
$$\Rightarrow I_1(s) = I_2(s)\left[R_2 j\omega C_1 + 1 + C_1/C_2\right]$$
$$Y(s) = \frac{1}{j\omega C_2} I_2(s)$$





$$\frac{Y(s)}{X(s)} = H(j\omega) = \frac{-j\omega C_1}{\left[1-\left(R_1+\frac{1}{j\omega C_1}\right)\left(R_2 j\omega C_1 + C_1/C_2 + 1\right)\right]} \; \frac{1}{j\omega C_2}$$
$$= \frac{C_1}{C_2\left[\left(R_1+\frac{1}{j\omega C_1}\right)\left(R_2 j\omega C_1 + \frac{C_1}{C_2} + 1\right) - 1\right]}$$

TEST # 1; CRAM FORMULAS

I) CHAPTER 2: SYSTEM REPRESENTATION AND ANALYSIS

A) n^{TH} ORDER DIFFER. EQ. (LINEAR):

$$a_n(t)\frac{d^nY(t)}{dt^n}+a_{n-1}(t)\frac{d^{n-1}Y(t)}{dt^{n-1}}+\dots+a_1(t)\frac{dY(t)}{dt}+a_0(t)Y(t)$$
$$=b_m(t)\frac{d^mX(t)}{dt^m}+b_{m-1}\frac{d^{m-1}X(t)}{dt^{m-1}}+\dots+b_1(t)\frac{dx}{dt}+b_0(t)X(t)$$

B) DEFINITION OF LINEAR SYSTEM:

$X_1(t)\Rightarrow Y_1(t)$ AND $X_2(t)\Rightarrow Y_2(t)$, THEN $aX_1(t)+bX_2(t)\Rightarrow aY_1(t)+bY_2(t)$

C) 1) $V(t)=L\frac{di(t)}{dt} \Leftrightarrow i(t)=\frac{1}{L}\int_{-\infty}^{t}V(t)dt$

2) $i(t)=C\frac{dV(t)}{dt} \Leftrightarrow V(t)=\frac{1}{C}\int_{-\infty}^{t}i(t)dt$

D) NORMAL FORM:

$$\frac{dq_1(t)}{dt}=a_{11}(t)q_1(t)+a_{12}(t)q_2(t)+\dots+a_{1n}(t)q_n(t)+b_1(t)X(t)$$
$$\frac{dq_2(t)}{dt}=a_{21}(t)q_1(t)+a_{22}(t)q_2(t)+\dots+a_{2n}(t)q_n(t)+b_2(t)X(t)$$
$$\vdots$$
$$\frac{dq_n(t)}{dt}=a_{n1}(t)q_1(t)+a_{n2}(t)q_2(t)+\dots+a_{nn}(t)q_n(t)+b_n(t)X(t)$$
$$Y(t)=C_1(t)q_1(t)+C_2(t)q_2(t)+\dots+C_n(t)q_n(t)$$

II) CHAPTER 3: REPRESENTATION OF SIGNALS

A) 1) ENERGY SIGNAL $\Rightarrow -\infty<\int_{-\infty}^{\infty}X^2(t)dt<\infty$

$E=\int_{t_1}^{t_2}X^2(t)dt$

2) POWER SIGNAL $\Rightarrow 0<\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}X^2(t)dt<\infty$

$P_{AVE}=\frac{1}{t_2-t_1}\int_{t_1}^{t_2}X^2(t)dt$

B) 1) $X(t)=\sum_{n=0}^{M}a_n\phi_n(t)$

2) ORTHOGONAL $\Rightarrow \int_{t_1}^{t_2}\phi_n(t)\phi_K(t)dt = 0 \;;K\neq N$

$\hookrightarrow$ FINALITY OF COEFFICIENTS $=\lambda_n \;;K=N$

3) $a_n=\frac{1}{\lambda_n}\int_{t_1}^{t_2}\phi_n(t)X(t)dt$

4) INTEGRAL SQUARE ERROR $\Rightarrow I=\int_{t_1}^{t_2}(X(t)-\hat{X}(t))^2dt$

5) ERROR ENERGY/SIGNAL EN $=1-\frac{1}{E}\sum_{n=0}^{M}\lambda_n a_n^2$

6) FOURIER SERIES:

a) $e^{\pm jn\omega_0 t}=\cos n\omega_0 t\pm j\sin n\omega_0 t$

b) $\omega_0=\frac{2\pi}{T}$

c) $\phi_n(t)=e^{jm\omega_0 t}$

d) $\lambda_n=T$

e) $\alpha_n=\frac{1}{T}\int_{t_1}^{t_1+T}X(t)e^{-jn\omega_0 t}dt$

f) $X(t)=\sum_{n=-\infty}^{\infty}\alpha_n e^{jn\omega_0 t}$ $\left(F.S.E=1-\frac{1}{E}\sum_{n=-M}^{M}\lambda_n|a_n|^2\right)$

(CONT.)

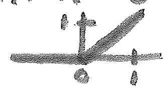
III) FROM CLASS NOTES

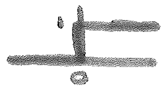
PARSEVAL'S THEOREM

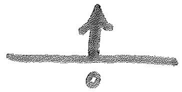
$$\int_{t_1}^{t_2} x^2(t)\,dt = \sum_{n=0}^{\infty} a_n^2 \lambda_n = E$$

TEST # 2; CRAM FORMULAS

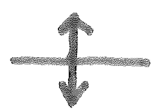
IV) CHAPT. 3: SINGULARITY FUNCTIONS

A)  $r(t)$

B)  $\mu(t) = \frac{dr(t)}{dt}$ $\quad \int_{-\infty}^{\infty} \mu(t) = r(t)$

C)  $\delta(t) = \frac{du(t)}{dt}$ $\quad \int_{-\infty}^{\infty} \delta(t)\,dt = 1 = \mu(t)$

$\delta(bt) = \frac{1}{|b|}\,\delta(t)$

D)  $\delta'(t) = d\delta(t)/dt$

V) CHAPT. 4: CONVOLUTION

A) $f(t) = \int_{-\infty}^{\infty} f(\lambda)\,\delta(t-\lambda)\,d\lambda$

B) $f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\lambda)\, f_2(t-\lambda)\,d\lambda$

C) TRAPIZOIDAL INTEGRATION PLUG

$$\int = \frac{\Delta t}{2}\left[Y_0 + 2Y_1 + 2Y_2 + 2Y_3 + \ldots + 2Y_{n-1} + Y_n\right]$$

D) CONVOLUTION ALGEBRA

1) $f_1 * f_2 = f_2 * f_1$
2) $f_1 * (f_2 * f_3) = (f_1 * f_2) * f_3$
3) $f_1 * (f_2 + f_3) = f_1 * f_2 + f_1 * f_3$
4) $f(t) * \delta(t) = f(t)$
5) $f(t) * \mu(t) = \int_{-\infty}^{t} f(\lambda)\,d\lambda$
6) $f(t) * \delta'(t) = f'(t)$

E) SUPERPOSITION INTEGRAL

1) $w(t) = \mu(t) * h(t)$
2) $Y(t) = x'(t) * w(t) = x(t) * w'(t) = x''(t) * \int_{-\infty}^{t} w(\lambda)\,d\lambda$

VI) FOURIER TRANSFORMS

A) $\mathcal{F}[f(t)] = F(jw) = \int_{-\infty}^{\infty} f(t)\, e^{-jwt}\,dt$

B) $\mathcal{F}^{-1}[F(jw)] = f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(jw)\, e^{jwt}\,dw$

C) $F(jw) = A(w)\, e^{j\theta(w)} \ni A(w) = |F(jw)|$

$\quad \& \ \theta(w) = \tan^{-1}\left[\frac{\mathrm{Im}\, F(jw)}{\mathrm{Re}\, F(jw)}\right]$

D) $\underbrace{\frac{\int_{-\infty}^{\infty} f(t)\,dt}{f(0)}}_{} \cdot \frac{\int_{-\infty}^{\infty} F(jw)\,dw}{F(0)} = 2\pi$ (DURATION · BW)

E) $\mathcal{F}\{d^n f(t)/dt^n\} = (jw)^n F(jw)$; $\mathcal{F}[\delta(t-t_0)] = e^{-jwt_0}$

F) ELEMENTARY PROPERTIES OF FOURIER TRANSFORMS:

1) $f(t) = f_e(t) + f_o(t)$

$$f_e(t) = \frac{f(t) + f(-t)}{2}$$

$$f_o(t) = \frac{f(t) - f(-t)}{2}$$

2) FOR CAUSAL SYSTEMS

a) $f_e(t) = f_o(t) = f(t)/2$

b) $|F(j\omega)| \neq 0$

TEST # 3 ; CRAM FORMULAS

I) CHAPTER 4: FOURIER TRANSFORMS

A) 1) FOURIER TRANSFORM: $\mathcal{F}\{f(t)\} = F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$

$F(j\omega) = A(\omega)\, e^{j\theta(\omega)} \Leftrightarrow A(\omega) = |F(j\omega)|$; $e^{j\theta(\omega)} = \tan^{-1}\left[\frac{\mathrm{IM}\, F(j\omega)}{\mathrm{Re}\, F(j\omega)}\right.$

2) INVERSE TRANSFORM: $\mathcal{F}^{-1}[F(j\omega)] = f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\, e^{j\omega t}\, d\omega$

3) DIRICHLET CONDITIONS:

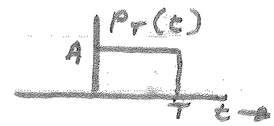
a) $f(t)$ MUST BE ABSOLUTELY INGTEGRABLE $\Rightarrow \int_{-\infty}^{\infty} |f(t)dt| < \infty$

b) $f(t)$ MUST HAVE FINITE MAX & MIN IN FINITE ~~AREA~~ INTERVAL

c) $f(t)$ MUST HAVE FINITE DISCONTINUITIES IN FINITE INTERVAL

B) SIMPLE TRANSFORMS

1) $p_T(t) \Leftrightarrow AT e^{-j\frac{\omega T}{2}} \frac{\sin \omega T/2}{\omega T/2}$



2) $$\underbrace{\frac{\int_{-\infty}^{\infty} f(t)dt}{f(0)}}_{\text{EQUIVILENT DURATION}} ; \cdot \underbrace{\frac{\int_{-\infty}^{\infty} F(j\omega)d\omega}{F(0)}}_{\text{EQUIVILENT BANDWIDTH}} = 2\pi$$

3) $e^{-\alpha t} u(t) \Leftrightarrow \frac{1}{j\omega + \alpha}$

4) $e^{-|\alpha| t} \Leftrightarrow \frac{2\alpha}{\omega^2 + \alpha^2}$

5) $\mathcal{F}\left[\frac{d^n}{dt^n} f(t)\right] = (j\omega)^n F(j\omega)$

6) $\left.\begin{aligned} F_e(j\omega) &= 2 \cdot \mathrm{Re}\,[F_c(j\omega)] \\ F_o(j\omega) &= 2 \cdot \mathrm{Im}\,[F_c(j\omega)] \end{aligned}\right\}$ $f_c(t)$ IS CAUSAL

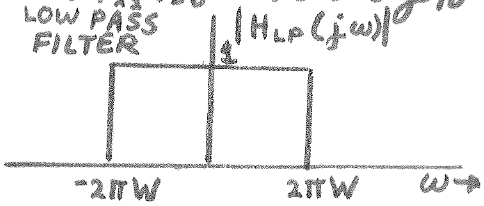
C) SYSTEM FUNCTION

1) $Y(j\omega) = X(j\omega) H(j\omega) \Rightarrow \mathcal{F}\{f_1(t) * f_2(t)\} = F_1(j\omega) F_2(j\omega)$

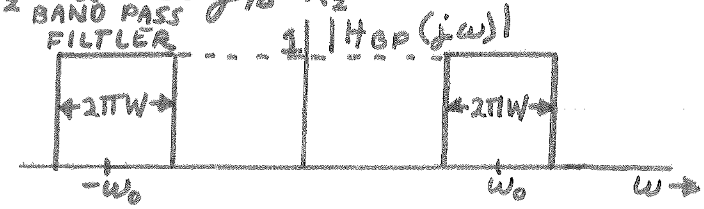
2) $\delta(t) \Leftrightarrow 1$

3) $P_{x_1}/P_{x_2}\big|_{DB} = 10 \log_{10} \frac{P_{x_1}}{P_{x_2}} = 20 \log_{10} \frac{X_1}{X_2}$

4) LOW PASS FILTER



BAND PASS FILTLER



D) ENERGY SPECTRUM

1) PARSEVAL'S THEOREM $\Rightarrow E = \int_{-\infty}^{\infty} [f(t)]^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$

2) $|F(j\omega)|^2 \Rightarrow$ ENERGY SPECTRUM

3) $|Y(j\omega)|^2 = |H(j\omega)|^2 \, |X(j\omega)|^2$

4) $|H(j\omega)|^2 \Rightarrow$ ENERGY TRANSFER FUNCTION

D) FOURIER TRANSFORMS MATHEMATICALLY

1) $\mathcal{F}\{f(at)\} \Leftrightarrow \frac{1}{|a|} F\left(\frac{j\omega}{a}\right)$

2) DELAY: $\mathcal{F}\{f(t-t_0)\} \Leftrightarrow e^{-j\omega t_0} F(j\omega)$

3) MODULATION: $e^{j\omega_0 t} f(t) \Leftrightarrow F[j(\omega-\omega_0)]$

4) REVERSAL: $f(-t) \Leftrightarrow F(-j\omega)$

5) SYMMETRY: $F(jt) \Leftrightarrow 2\pi f(-\omega)$

$\frac{1}{2\pi} F(-t) \Leftrightarrow f(j\omega)$

E) FOURIER TRANSFORMS OF POWER SIGNALS

1) $1 \Leftrightarrow 2\pi\,\delta(\omega)$

2) $\sum_{n=-\infty}^{\infty} \delta(t-nT) \Leftrightarrow \frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi n}{T}\right)$

F) CONVOLUTION ON THE FREQ. DOMAIN

$f_1(t) f_2(t) = F_1(j\omega) \circledast F_2(j\omega) \frac{1}{2\pi}$

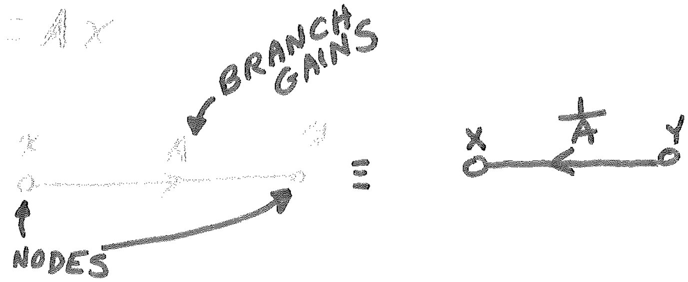
$f_1(t) f_2(t) \Leftrightarrow F_1(f) * F_2(f)$

G) SAMPLING THEOREM

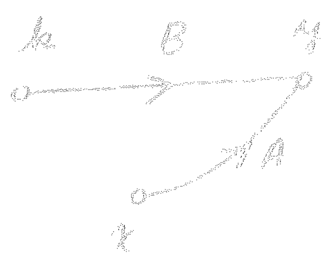
1) SAMPLING $> 2W$ HZ $\Rightarrow$ 2) SAMPLING $< \frac{1}{2W}$ SEC

Signal flow graphs

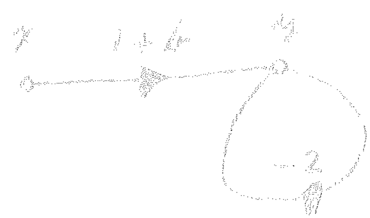
Graph of $y = Ax$



graph of $y = Ax + Bk$



graph of $y = (1+k)x - 2y$



$$e_1 = z_{11} i_1 + z_{12} i_2$$

$$e_2 = z_{21} i_1 + z_{22} i_2$$

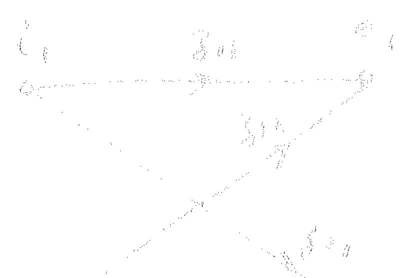
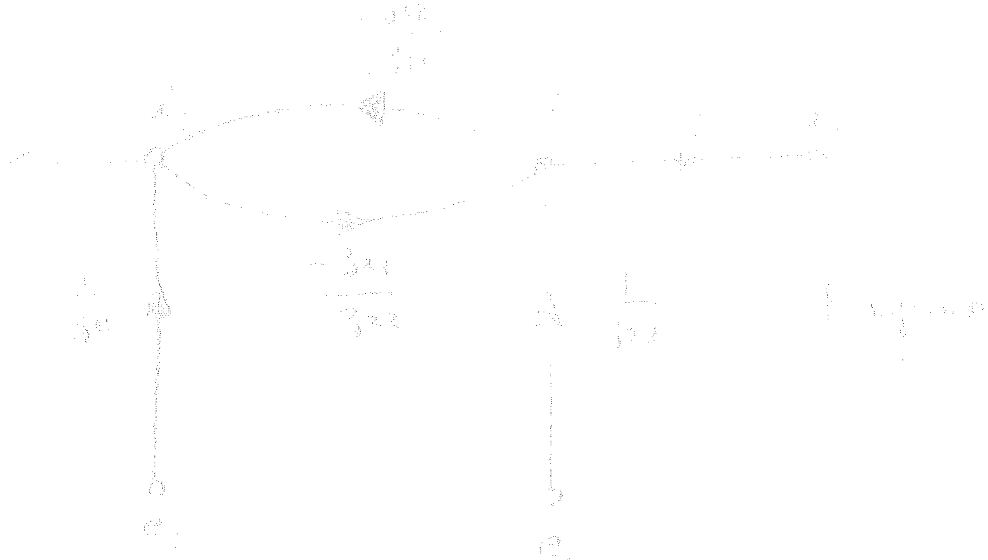


Figure 4



[illegible]

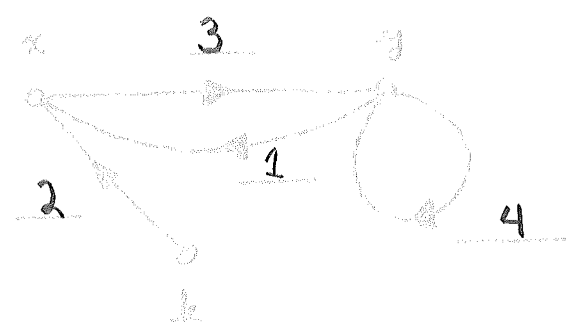
[illegible] Two graphs are no different because the graph[illegible] the inputs are the two currents and the outputs are the two voltages. Find the [illegible] in this graph.

A more complicated set of equations.

$$y = 3x + 4y$$

$$x = 2k + y$$

The graph would be

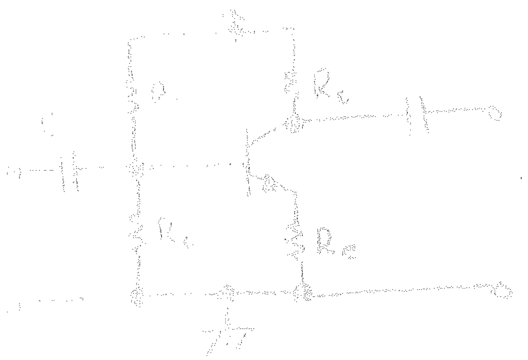


[illegible] the blanks in this graph using the equations above.

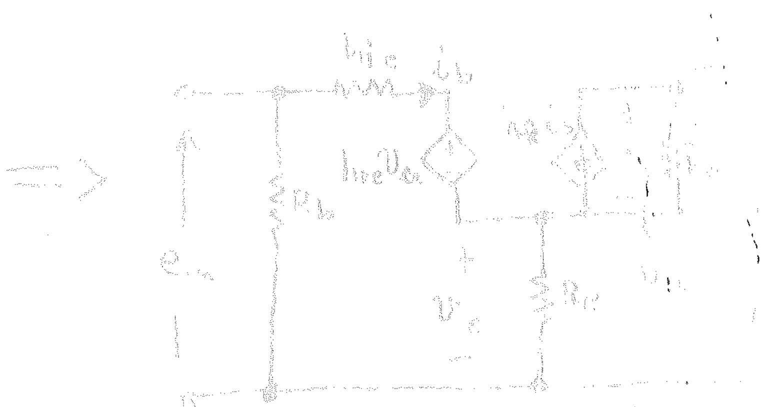
[illegible] that this graph has one input. This is [illegible] the variable k.

Let us investigate the gain of a common emitter amplifier when the emitter bypass capacitor is removed.

CIRCUIT



MODEL



Equations

(1) $e_o = -i_c R_L$

(2) $i_c = h_{fe} i_b + h_{oe} v_{ce}$

(3) $v_{ce} = e_o - v_e$

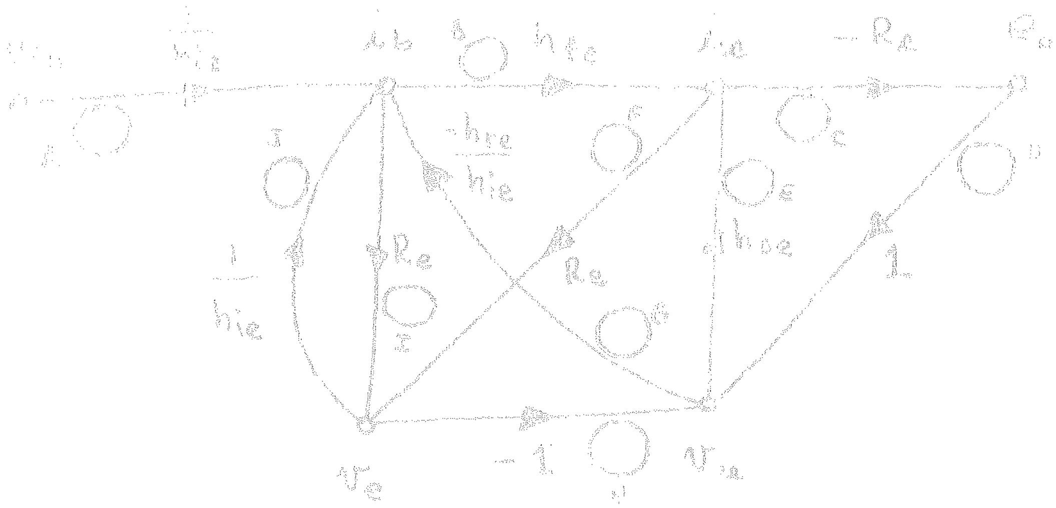
(4) $v_e = (i_b + i_c) R_e$

(5) $i_b = \dfrac{e_{in} - h_{re} v_{ce} - v_e}{h_{ie}}$

note here that [illegible] with 5 unknowns [illegible] each equation has written to yield [illegible] the unknowns in [illegible] equations above [illegible] all unknowns [illegible] satisfied.

As a step in solving this system of equations, we will draw the signal flow graph.

[illegible] one equation at a time.



Shown is the complete signal flow graph for the circuit. Study it carefully. Note there are 5 unknowns and one known quantity, e_{in}.

On the circles drawn for each signal path place the equation number from the equations on previous page.

The numbers in the circles in the previous page:

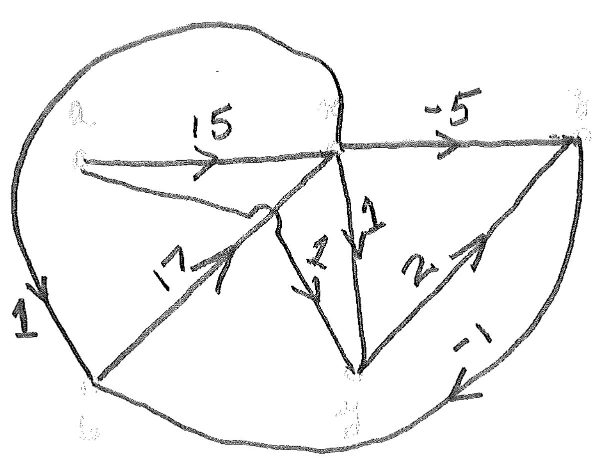
Ans:

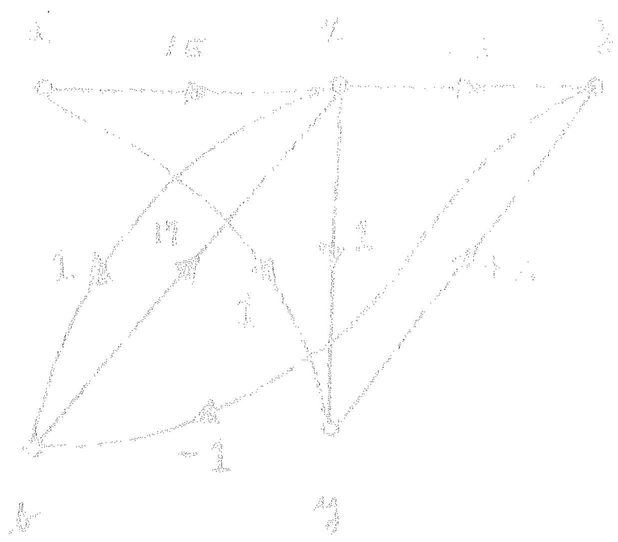
A - 5
B - 2
C - 1
D - 3
E - 2
F - 4
G - 5
H - 3
I - 4
J - 5

Draw the signal flow graph for the equations shown.

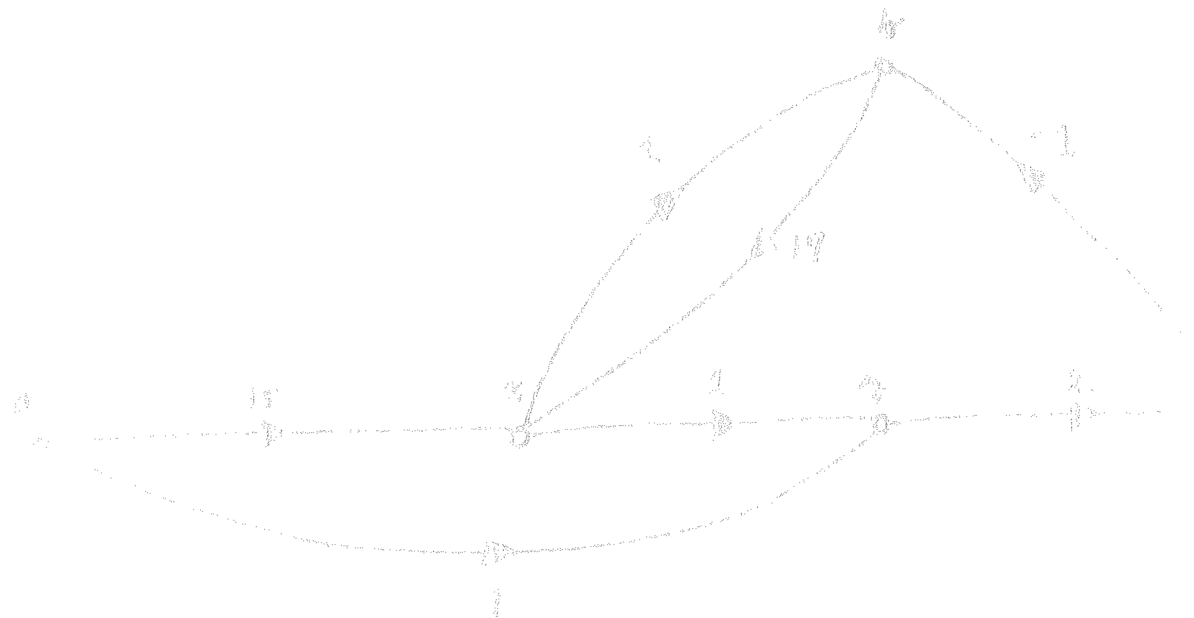
$y = -5x + 2z$
$x = 15a + 17b$
$z = a + x$
$b = a - y$

For these equations, the input parameter is ______

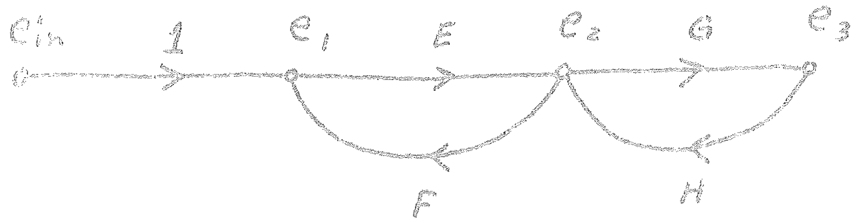




In vertices this may be redrawn below.



Flow graph notes cont.



$$e_1 = e_{in} + F e_2 \qquad E e_1 = E e_{in} + EF e_2$$

$$e_2 = E e_1 + H e_3 = E e_{in} + EF e_2 + H e_3$$

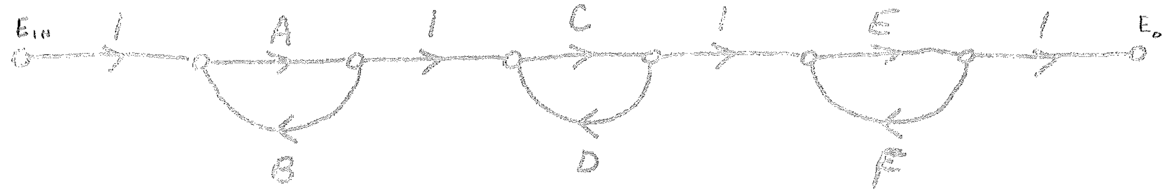
$$e_3 = G e_2 \qquad \frac{e_0}{G} = E e_{in} + EF e_2 + H e_0$$

$$\frac{e_0}{e_{in}} = \frac{E}{\frac{1}{G} - \frac{FE}{G} - H} = \frac{EG}{1 - (EF + GH)}$$

Rule for finding gain:

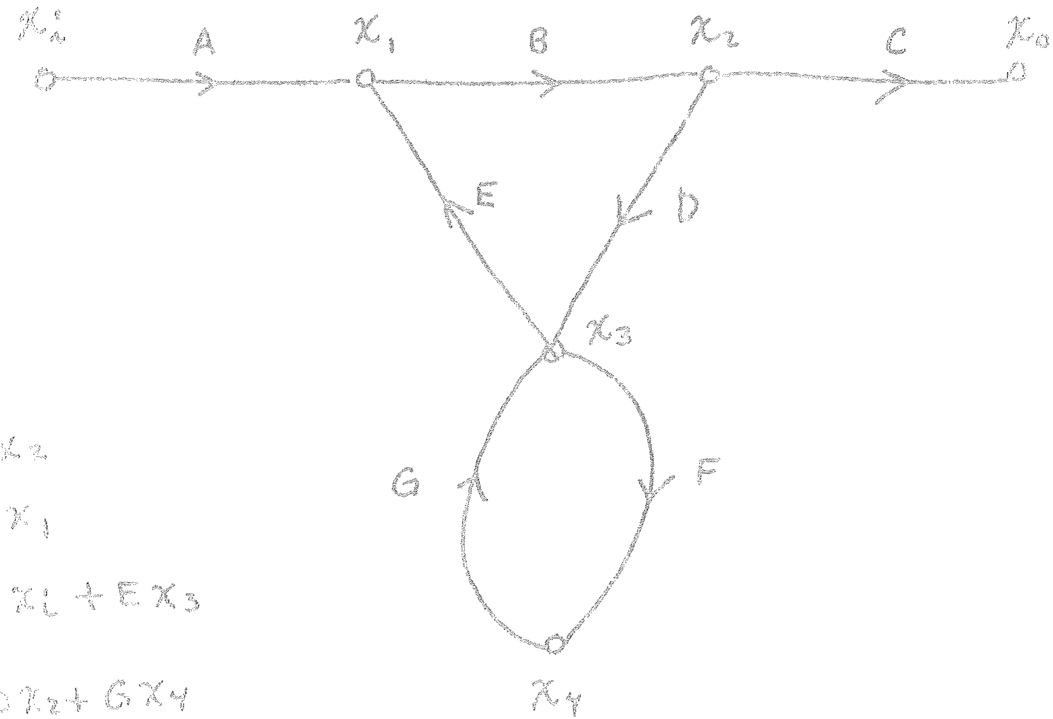
$$\text{"Gain"} = \frac{\text{Forward gain}}{1 - \sum(\text{loop gains})_{\text{1 at a time}} + \sum(\text{loop gains})_{\text{2 at a time, nontouching}} - \sum(\text{loop gains})_{\text{3 at a time, non touching}} + \sum(\text{loop g})_{\text{4 at a} \ldots}}$$

Example



$$\frac{E_o}{E_{in}} = \frac{A}{1-AB} \cdot \frac{C}{1-CD} \cdot \frac{E}{1-EF}$$

$$= \frac{ACE}{1-(AB+CD+EF)+(ABCD+ABEF+CDEF)-(ABCDEF)}$$



$x_o = Cx_2$

$x_2 = Bx_1$

$x_1 = Ax_i + Ex_3$

$x_3 = Dx_2 + Gx_4$

$x_4 = Fx_3$

$$\frac{x_o}{x_i} = \frac{ABC(1-GF)}{1-(BED+GF)}$$

loop not connected with a forward path

all terms in denominator not connected to forward path.

(11)

Final expression for Gain

$$\text{Gain} = \frac{G_1 \Delta_1 + G_2 \Delta_2 + \ldots}{\Delta} = \frac{\sum_{i=1}^{n_F} G_i \Delta_i}{\Delta}$$

Δ = same denominator as before

G_i = forward path gain

Δ_i = all terms in denominator made up of loops not ~~considered~~ touching $i^{\underline{th}}$ forward path

$\frac{K_1 2(s+2)}{(s+2)^2}$ $\quad K_2 \quad (s+5) \quad K_3 \quad (s+2)$

$\frac{d}{ds} \quad = 1$

$0K_2 + K_3 = 1$

$s = -5 \Rightarrow$

$-4 = 9K_1 - 3K_3$

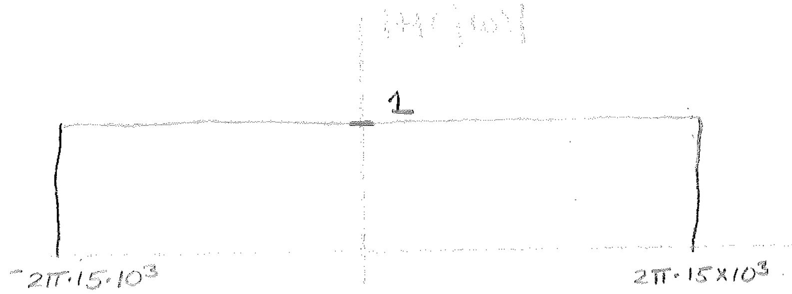
$s = 2 \Rightarrow -1 = 3K_2 \Rightarrow K_2 = \frac{-1}{3}$

X

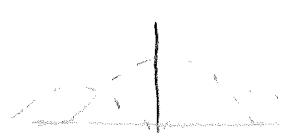
0

$$= \frac{s+\frac{1}{2}}{(s+1)^2+9} = \frac{s+\frac{1}{2}}{(s+1-j3)(s+1+j3)}$$

$$e^{j\theta} = \tan^{-1}\frac{j9}{2} + \tan^{-1} j(9+3) - \tan^{-1} j(9-3)$$



$$> 30 \times 10^3 \text{ HZ} \Rightarrow < \frac{1}{30} \times 10^{-3} \frac{\text{SEC}}{\text{SAMPLE}}$$



$\Rightarrow$ WILL HAVE COMPLETE JUNE'S OVERLAP, WITH THE HIER FREQUENCIES PROBABLY SUFFERING MOST

$$\int_0^\infty t e^{-(s+\alpha)t} dt$$

$$\frac{-t}{s+\alpha} e^{-(s+\alpha)t}\Big|_0^\infty + \int_0^\infty \frac{1}{s+\alpha} e^{-(\alpha+\alpha)t} dt$$

$$e^{-j\omega(\infty)} e^{-\sigma(\infty)} e^{-\alpha\infty} \qquad \sigma+\alpha>0$$

CONV $\sigma > -\alpha$

$$\int_0^t \lambda e^{-3\lambda} d\lambda$$

$$\frac{1}{3}e^{-3\lambda}$$

$$= \frac{-\lambda}{3} e^{-3\lambda}\Big]_0^t + \int_0^t \frac{1}{3} e^{-3\lambda}$$

$$= \frac{-\lambda}{3} e^{-3\lambda}\Big]_0^t - \frac{1}{9} e^{-3\lambda}\Big]_0^t$$

$$= -\left[\frac{t}{3} e^{-3t}\right] - \frac{1}{9}\left[e^{-3t}-1\right]$$

$$= -\frac{t}{3} e^{-3t} - \frac{1}{9} e^{-3t} + \frac{1}{9}$$

$$\mathcal{L}\{\} = \frac{d}{dt}\frac{1}{s+3} - \frac{1}{9(s+3)} + \frac{1}{9(s)}$$

$$\frac{-1}{(s+3)^2} - \frac{1}{9(s+3)} + \frac{1}{9s}$$

$$\frac{d}{dt} t e^{-3t} = t e^{-3t} \overset{=0}{\cancel{\delta(t)}} + \left(\frac{d}{dt} t e^{-3t}\right) \cancel{\mu(t)} = (-3te^{-3t} + e^{-3t})\mu(t)$$

1) $\mathcal{L}\{t e^{-st}\} = -\frac{d}{ds} \frac{1}{s+\alpha} = \frac{+1}{(s+\alpha)^2} = \frac{1}{(s+3)^2}$

2) $\mathcal{L}\{t e^{-st}\} = \int_0^\infty t\, e^{-st} e^{-3t} dt$

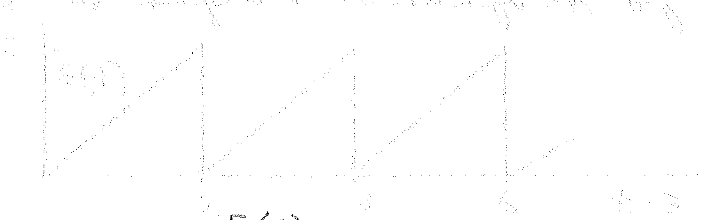
$$= \int_0^\infty t\, e^{-t(s+3)} dt$$

$u = t \quad dv = e^{-t(s+3)} dt$

$du = dt \quad v = \frac{e^{-t(s+3)}}{-(s+3)}$

$$\Rightarrow F(s) = \left.\frac{-t\, e^{-t(s+3)}}{(s+3)}\right]_0^\infty + \int_0^\infty \frac{e^{-t(s+3)}}{(s+3)} dt$$

$$= \left.\frac{-e^{-t(s+3)}}{(s+3)^2}\right]_0^\infty = \frac{1}{(s+3)^2}$$



$$F(s) = \frac{F_1(s)}{1 - e^{-sT}}$$

$F_1(t) = \frac{5}{2} t - \frac{5}{2} t \mu(t-2)$

$$\mathcal{L}\{F_1(t)\} = \int_0^2 \frac{5}{2} t\, e^{-st} dt$$

$u = \frac{5}{2} t \quad dV = e^{-st} dt$

$du = 5/2 \quad V = -e^{-st}/s$

$$\left.\frac{-5}{2s} t e^{-st}\right]_0^2 + \frac{5}{2s} \int_0^2 e^{-st}$$

$$\frac{-5}{s} e^{-2s} - \frac{5}{2s^2}\left[e^{-2s} - 1\right]$$

$$\therefore F(s) = \frac{-e^{-2s}\left[\frac{5}{s} + \frac{5}{2s^2}\right] + \frac{5}{2s^2}}{1 - e^{-2s}} = \frac{-e^{-2s}[10s + 5] + 5}{2s^2[1 - e^{-2s}]}$$

$$H(s)\cdot(s+4)(s^2+16) = s(s+2)$$

$$\frac{s(s+2)}{(s+4)(s+j4)(s-j4)} = \frac{A}{s+4} + \frac{B}{s+j4} + \frac{C}{s-j4}$$

$$s(s+2) = A(s+j4)(s-j4) + B(s+4)(s-j4) + C(s+4)(s+j4)$$

$$s = -4 \Rightarrow 8 = A(32) \Rightarrow A = \frac{1}{4}$$

$$s = j4 \Rightarrow j4(j4+2) = -8 + j8 = C(j4+4)(j8)$$
$$= -32C + j32C \Rightarrow C = \frac{1}{4}$$

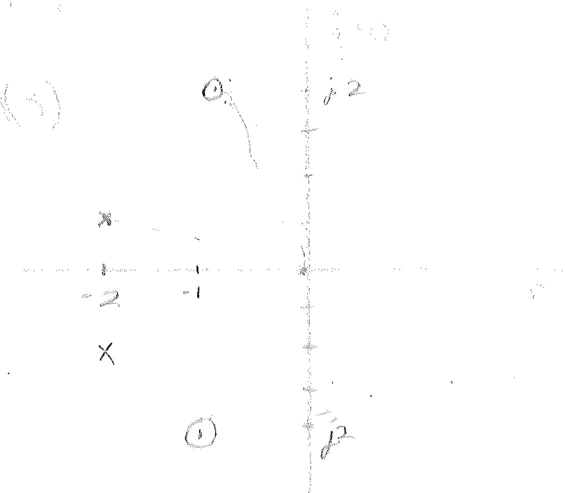
$$s = -j4 \Rightarrow -j4(-j4+2) = -16 - j8 = B(-j4+4)(-j8)$$
$$= jB8(4-j4) = jB8$$

$$\lim_{t\to 0^+} f(t) = \lim_{s\to +\infty} sF(s)$$
$$= \lim_{s\to +\infty} \frac{s^2(s+2)}{(s+4)(s^2+16)} \to \frac{s^2 s}{s\, s^2} = 1$$

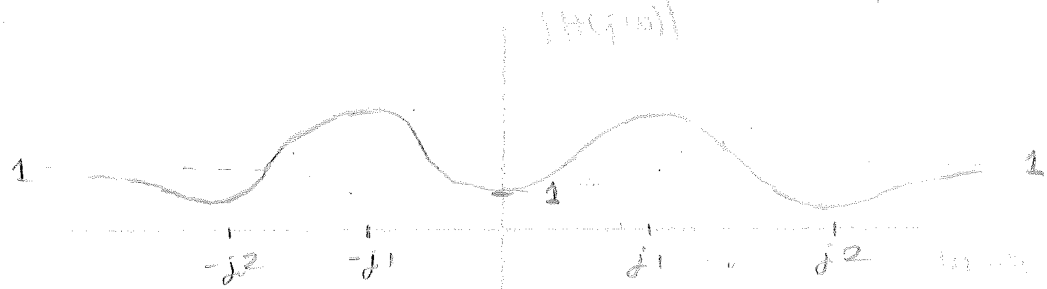
[illegible] $H(s) = $ [illegible]

[illegible] pole-zero plot of $H(s)$

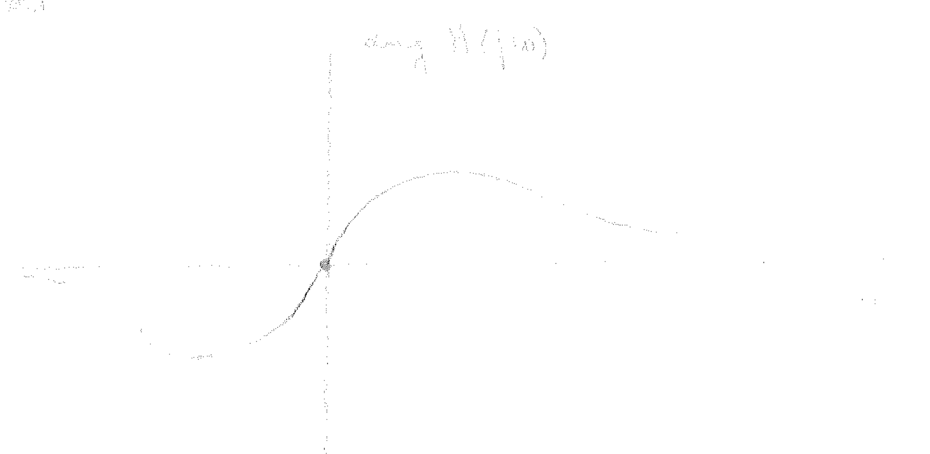
$$\frac{(s+1+j2)(s+1-j2)}{(s+2+j)(s+2-j)}$$



[illegible] $|H(j\omega)|$ [illegible] showing values at the origin [illegible]. Indicate roughly locations of maxima and minima.



[illegible] $H(j\omega)$ [illegible] showing [illegible]



$$\frac{s(s+2)}{(s+1)(s-j2)(s+j2)} = \frac{A}{s+1} + \frac{B}{s-j2} + \frac{C}{s+j2}$$

$$s(s+2) = A(s^2+4) + B(s+j2)(s+1) + C(s-j2)(s+1)$$

$$s=-1 \Rightarrow -1 = A(5) \Rightarrow A = -1/5$$

$$s=j2 \Rightarrow j2(j2+2) = -4+j4 = Bj2(j2+1) = Bj4 + B4 \Rightarrow B = -1$$

$$s=-j2 \Rightarrow +j2(2-j2) = 4j+4 = C(+j4)(1-j2) = Cj4 + C8 \Rightarrow C = 0$$

$$\therefore F(s) = \frac{-1}{5(s+1)} - \frac{1}{s-j2}$$

$$\Rightarrow f(t) = \mathcal{L}^{-1}\{F(s)\} = -\frac{1}{5}e^{-t} - e^{j2}$$

$$\frac{s(s+2)}{(s+1)(s^2+4)} = \frac{A}{s+1} + \frac{B}{s^2+4}$$

$$s(s+2) = A(s^2+4) + B(s+1)$$

$$s=-1 \Rightarrow -1 = 5A \Rightarrow A = -1/5$$

$$s=j2 \Rightarrow j2(j2+2) = B(j2+1)$$

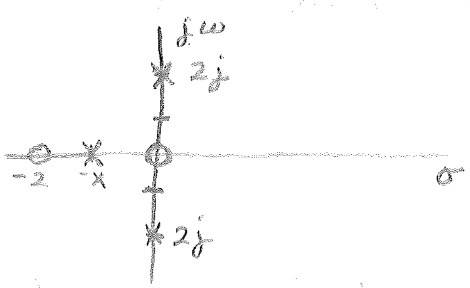
$$-4+j4 = B + j2B$$

$$s+s+2 = 2s+2 = 2As + B$$

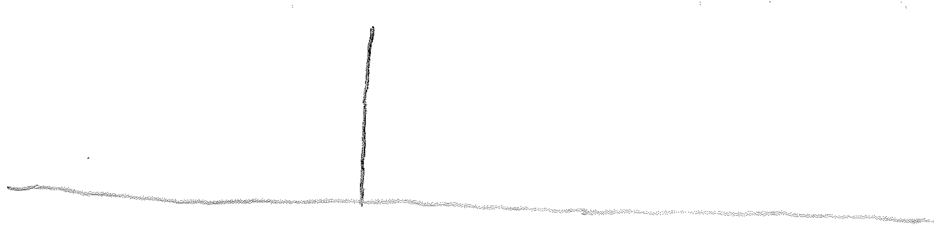
$$2s+2 = -\frac{2}{5}s + B$$

$$\frac{12}{5}s + 2 = B$$

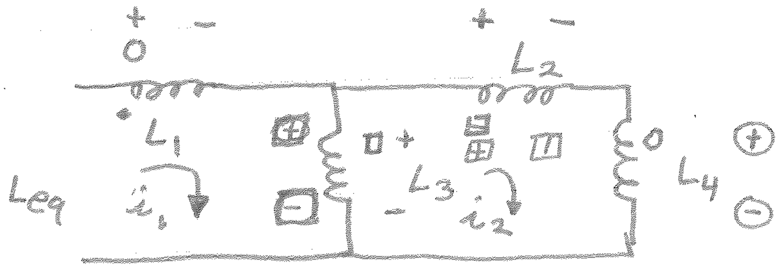
ARG!



$$s^2+4 \Rightarrow (s+j2)(s-j2)$$



X-38)



I) SIGNALS & SYSTEMS

A) INTRODUCTION - WHILE GENERAL DEFINITIONS OF SIGNALS & SYSTEMS IS VAGUE; ENGINEER SHOULD BE FAMILIAR WITH IN SPECIFIC UNDERTAKINGS

B) SYSTEMS - WHAT THEY ARE AND WHAT THEY DO

1) GENERAL DEFINITION: IT CONSISTS OF A GROUP OF OBJECTS WHICH INTERACT WITH ONE ANOTHER AND ARE ASSEMBLED IN A MANNER TO ACHEIVE A DESIRED OBJECTIVE.

2) MOST SYSTEM ANALYSIS CONSISTS OF A PORTION OF A MORE COMPLEX SYSTEM

3) ENGINEER IS RESPONSIBLE FORE DESIGN, PRODUCTION, & ANALYSIS OF SYSTEMS THRU MATHEMATICAL TOOLS

C) SIGNALS

1) SIGNALS DIRECT THE SYSTEM

2) INPUT/OUTPUT TYPE

3) MUST BE ABLE TO REPRESENT SIGNAL MATHEMATICALLY

D) SYSTEM ANALYSIS AND DESIGN

1) WHY ANALYSIS IS IMPORTANT

a) CHEAPER THAT EXPERIMENT

b) DETERMINING FEASIBILITY

c) MAY ASSUME IMPOSSIBLE OR DANGEROUS CONDITIONS

d) IMPORTANT PART OF DESIGN SYSTEM

2) PROBLEMS IN SYSTEM DESIGN

a) DETERMINE CHARACTERISTICS THAT WILL YEILD DESIRED RESPONSE FROM AN INPUT

b) DETERMINING BEST HARDWARE TO FIT MATHEMATICAL MODEL

II) SYSTEM REPRESENTATION AND ANALYSIS

A) INTRODUCTION

B) SYSTEM REPRESENTATION

1) NESESARY TO REPRESENT SYSTEM MATHEMATICALLY BY USE OF ~~A~~ MATHEMATICAL MODEL

2) MATHEMATICAL MODEL ALWAYS AN APPROXIMATION

3) MATH MODEL IS A COMPROMISE BETWEEN ACCURACY & MATHEMATICAL SIMPLICITY

C) THE MATHEMATICAL MODEL

1) BLOCK DIAGRAM

a) $X(t) \xrightarrow{\text{EXCITATION OR INPUT}} \boxed{\text{SYSTEM}} \xrightarrow[\text{OUTPUT SIGNAL}]{\text{RESPONSE OR}} Y(t)$

b) MATH MODEL FOR SYSTEM IS EQUATION RELATING $X(t)$ AND $Y(t)$ $[Y(t) = f[X(t)]]$

2) MOST BASIC MATH MODEL IS DIFF. EQUATION

3) FIRST ORDER DIFF. EQUATION KNOWN AS "NORMAL FORM" OF SYSTEMS EQUATION

D) CLASSIFICATION OF SYSTEMS

1) ORDER OF SYSTEM - REFERS TO ORDER OF DIFF. EQUATION USED TO DESCRIBE THE SYSTEM

2) CAUSAL & NON-CAUSAL

a) CAUSAL SYSTEM - RESPONSE IS NOT A FUNCTION OF FUTURE INPUT (ASSUMED TO APPLY TO SYSTEMS EQUATION)

b) NON-CAUSAL - (NON PHYSICAL OR ANTICIPATORY) DON'T EXIST PHYSICALLY.

3) LINEAR & NON-LINEAR (D.E.)

a) ~~LINEAR~~ $\Leftrightarrow$ $Y_1(t) = f[X_1(t)]$ AND $Y_2(t) = f[X_2(t)]$ THEN $aY_1(t) + bY_2(t) = af[X_1(t)] + bf[X_2(t)]$

b) SUPORPOSITION APPLIES TO LINEAR SYSTEMS)

4) FIXED VS. TIME VARIENT SYSTEMS

a) FIXED SYSTEMS HAVE CONSTANT COEFFICIENTS IN THEIR DIFFERINTIAL EQUATIONS.

ALSO $Y(t) \leftarrow X(t) \Longrightarrow Y(t+\tau) \leftarrow X(t+\tau)$

5) LUMPED PARAMETER VS DISTRIBUTED PARAMETER SYSTEM
a) LUMPED ⇒ DIFFERINTIAL EQUATION
b) DIST ⇒ PARTIAL DIFFER. EQUATIONS
6) CONTINUOUS VS. DISCRETE TIME SYSTEM
a) CONTINUOUS ⇒ DIFFERINTIAL EQUA.
b) DISCRETE TIME ⇒ DIFFERENCE EQN. (FOR SHORT INSTANCES IN TIME)
7) INSTANTANEOUS VS. DYNAMIC SYSTEM
a) INSTANTANEOUS SYSTEM DOES NOT DEPEND ON FUTURE OR PAST EXCITATIONS OF THE SYSTEM
b) DYNAMIC SYSTEMS HAVE MEMORY (L & C)
8) THOSE SYSTEMS DISCUSSED IN THIS COURSE ARE 1) CAUSAL, 2) LINEAR, 3) FIXED, 4) LUMPED PARAMETER 5) CONTINUOUS TIME, 5) & DYNAMIC
MATH-WISE 1) ORDINARY 2) LINEAR, 3) D.E. WITH CONSTANT COEFF.

E) EXAMPLES OF MATHEMATICAL MODELS OF SYSTEMS
1) REVIEW
a) RESISTANCE

$$v(t) = R\,i(t)$$
$$i(t) = G\,v(t) \quad ; \quad G = 1/R$$

b) INDUCTANCE

$$v(t) = L\, di(t)/dt$$
$$i(t) = \Gamma \int_{-\infty}^{t} v(\lambda)\,d\lambda \; ; \; \Gamma = 1/L$$

c) CAPACITANCE

$$v(t) = S \int_{-\infty}^{t} i(\lambda)\,d\lambda \; ; \; S = 1/C$$
$$i(t) = C\, dv(t)/dt$$

d) AMPLIFIER

$$v_2(t) = K\, v_1(t)$$

e) DELAY; $v_2(t) = v_1(t-\tau)$; $\tau > 0$

f) INTEGRATOR: $v_2(t) = \int_{-\infty}^{t} v_1(\lambda)\,d\lambda$

2) AMPLIFIER EXAMPLE

a)

b) $v_1(t) = v_2(t) + \frac{1}{C}\int_{-\infty}^{t} i(\lambda)\,d\lambda$

$v_i(t) = v_1(t) - Ri(t)$

$v_2(t) = -K\,v_i(t)$

c) $\therefore\ (K+1)RC\,\frac{dv_2(t)}{dt} + v_2(t) = -K\,v_1(t)$

THIS IS THE MATH MODEL FOR THE SYSTEM

F) THE NORMAL FORM OF SYSTEM EQUATIONS

1) FOR n EQUATIONS, MUST BE n UNKNOWNS

a) UNKNOWN TIME FUNCTIONS CALLED "STATE VARIABLES"

b) IN AN ELECTRICAL SYSTEM, STATE VARIABLES REPRESENT v & i

2) IF ONLY ONE INPUT, WITH $q_n(t)$ AS STATE VARIABLES:

a) $dq_1/dt = a_{11}(t)q_1(t) + a_{12}(t)q_2(t) + \ldots + a_{1n}(t)q_n(t) + b_1(t)x(t)$

$$\frac{dq_2}{dt} = a_{21}(t)q_1(t) + a_{22}(t)q_2(t) + \ldots + a_{2n}(t)q_n(t) + b_2(t)x(t)$$

$$\vdots$$

$$\frac{dq_n}{dt} = a_{n1}(t)q_1(t) + a_{n2}(t)q_2(t) + \ldots + a_{nn}(t)q_n(t) + b_n(t)x(t)$$

b) IF THERE IS A SINGLE OUTPUT $y(t)$, THEN

$y(t) = c_1(t)q_1(t) + c_2(t)q_2(t) + \ldots c_n(t)q_n(t)$

c) THE ABOVE EQUATIONS FORM A MATHEMATICAL MODEL FOR THE GENERAL n^{TH} ORDER, TIME VARYING SYSTEM (LINEAR)

d) IF SYSTEM IS FIXED $a_i(t)$, $b_i(t)$ & $c_i(t)$ ARE FIXED.

3) WHY THIS FORM IS USEFUL
   a) MAY USE MATRICES
   b) MAY BE APLIED TO NON-LINEAR
   c) STRAIGHTFORWARD SOLUTION
   d) BETTER UNDERSTANDING OF WHAT'S HAPPENING IN SYSTEM
   e) COMPUTOR USE
   f) MULTIPLE INPUT/OUTPUT

G) INITIAL CONDITIONS DETERMINATION
  1) SUPERPOSITION - ASSUME INITIAL CONDITIONS ARE 0, AND ADD RESPONSE TO RESULT
  2) DERIVITIVES OF INPUT SIGNAL, USED WITH n^{TH} ORDER EQUATIONS
  3) INITIAL CONDITIONS ARISE WHEN SOLVING NORMAL FORM

III) REPRESENTATION OF SIGNALS

A) GENERAL METHODS OF SIGNAL REPRESENTATION - SHOULD BE QUANTITATIVE

B) CLASSIFICATION OF SIGNALS

1) PERIODIC & NON-PERIODIC

2) RANDOM & NON-~~[illegible]~~ RANDOM (PREDICTION)

3) ENERGY & POWER SIGNALS

a) FOR ~~CURRENT~~ VOLTAGE

$$E = \int_{t_1}^{t_2} \frac{e^2(t)}{R}\,dt \quad \text{WATT SECONDS}$$

FOR CURRENT

$$E = \int_{t_1}^{t_2} R\,i^2(t)\,dt \quad \text{WATT SECONDS}$$

b) ENERGY SIGNAL DEFINED WHEN $x(t)$ FITS THE CONDITION:

$$\int_{-\infty}^{\infty} x^2(t)\,dt < \infty$$

c) POWER SIGNAL:

(1) $$P = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} x^2(t)\,dt$$

(2) MUST SATISFY CONDITION:

$$0 < \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt < \infty$$

d) SOME SIGNALS FIT BOTH

C) REPRESENTATION IN TERMS OF ELEMENTARY SIGNALS

1) BASIC FUNCTIONS - ELEMENTARY FUNC. CHOSEN FOR THEIR SIMPLICITY & PROPERTIES

2) LINEAR COMBINATIONS OF BASIC FUNC.

a) WRITTEN $x(t) = \sum_{n=0}^{N} a_n \phi_n(t)$

b) FINALITY OF COEFFICIENTS: ANY a_n MAY BE COMUTED INDEPENDENT OF ANY OTHER a_n

3) ORTHONORMAL:

a) $\int_{t_1}^{t_2} \phi_n(t)\,\phi_K(t)\,dt \begin{cases} =0 & \text{FOR } k \neq n \\ =\lambda_g & \text{FOR } k=n \end{cases}$

(LIMITS OF INTEGRATION MAY BE $\pm\infty$)

b) COEFFICIENTS MAY BE DETERMINED:

$$a_j \lambda_j = \int_{t_1}^{t_2} \phi_j(t)\,x(t)\,dt$$

WHEN BASIC FUNCTION IS REAL AND ORTHONORMAL

4) CHECKING ERROR IN APPROXIMATION

a) $x(t) = \sum_{n=0} a_n \phi_n(t)$; $\hat{x}(t) = \sum_{n=0}^{M} \hat{a}_n \phi_n(t)$

b) USE INTEGRAL SQUARED ERROR

(1) $I = \int_{t_1}^{t_2} [x(t) - \hat{x}(t)]^2\,dt$

$$= \int_{t_1}^{t_2} \left[x(t) - \sum_{n=0}^{M} \hat{a}_n \phi_n(t)\right]^2 dt$$

(2) WISH TO FIND VALUES OF $\hat{a}_n$ TO GIVE I MINIMUM VALUE

(3) REARRANGING TERMS YIELDS:

$$I = \int_{t_1}^{t_2} \Big[x^2(t) - 2x(t) \sum_{n=0}^{M} \hat{a}_n \phi_n(t)$$

$$+ \sum_{n=0}^{M} \hat{a}_n \phi_n(t) \sum_{n=0}^{M} \hat{a}_K \phi_K(t)\Big]\,dt$$

$$= \int_{t_1}^{t_2} x^2(t)\,dt - 2 \sum_{n=0}^{M} \hat{a}_n \int_{t_1}^{t_2} x(t)\phi_n(t)\,dt$$

$$+ \sum_{n=0}^{M} \sum_{K=0}^{M} \hat{a}_n \hat{a}_K \int_{t_1}^{t_2} \phi_n(t)\,\phi_K(t)\,dt$$

FIRST TERM INDEP. OF a_n

SECOND TERM $= 2\hat{a}_n \lambda_n$

$$\therefore I = K - 2 \sum_{n=0}^{M} \hat{a}_n a_n \lambda_n + \sum_{n=0} \hat{a}_n^2 \lambda_n$$

$$= K - \sum_{n=0}^{M} \lambda_n a_n^2 + \sum_{n=0}^{M} \lambda_n (\hat{a}_n - a_n)^2$$

$\Rightarrow I$ WILL BE MINIMUM WHEN $\hat{a}_n = a$

(4) $\frac{\text{ERROR ENERGY}}{\text{SIGNAL ENERGY}} = 1 - \frac{1}{E} \sum_{n=0}^{N} \lambda_n a_n^2$

5) EXAMPLE OF BASIS FUNCTION USE

a) USE RECTANGULAR WAVEFORMS

SYSTEM ORTHONORMAL $\Rightarrow \lambda_K = 1$ FOR $k \geq 0$

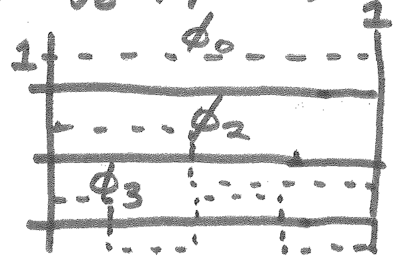
b) LET $\phi_0(t) = 1 \quad 0 \leq t \leq 1$

$= 0$ ELSEWHERE

c) ORTHONORMALTY REQUIRES:

$$\int_0^1 \phi_0(t)\, \phi_1(t)\, dt = 0$$

AND $\int_0^1 \phi_1^2(t) = 1$



d) LET $x(t) = 6(t) \quad 0 \leq t \leq 1$

$= 0$ ELSEWHERE

e) $a_0 = \int_0^1 \frac{1}{\lambda_K} x(t)\, dt = \int_0^1 (1)\, 6t\, dt = 3$

SIMILARLY $a_1 = \int_0^{\frac{1}{2}} (1)\, 6t\, dt = -\frac{3}{2}$

AND $a_2 = -3/4$; $a_3 = -3/8$

f) $\therefore \hat{x}(t) = 3\phi_0(t) - \frac{3}{2}\phi_1(t) - \frac{3}{4}\phi_2(t) \ldots$

g) COMPUTING ERROR APROXIMATION

$$E = \int_0^1 (6t)^2 dt = 12$$

FOR FIRST 4 ENTRIES:

$\frac{\text{ERROR EN}}{\text{SIGNAL EN}} = 1 - \frac{1}{12}\left(3^2 + \left(-\frac{3}{2}\right)^2 + \left(-\frac{3}{4}\right)^2 + \left(-\frac{3}{8}\right)^2\right) = \frac{1}{256}$

D) FOURIER SERIES REPRESENTATION

1) FUNCTION MUST FIT DIRICHLET CONDITIONS

a) $x(t)$ BE SINGLE VALUED

b) FINITE MAX AND MIN

c) FINITE # DISCONTINUITIES

d) $\int_{t_1}^{t_2} |x(t)|\,dt < \infty$

2) $\phi_n(t) = e^{jn\omega_0 t}$; $n = 0, \pm 1, \pm 2 \ldots$

$\omega_0 = 2\pi/T$

3) BASIS FUNCTION ORTHOGONOL:

$$\int_{t_1}^{t_1+T} e^{jn\omega_0 t} e^{-jk\omega_0 t}\,dt \begin{cases} = 0 & n \neq k \\ = T & n = k \end{cases}$$

4) $a_n = \frac{1}{T}\int_{t_1}^{t_1+T} x(t)\, e^{-jn\omega_0 t}\,dt$

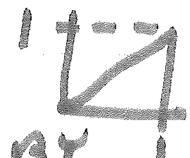
5) $x(t) = \sum a_n e^{jn\omega_0 t}$

E) SINGULARITY FUNCTIONS

1) UNIT RAMP FUNCTION

$r(t) = t \qquad t \geq 0$

$\quad\;\; = 0 \qquad t > 0$

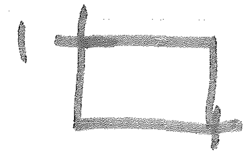


SLOPE MAY BE CHANGED BY MULTIPLICATION

2) UNIT STEP FUNCTION

$\mu(t) = 1 \qquad t \geq 0$

$\quad\;\; = 0 \qquad t < 0$



3) $r(t) = \int_{-\infty}^{t} \mu(\lambda)\,d\lambda$

FOURIER TRANSFORMS

I) THE FOURIER TRANSFORM

A) CONSIDER FOURIER SERIES

$$f(t)=\sum_{n=-\infty}^{\infty}\alpha_n e^{jn\omega_0 t}=\sum_{n=-\infty}^{\infty}\alpha_n e^{\frac{j2\pi nt}{T}}$$

$$\alpha_n=\frac{1}{T}\int_{-T/2}^{T/2} f(t)\,e^{\frac{-j2\pi nt}{T}}$$

$$\Rightarrow f(t)=\sum_{n=-\infty}^{\infty} e^{\frac{j2\pi nt}{T}}\left[\frac{\omega_0}{2\pi}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\,e^{-jn\omega_0}\,dt\right]$$

LET $T\to\infty \Rightarrow \omega_0\to d\omega \Rightarrow n\to\infty$

$$f(t)=\int_{-\infty}^{\infty} e^{j\omega t}\left[\frac{d\omega}{2\pi}\int_{-\infty}^{\infty} f(t)\,e^{-jnt}\,dt\right]$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\underbrace{\left[\int_{-\infty}^{\infty} f(t)\,e^{-j\omega t}\,dt\right]}_{\text{FOURIER TRANSFORM}} e^{j\omega t}\,d\omega$$

$$\mathcal{F}[f(t)]=\int_{-\infty}^{\infty} f(t)\,e^{-j\omega t}\,dt=F(j\omega)$$

$$\mathcal{F}^{-1}[F(j\omega)]=f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(j\omega)\,e^{j\omega t}\,d\omega$$

$$F(j\omega)=A(\omega)\,e^{j\theta(\omega)}$$

AMPLITUDE SPECTRUM $A(\omega)=|F(j\omega)|$

PHASE SPECTRUM $e^{j\theta(\omega)}=\tan^{-1}\left[\frac{\mathrm{IM}\,F(j\omega)}{\mathrm{Re}\,F(j\omega)}\right]$

B) DIRICHLET CONDITIONS (ASSURE CONVERGENCE), i.e.

1) $f(t)$ MUST BE ABSOLUTELY INTEGRABLE, i.e.

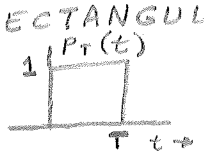
$$\int_{-\infty}^{\infty}|f(t)|\,dt<\infty$$

2) $f(t)$ MUST HAVE FINITE NUMBER OF MAXIMA AND MINIMA IN ANY FINITE INTERVAL

3) $f(t)$ MUST HAVE A FINITE NUMBER OF DISCONTINUITIES IN A FINITE INTERVAL

II) CALCULATIONS OF SIMPLE TRANSFORMS

A) RECTANGULAR PULSE



$$P_T(j\omega)=\int_{-\infty}^{\infty}P_T(t)\,e^{-j\omega t}\,dt$$

$$=\int_0^T e^{-j\omega t}\,dt=\left.\frac{e^{-j\omega t}}{-j\omega}\right]_0^T$$

$$=\frac{1-e^{-j\omega T}}{j\omega}$$

$$=\frac{e^{-j\omega T/2}}{\omega/2}\left[\frac{e^{j\omega T/2}-e^{-j\omega T/2}}{2j}\right]$$

$$=T\,e^{-\frac{j\omega T}{2}}\,\frac{\sin\omega T/2}{\omega T/2}$$

$$=T\,e^{-\frac{j\omega T}{2}}\,\mathrm{sinc}\,\frac{\omega T}{2}$$

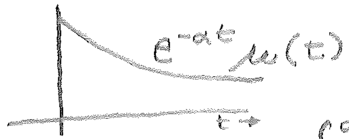
B) NOTES

1) MORE COMPACT IN TIME, THE MORE SPEAD OUT THE FREQUENCY AND VISA VERSA

2) $$\underbrace{\frac{\int_{-\infty}^{\infty}f(t)\,dt}{f(0)}}_{\text{EQUIVILENT DURATION}}\cdot\underbrace{\frac{\int_{-\infty}^{\infty}F(j\omega)\,d\omega}{F(0)}}_{\text{EQUIVILENT BANDWIDTH}}=2\pi$$

C) ONE SIDED EXPODENTIAL PULSE

$$f(t) = e^{-\alpha t}\mu(t)$$



$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$F(j\omega) = \int_{0}^{\infty} e^{-\alpha t} e^{-j\omega t} dt$$

$$= \left. \frac{e^{-(\alpha + j\omega)t}}{-(\alpha + j\omega)} \right]_0^{\infty} = \frac{1}{j\omega + \alpha}$$

D) TWO SIDED EXPODENTIAL PULSE

$$F(j\omega) = \int_{-\infty}^{\infty} e^{\alpha t} e^{-j\omega t} dt + \int_{0}^{\infty} e^{-\alpha t} e^{-j\omega t} dt$$

$$= \frac{1}{\alpha - j\omega} + \frac{1}{\alpha + j\omega} = \frac{2\alpha}{\omega^2 + \alpha^2}$$

E) DERIVITIVE OF A TIME FUNCTION

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

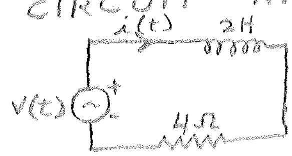
$$\frac{d}{dt} f(t) = \frac{d}{dt}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega\right]$$

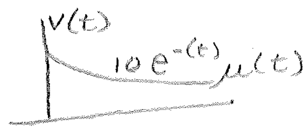
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} j\omega F(j\omega) e^{j\omega t} d\omega$$

$$\mathcal{F}^{-1}\{j\omega F(j\omega)\} = \frac{df(t)}{dt}$$

$$\mathcal{F}\left\{\frac{df(t)}{dt}\right\} = j\omega F(j\omega)$$

F) CIRCUIT ANALYSIS





$$V(t) = 2\frac{di(t)}{dt} + 4i(t)$$

$$\frac{10}{j\omega + 1} = j2\omega I(j\omega) + 4I(j\omega) \Rightarrow I(j\omega) = \frac{5}{(j\omega+1)(j\omega+2)}$$

$$I(j\omega) = \frac{5}{j\omega + 1} - \frac{5}{j\omega + 2} \Rightarrow i(t) = (5e^{-t} - 5e^{-2t})\mu(t)$$

III) ELEMENTRY PROPERTIES OF THE FOURIER TRANSFORMS

A) SYMMETRY OF $f(t)$ ABOUT THE TIME ORIGIN

$$f(t) = f_e(t) + f_o(t)$$

$$f_e(t) = \frac{f(t) + f(-t)}{2} \;;\; f_o = \frac{f(t) - f(-t)}{2}$$

$$\Rightarrow F(j\omega) = \underbrace{2\int_{-\infty}^{\infty} f_e(t)\cos\omega t \, dt}_{\text{REAL}} \underbrace{- j2\int_{-\infty}^{\infty} f_o(t)\sin\omega t \, dt}_{\text{IMAGINARY}}$$

EVEN $f(t) \longrightarrow$ REAL $F(j\omega)$

ODD $f(t) \longrightarrow$ IMAG $F(j\omega)$

$|F(j\omega)|$ IS EVEN

$e^{j\theta(\omega)}$ IS ODD

B) SYMMETRY OF $F(j\omega)$ ABOUT FREQ. ORIGIN

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[F_e(j\omega)+F_o(j\omega)\right]e^{j\omega t}dt$$

$$f_e(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F_e(j\omega)\cos\omega t\,d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}\operatorname{Re}F(j\omega)\cos\omega t\,d\omega$$

$$f_o(t)=\frac{j}{2\pi}\int_{-\infty}^{\infty}F_o(j\omega)\sin\omega t\,d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}\operatorname{Im}F(j\omega)\sin\omega t\,d\omega$$

C) CAUSAL TIME FUNCTIONS

$$\left.\begin{aligned}f_e&=\frac{f(t)+f(-t)}{2}=\tfrac{1}{2}f(t)\\ f_o&=\frac{f(t)-f(-t)}{2}=\tfrac{1}{2}f(t)\end{aligned}\right\}\Rightarrow f(t)=2f_e(t)=2f_o(t)$$

$$f(t)=\frac{1}{\pi}\int_{-\infty}^{\infty}F_e(j\omega)\cos\omega t\,d\omega=\frac{1}{\pi}\int_{-\infty}^{\infty}\operatorname{Re}F(j\omega)\cos\omega t\,d\omega$$

$$=\frac{j}{\pi}\int_{-\infty}^{\infty}F_o(j\omega)\sin\omega t\,d\omega=\frac{1}{\pi}\int_{-\infty}^{\infty}\operatorname{Im}F(j\omega)\sin\omega t\,dt$$

PALEY-WIENER CRITERIA FOR CAUSAL FUNCTIONS $\rightarrow \int_{-\infty}^{\infty}\frac{|\ln A(\omega)|}{1+\omega^2}d\omega<\infty$

IV) SYSTEM FUNCTIONS

A) $\rightarrow x(t)\rightarrow \boxed{h(t)} \rightarrow Y(t)\rightarrow$

CONVOLUTION $\rightarrow$ $Y(t)=\int_{-\infty}^{\infty}h(\lambda)\,x(t-\lambda)\,d\lambda$

$$Y(j\omega)=\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}h(\lambda)\,x(t-\lambda)\,d\lambda\right]e^{-j\omega t}dt$$

$$=\int_{-\infty}^{\infty}h(\lambda)\left[\int_{-\infty}^{\infty}x(t-\lambda)\,e^{-j\omega t}dt\right]d\lambda$$

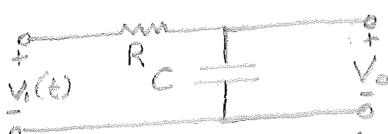
$$=X(j\omega)\,H(j\omega)$$

ie $\mathcal{F}\{f_1(t)*f_2(t)\}=F_1(j\omega)\,F_2(j\omega)$

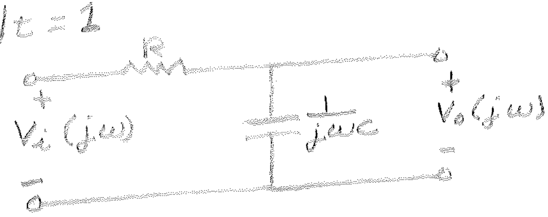
B) $H(j\omega)=\mathcal{F}[h(t)]=\frac{Y(j\omega)}{X(j\omega)}$

$\mathcal{F}\{\delta(t)\}=\int_{-\infty}^{\infty}\delta(t)e^{-j\omega t}dt=1$

C)



$\Leftrightarrow$



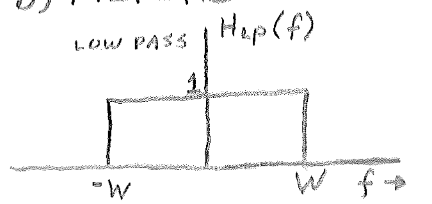
$$H(j\omega)=\frac{V_o(j\omega)}{V_i(j\omega)}=\frac{\frac{1}{j\omega C}}{R+1/j\omega C}$$

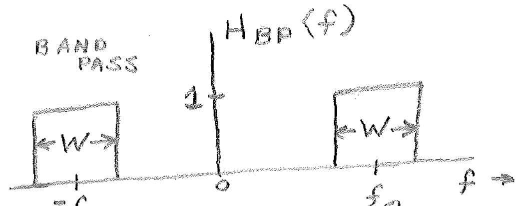
$$=\frac{1/RC}{j\omega+1/RC}$$

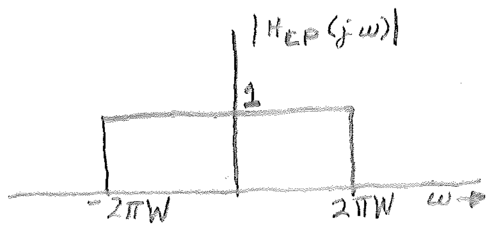
ON A ONE OHM BASIS $|H(j\omega)|=\frac{1}{\sqrt{\omega^2+1}}$

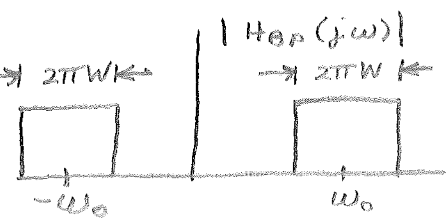
$$|H(j\omega)|_{DB}=10\log_{10}|H(j\omega)|^2=10\log\left(\frac{1}{\omega^2+1}\right)=-10\log(\omega^2+1)$$

D) FILTERS







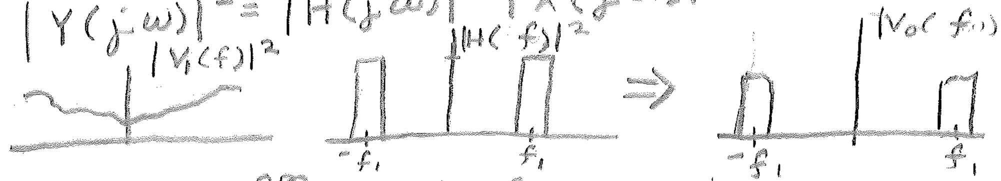


V) ENERGY SPECTRA

A) $E=\int_{-\infty}^{\infty}[f(t)]^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|F(j\omega)|^2 d\omega$ ⇐ PARSEVAL'S THEM.

$$|Y(j\omega)|^2 = |H(j\omega)|^2\,|X(j\omega)|^2$$

B)



$$E_o=\int_{-\infty}^{\infty}|V_o(f)|^2 df$$
$$= 2\int_{f_1-\frac{W}{2}}^{f_1+\frac{W}{2}}|V_i(f)|^2 df$$

$\Rightarrow |V_i(f_1)|^2 = \frac{E_o}{2W}$ ⇒ ENERGY PER UNIT BANDWIDTH

VI) MATHEMATICAL OPERATIONS ON FOURIER TRANSFORMS

A) SCALING
$$f(at) \Leftrightarrow \frac{1}{|a|}F\left(\frac{j\omega}{a}\right)$$

B) DELAY
$$f(t-t_0) \Leftrightarrow e^{-j\omega t_0}F(j\omega)$$

C) MODULATION
$$e^{j\omega_0 t}f(t) \Leftrightarrow F[j(\omega-\omega_0)]$$
$$f(t)\cos\omega_0 t \Leftrightarrow \tfrac{1}{2}F[j(\omega-\omega_0)]+\tfrac{1}{2}F[j(\omega+\omega_0)]$$

D) REVERSAL
$$f(-t) \Leftrightarrow F(-j\omega)$$

E) SYMMETRY
$$F(jt) \Leftrightarrow 2\pi f(-\omega)$$
$$\frac{1}{2\pi}F(-t) \Leftrightarrow f(j\omega)$$

VII) FOURIER TRANSFORMS OF POWER SIGNALS

A) $\mathcal{F}\{sgn(t)\} = \lim_{\alpha\to 0}\int_{-\infty}^{\infty} e^{-\alpha|t|}sgn(t)e^{-j\omega t}dt$

$$sgn(t) \Leftrightarrow \frac{2}{j\omega}$$

B) $\mathcal{F}\{1\} = \lim_{\alpha\to 0} e^{-\alpha|t|}e^{-j\omega t}dt$

$$= \lim_{\alpha\to 0}\left[\frac{2\alpha}{\alpha^2+\omega^2}\right] \quad \begin{matrix}\omega\neq 0 \Rightarrow 0\\ \omega=0\Rightarrow\infty\end{matrix}$$

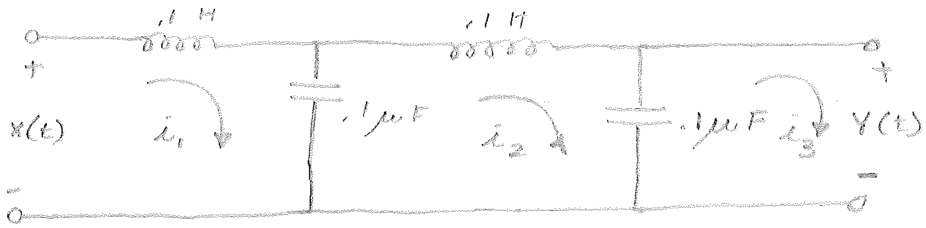
$$\mathcal{F}\{1\} \Leftrightarrow 2\pi\delta(\omega)$$

C) $\mathcal{F}\{\mu(t)\} = \mathcal{F}\left\{\frac{1}{2}+\frac{1}{2}sgn(t)\right\} = \pi\delta(\omega)+\frac{1}{j\omega}$

VIII) $f_1(t)f_2(t) = \frac{1}{2\pi}F_1(j\omega) \circledast F_2(j\omega)$

Pg 24-25   9-17-70

9)



$$x(t) = (.1)\frac{di_1}{dt} + 10^7\int_{-\infty}^{t} i_1(t)dt - 10^7\int_{-\infty} i_2(t)dt$$
$$0 = 10^7\int_{-\infty}^{t} i_2(t)dt + 2\frac{di_2}{dt} - \left(\frac{di_1}{dt} + \frac{di_3}{dt}\right)(.1)$$
$$+Y(t) = -10^7\int_{-\infty}^{t} i_3(t)dt + 10^7\int_{-\infty}^{t} i_2(t)dt$$

$$x'(t) = (.1)\frac{d^2 i_1}{dt^2} + 10^7 i_1(t) - 10^7 i_2(t)$$
$$0 = 10^7 i_2(t) + 2\frac{d^2 i_2}{dt} - (.1)\frac{d^2 i_1}{dt^2} - \frac{d^2 i_3}{dt^2}$$
$$\underline{Y}'(t) = 10^7 i_2(t) - 10^7 i_3(t)$$
$$x'(t) = (.1)\frac{d^2 i_1}{dt^2} + 10^7 i_1(t) - 10^7 i_3(t) - Y'(t)$$

ARG!

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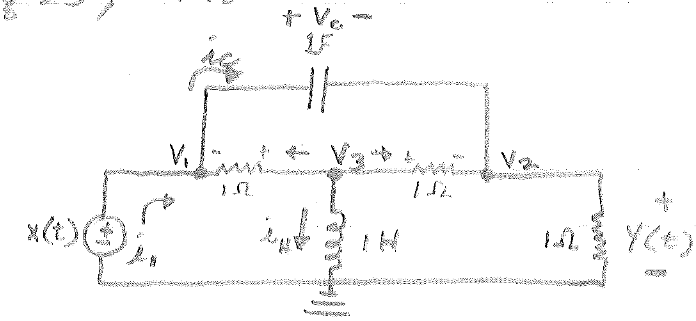
2-10)



$V_1 = x(t)$; $V_2 = Y(t)$

$0 = (V_3 - V_2) + (V_3 - V_1) + \frac{dV_3}{dt} \Rightarrow 2V_3 + \frac{dV_3}{dt} - V_2 - V_1 = 0$

$V_c = V_1 - V_2 = x(t) - Y(t)$

$i_H = dV_3/dt$

Let $V_3 = q_1$; $V_c = q_2$ $\qquad 0 = 2V_3 + \frac{dV_3}{dt} - Y(t) - x(t)$

$i_H = dV_3/dt$; $q_2 = x(t) - Y(t)$

$i_H = dV_3/dt$; $V_c = x(t) - Y(t)$

$V_c = x(t) - Y(t)$; $\quad 2V_3 + \frac{dV_3}{dt} - V_2 - V_1 = 0$

$2V_3 + \frac{dV_3}{dt} - Y(t) - x(t) = 0$

$- V_c - Y(t) + x(t) = 0$

$2V_3 + \frac{dV_3}{dt} + V_c - 2x(t) = 0$

$\frac{dq_1}{dt} = \frac{dV_3}{dt} = -2V_3 - V_c + 2x(t)$

$\cancel{V_c = x(t) - Y(t)}$

$\Rightarrow \cancel{x(t) - Y(t) - V_c = 0}$

$i_c = \frac{dV_c}{dt} \qquad i_c = V_3 - V_1 = V_3 - x(t)$

$\therefore \frac{dV_c}{dt} = V_3 - x(t)$

$\frac{dV_3}{dt} = -2V_3 - V_c + 2x(t)$

$\frac{dV_c}{dt} = V_3 - x(t)$

$Y(t) = x(t) - V_c$

Pg 50 9-23-70

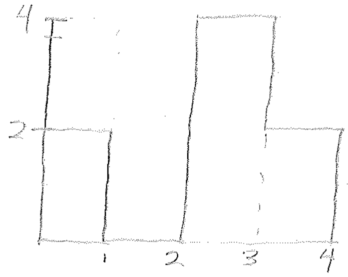
3-8) a) $f(t) = \sum_{n=-\infty}^{\infty} a_n \phi_n(t)$; $g(t) = \sum_{n=-\infty}^{\infty} b_n \phi_n(t) dt$

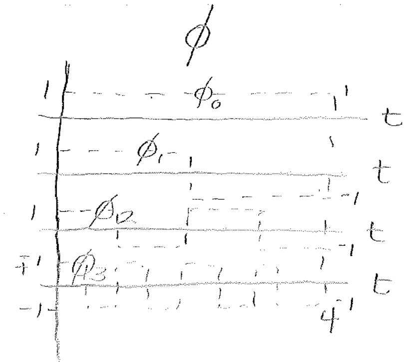
$$I = \int_{t_1}^{t_2} f(t) g(t) dt$$
$$= \int_{t_1}^{t_2} \left[\sum_{-\infty}^{\infty} a_n \phi_n(t) \sum_{n=-\infty}^{\infty} b_n \phi_n(t) \right] dt$$
$$= \sum_{i=-\infty}^{\infty} a_i b_i \int_{t_1}^{t_2} \phi_k(t) \phi_n(t) dt$$
$$= \sum_{i=-\infty}^{\infty} a_i b_i$$

b) $$E = \int_{t_1}^{t_2} i^2(t) dt$$
$$= \int_{t_1}^{t_2} \sum_{i=-\infty}^{\infty} a_i b_i = \sum_{i=-\infty}^{\infty} a_i b_i$$

Pp 50-51 9-23-70

3-10) a)





$$x(t) = \sum_{n=0}^{N} a_n \phi_n(t)$$

$$a_n = \frac{1}{\lambda_n} \int_{t_1}^{t_2} \phi_n(t)\, x(t)$$

$$\frac{1}{\lambda_n} = 1$$

$$a_0 = \int_0^4 (1)\, x(t) = \int_0^1 2\,dt + \int_1^2 0 + \int_2^3 4\,dt + \int_3^4 2\,dt = 8$$

$$a_1 = \int_0^1 (1)\, 2\,dt + \int_2^3 (-1)\, 4\,dt + \int_3^4 (-1)\, 2\,dt$$

$$= 2 - 4 - 2 = -4$$

$$a_2 = \int_0^1 (1)(2)\,dt + \int_2^3 (1)(4)\,dt - \int_3^4 2\,dt$$

$$= 2 + 4 - 2 = 4$$

$$a_3 = a_4 = a_n \ (\text{FOR } n > 2) = 0$$

$$\therefore\ x(t) = 8\phi_0(t) - 4\phi_1(t) + 4\phi_2(t)$$

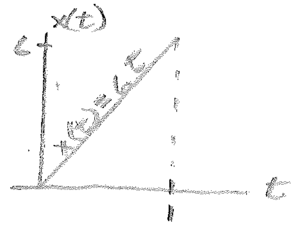
b) $\dfrac{\text{ERROR ENERGY}}{\text{SIGNAL ENERGY}} = 1 - \dfrac{1}{E} \displaystyle\sum_{n=0}^{M} \lambda_n a_n^2$

$$E = \int_{t_1}^{t_2} x^2(t)\,dt = 64$$

$$\therefore\ ER = 1 - \frac{1}{64}\left[64 + 16 + 16\right]$$

Pg 53   9-25-70

3-16) a)



$x(t) =$

$$\alpha_n = \frac{1}{T}\int_{t_1}^{t_1+T} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$\alpha_n = \int_0^1 6t\, e^{-jn2\pi t}\, dt$$

$U = 6t \qquad dV = e^{-jn2\pi t}\, dt$

$dU = 6 \qquad V = \frac{-e^{-jn2\pi t}}{jn2\pi}$

$$\alpha_n = \left[-6t\, e^{-jn2\pi t}\, \frac{1}{jn2\pi}\right]_0^1 + \int_0^1 \frac{6e^{-jn2\pi t}}{jn2\pi}$$

$$= \left[-6te^{-jn2\pi t}\, \frac{1}{jn2\pi}\right]_0^1 - \frac{6}{jn2\pi}\left[\frac{e^{-jn2\pi t}}{jn2\pi}\right]_0^1$$

$$= \frac{-6}{jn2\pi}\left[te^{-jn2\pi t}\right]_0^1 + \frac{6}{n^2 4\pi^2}\left[e^{-jn2\pi t}\right]_0^1$$

$$= \frac{-6}{jn2\pi}\left[e^{-jn2\pi}\right] + \frac{6}{n^2 4\pi^2}\left[e^{-jn2\pi} - 1\right]$$

$$= \cancel{\frac{-6}{jn2\pi}\left[\cos n2\pi - j\sin n2\pi\right] + \frac{6}{n^2 4\pi^2}}$$

$$= e^{-jn2\pi}\left[\frac{-6}{jn2\pi} + \frac{6}{n^2 4\pi^2}\right] - \frac{6}{n^2 4\pi^2}$$

$$= \left[\cos n2\pi - j\sin n2\pi\right]\left[\frac{-6}{jn2\pi} + \frac{6}{n^2 4\pi^2}\right] - \frac{6}{n^2 4\pi^2}$$

$$= \left[1 - 0\right]\left[\frac{-6}{jn2\pi} + \frac{6}{n^2 4\pi^2}\right] - \frac{6}{n^2 4\pi^2} = \frac{-6}{jn2\pi}$$

$$\therefore\ x(t) = \sum_{-\infty}^{\infty} \frac{-3}{jn\pi}\, e^{jn2\pi t}$$

b) $E = \frac{1}{T}\int_0^1 36t^2\,dt$

$= 36\frac{t^3}{3} = \left[12t^2\right]_0^1 = 12$

$$\frac{EAR}{SIG} = 1 - \frac{1}{E}\sum_{n=0}^{M} \lambda_n a_n^2$$

$$\alpha_n = \frac{-6}{jn2\pi}$$

$$\alpha_{-3} = \frac{+6}{j6\pi} = -\frac{j}{\pi}$$

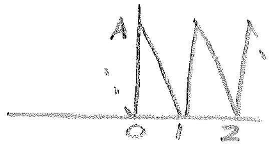
$$\alpha_{-2} = \frac{+6}{j4\pi} = -\frac{j3}{2\pi}$$

$$\alpha_{-1} = \frac{6}{j\pi} = -\frac{j}{\pi}$$

$$\alpha_0 = \infty \quad \text{ARG!}$$

$$\alpha_n = \frac{6}{n2\pi}$$

Pg 53 9-25-70

3-17) 

$$x(t) = -At + A$$

$$\alpha_n = \frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}\,dt$$

$$\alpha_n = \frac{1}{1}\int_0^1 (A-At)e^{-jn2\pi t}\,dt$$
$$= A\int_0^1 e^{-jn2\pi t}\,dt - A\int_0^1 t\,e^{-jn2\pi t}\,dt$$
$$\frac{\alpha_n}{A} = \left[\frac{e^{-jn2\pi t}}{-jn2\pi}\right]_0^1 - \int_0^1 t\,e^{-jn2\pi t}\,dt$$
$$M = \int_0^1 t\,e^{-jn2\pi t}\,dt$$
$$U = t \qquad dV = e^{-jn2\pi t}$$
$$dU = dt \qquad V = \frac{e^{-jn2\pi t}}{-jn2\pi}$$
$$M = \left[+t\,\frac{e^{-jn2\pi t}}{+jn2\pi}\right]_0^1 + \frac{1}{jn2\pi}\int_0^1 e^{-jn2\pi t}\,dt$$
$$= \frac{1}{jn2\pi}e^{-jn2\pi} - \frac{1}{4n^2\pi^2}\left[e^{-jn2\pi t}\right]_0^1$$
$$= \frac{1}{jn2\pi}e^{-jn2\pi} - \frac{1}{4n^2\pi^2}\left[e^{-jn2\pi} - 1\right]$$
$$= \frac{1}{jn2\pi}e^{-jn2\pi} - \frac{1}{4n^2\pi^2}e^{-jn2\pi} + \frac{1}{4n^2\pi^2}$$
$$\Rightarrow \frac{\alpha_n}{A} = \frac{j}{2n\pi}\left[e^{-jn2\pi} - 1\right] + \cancel{\frac{j}{2\pi n}e^{-jn2\pi}} + \frac{1}{4n^2\pi^2}e^{-jn2\pi} + \frac{1}{4n^2\pi^2}$$
$$\alpha_n = A\left[\frac{1}{j2n\pi} + \frac{1}{4n^2\pi^2}e^{-jn2\pi} - \frac{1}{4n^2\pi^2}\right]$$
$$= A\left[\frac{1}{j2n\pi} - \frac{1}{4n^2\pi^2} + \frac{1}{4n^2\pi^2}(\cos 2\pi n - j\sin 2\pi n)\right]$$
$$= A\left[\frac{1}{j2\pi n}\right]$$
$$= \frac{A}{j2\pi n}$$
$$\Rightarrow x(t) = \sum_{n=-\infty}^{\infty}\frac{A}{j2\pi n}e^{jn2\pi t}$$

Pg 49 9-27-70

3-2) a) $x(t) = 10 \sin 20\pi t \; ; \; t \geq 0$

$= 0 \quad ; t < 0$

$T = \frac{1}{10}$

$\lim_{T\to\infty} \frac{50}{T} \int_{-T}^{T} \sin^2 20\pi t \, dt$

$\frac{25}{T} \int_{-T}^{T} [1 - \cos 40\pi t] dt$

$\frac{25}{T} \left[t - \frac{\sin 40\pi t}{40\pi} \right]_{-T}^{T}$

$\frac{25}{T} \left[\left(T - \frac{\sin 40\pi T}{40\pi}\right) + \left(+T - \frac{\sin 40\pi T}{40\pi}\right) \right]$

$\frac{25}{T} \left[2T - \frac{\sin 40\pi T}{20\pi} \right]$

$50 - \frac{5 \sin 40\pi T}{4\pi T}$

$-1 \leq \sin 40\pi T \leq 1$

$\therefore 0 < \lim_{T\to\infty} < \infty$

POWER SIGNAL

$P_{AVE} = \frac{100}{T} \int_0^T \sin^2 20\pi t \, dt$

$= \frac{50}{T} \int_0^T (1 - \cos 40t) dt$

$= 500 \int_0^{1/10} (1 - \cos 40\pi t) dt$

$= 500 \left[t - \frac{\sin 40\pi t}{40\pi} \right]_0^{1/10}$

$= 500 \left[\frac{1}{10} - \frac{\sin 4\pi}{40\pi} \right] = 50$

b) $x(t) = 10 e^{-5t} \; ; \; t \geq 0$

$= 0 \quad ; t < 0$

$E = \int_{-\infty}^{\infty} 100 e^{-10t} dt$

$= 100 \left[\frac{e^{-10t}}{-10} \right]_{-\infty}^{\infty} \Rightarrow$ NOT E SIGN.

$\frac{100}{2T} \int_{-T}^{T} e^{-10t} dt$

$\frac{50}{-10T} \left[e^{-10t} \right]_{-T}^{T} = \frac{-5}{T} \left[e^{-10T} - e^{10T} \right]$

$= \frac{-5e^{-10T}}{T} + \frac{5e^{10T}}{T}$

$= 0 + \frac{50e^{10T}}{1} = \infty$

(CONT)

b) $x(t) = 10e^{-5t} \quad t \geq 0$

$= 0 \quad t < 0$

$100 \int_0^\infty e^{-10t}\,dt$

$100 \; \frac{-1}{10}\left[e^{-10t}\right]_0^\infty$

$-10\,[0-1] = 10$

$\Rightarrow E = 10$

c) $x(t) = 10\sin 20\pi t + 5\cos 22\pi t \;;\; -\infty < t < \infty$

$\omega_{0_1} = 20\pi = 2\pi f \Rightarrow 10 = f \Rightarrow T_1 = \frac{1}{10}$

$\omega_{0_2} = 22\pi = 2\pi f \Rightarrow 11 = f \Rightarrow T_2 = \frac{1}{11}$

T FOR $x(t) = 1.1$

$\frac{1}{1.1}\int_0^{1.1}(10\sin 20\pi t + 5\cos 22\pi t)^2\,dt$

$.911 \int_0^{1.1}(100\sin^2 20\pi t + 50\sin 20\pi t\cos 22\pi t + 25\cos^2 22\pi t)\,dt$

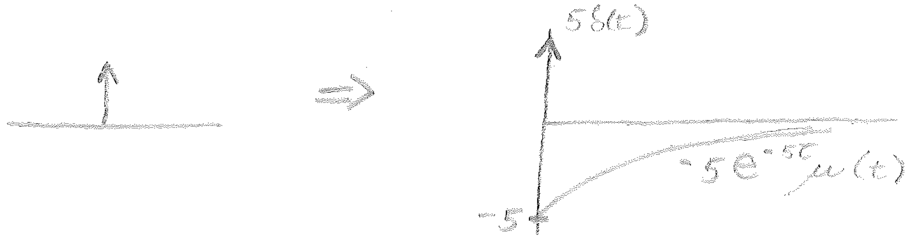
$a = 20\pi \; b = 22\pi$

$.911\left[-\frac{1}{2}\frac{\cos -2\pi t}{-2\pi} + \right.$ ARG!

d)

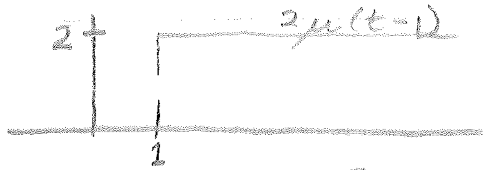
Pg 80A

4-2)



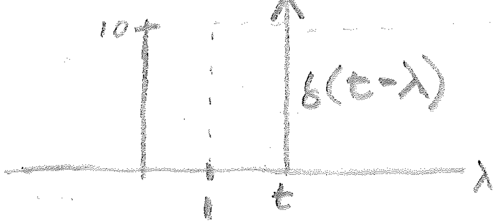
$$Y(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$

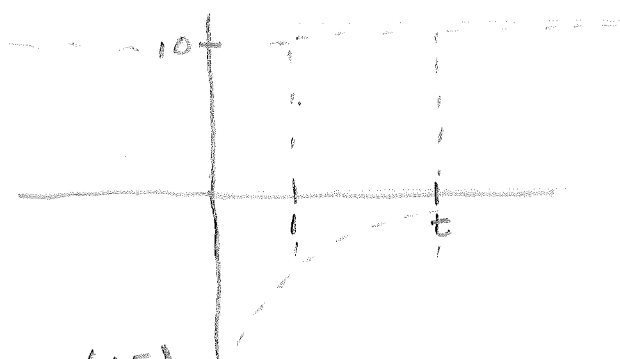
$x(t) \Rightarrow$



$$Y(t) = \int_{-\infty}^{\infty} 2\mu(\lambda-1)\left[5\delta(t-\lambda) - 5e^{-5(t-\lambda)}\mu(t-\lambda)\right]dt$$

$$Y(t) = \int_{-\infty}^{\infty} 10\mu(\lambda-1)\,\delta(t-\lambda)\,dt - \int_{-\infty}^{\infty} 10\mu(\lambda-1)\, e^{-5(t-\lambda)}\,\mu(t-\lambda)\,dt$$





$$Y(t) = \overset{t>1}{10} - \int_1^t 10e^{-5t+5\lambda}\,dt$$

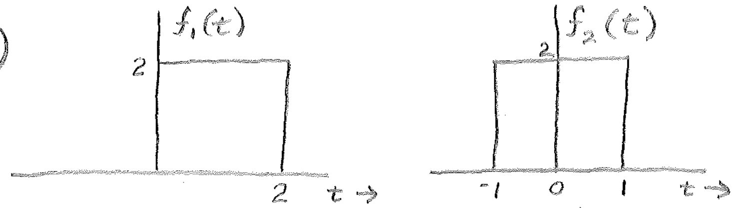
$$\underset{t>1}{=} 10 - 10\left[\frac{e^{-5t+5\lambda}}{5}\right]_1^t$$

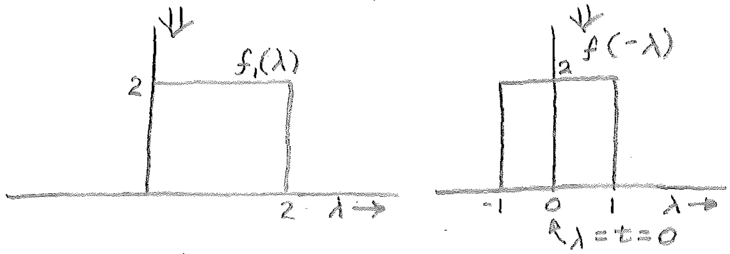
$$= 10 - 2\left[1 - e^{-5(t+1)}\right]$$

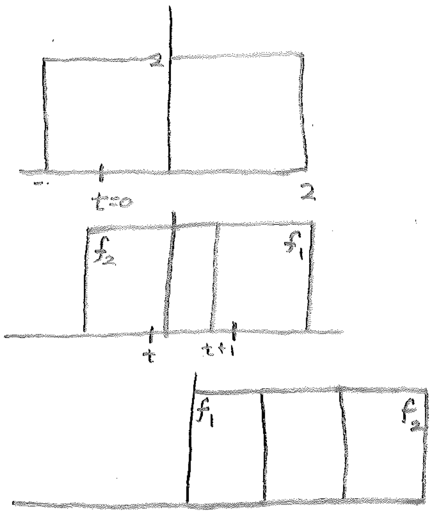
$$= \left[8 + 2e^{-5(t+1)}\right]\mu(t-1)$$

Pg. 80   10-2-70

4-3) a)





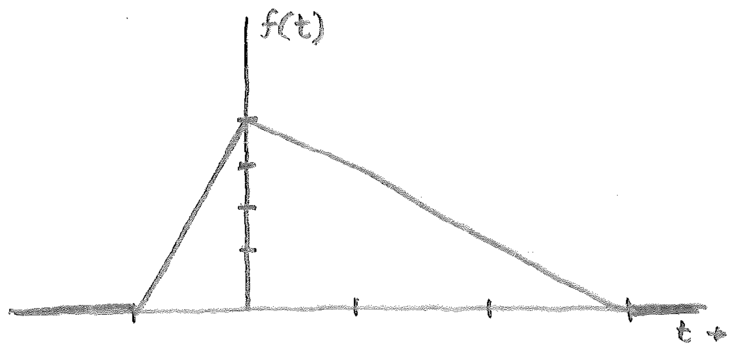


$t < -1 \Rightarrow \int_{-\infty} f_1(\lambda) f_2(\lambda)\, d\lambda = 0$

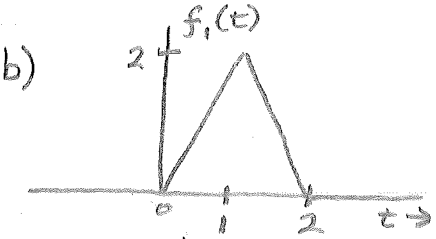
$$-1 < t < 1 \Rightarrow \int_0^{t+1} f_1(\lambda)\, f_2(\lambda)\, d\lambda$$
$$= \int_0^{t+1} 4\, d\lambda = \left|4\lambda\right|_0^{t+1}$$
$$= 4t + 4$$

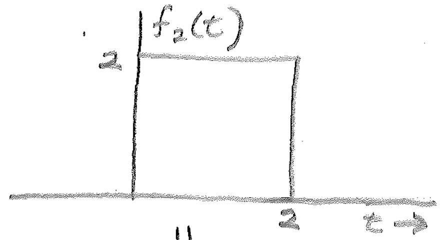
$$1 < t < 3 \Rightarrow \int_{t-1}^{2} 4\, d\lambda = \left[4\lambda\right]_{t-1}^{2}$$
$$= 8 - 4(t-1)$$
$$= 12 - 4t$$

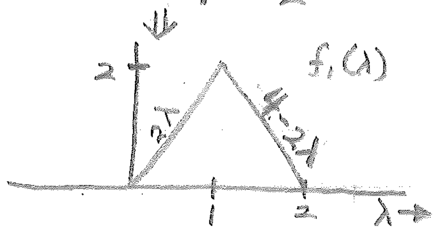
$3 < t \Rightarrow f_1 * f_2 = 0$

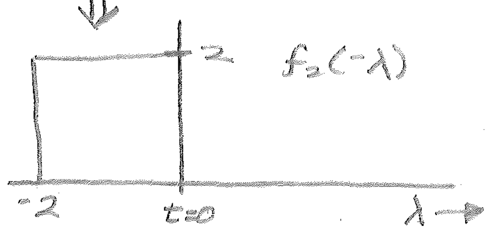


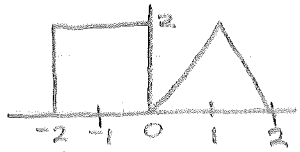
b)



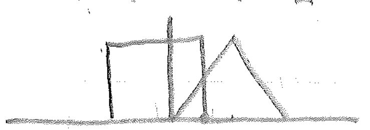








$t<0 \Rightarrow \quad f_1 * f_2 = 0$



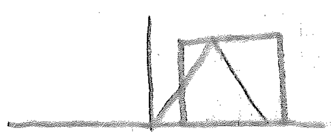
$0<t<1 \Rightarrow \int_0^t 4\lambda \, dY = \left. 2\lambda^2 \right]_0^t = 2t^2$

$1<t<2 \Rightarrow \int_0^1 4\lambda \, d\lambda + \int_1^t (8-4\lambda)\, d\lambda$

$= 2 + \left[8\lambda - 2\lambda^2 \right]_1^t$

$= 2 + (8t - 2t^2) - (8-2)$

$= -2t^2 + 8t - 4$



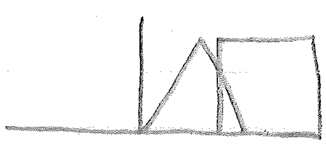
$2<t<3 \Rightarrow \int_{t-2}^1 4\lambda \, d\lambda + \int_1^2 (8-4\lambda)\, d\lambda$

$= \left[4\lambda^2 \right]_{t-2}^1 + \left[8\lambda - 2\lambda^2 \right]_1^2$

$= 4 - 4(t-2)^2 + (16-16) - 6$

$= -6 - (4t^2 - 16t + 16)$

$= -4t^2 + 16t - 22$



$3<t<4 \Rightarrow \int_{t-2}^2 (8-4\lambda)\, d\lambda$

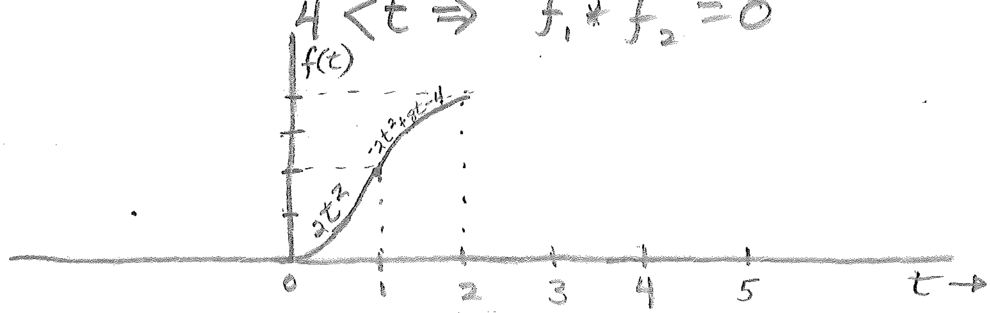
$\left[8\lambda - 2\lambda^2 \right]_{t-2}^2$

$(16-16) - 8(t-2) + 2(t-2)^2$

$= -8t + 16 + 2t^2 - 8t + 8$

$= 2t^2 - 16t + 24$

$4 < t \Rightarrow \quad f_1 * f_2 = 0$



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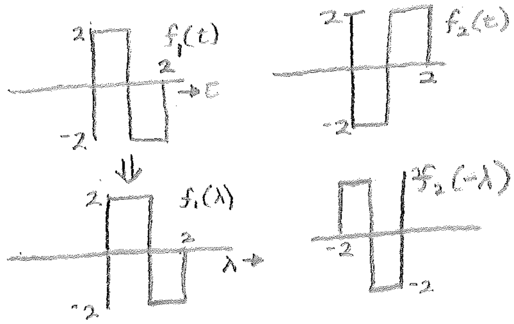
4-3) d)

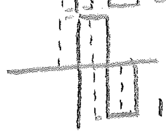


1) $t<0 \Rightarrow f_1 * f_2 = 0$

2) 

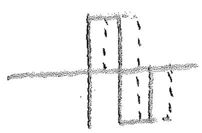
$0<t<1$

$$f_1 * f_2 = \int_0^t -4\,d\lambda = -4\lambda\Big]_0^t$$
$$-4t$$

3) 

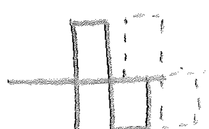
$1<t<2$

$$f_1 * f_2 = \int_0^{t-1} 4\,d\lambda + \int_{t-1}^{1} -4\,d\lambda + \int_1^t 4\,d\lambda$$
$$= 4\lambda\Big]_0^{t-1} + 4\lambda\Big]_{t-1}^{1} + 4\lambda\Big]_1^t$$
$$= 4(t-1) + [(4) + 4(t-1)] + 4t - 4$$
$$= 4t - 4 - 4 + 4t - 4 + 4t - 4$$
$$= 12t - 16$$

4) 

$2<t<3$

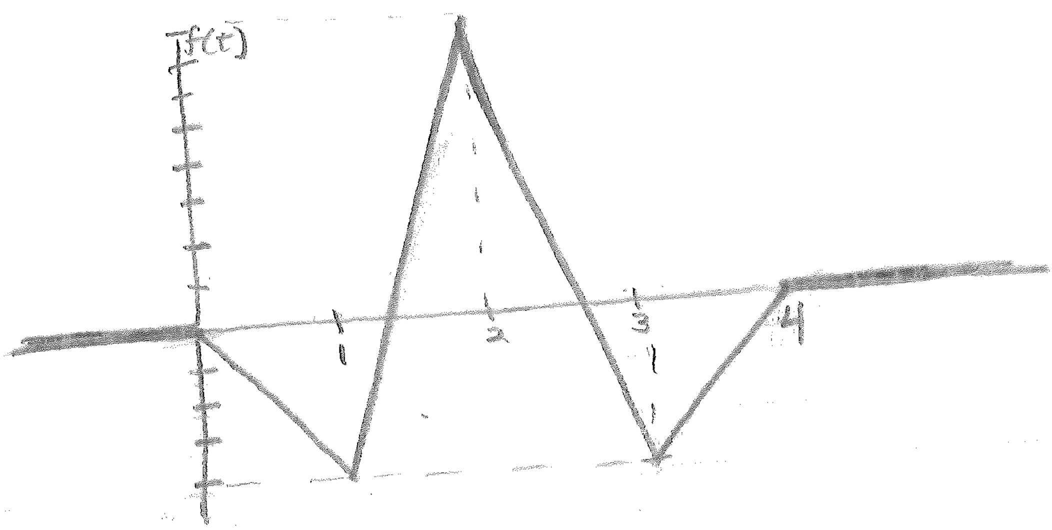
$$f_1 * f_2 = \int_{t-2}^{1} 4\,d\lambda - \int_1^{t-1} 4\,d\lambda + \int_{t-1}^{2} 4\,d\lambda$$
$$= 4\lambda\Big]_{t-2}^{1} - 4\lambda\Big]_1^{t-1} + 4\lambda\Big]_{t-1}^{2}$$
$$= [4 - 4(t-2)] + [4(t-1) + 4] + [8 - 4(t-1)]$$
$$= 4 - 4t + 8 - 4t + 4 + 4 + 8 - 4t + 4$$
$$= -12t + 32$$

5) 

$3<t<4$

$$f_1 * f_2 = \int_{t-2}^{2} -4\,d\lambda$$
$$= -4\lambda\Big]_{t-2}^{2}$$
$$= -8 + 4(t-2)$$
$$= 4t - 16$$

6) $t>4 \Rightarrow f_1 * f_2 = 0$



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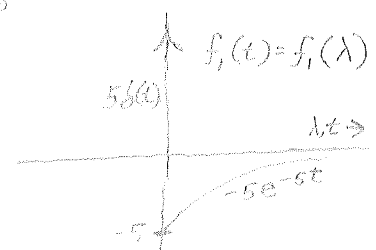
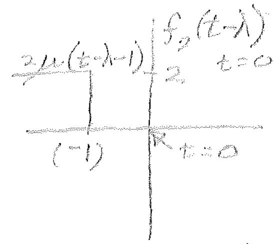
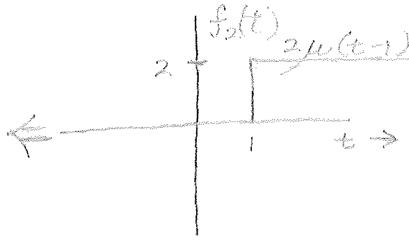
4-2)

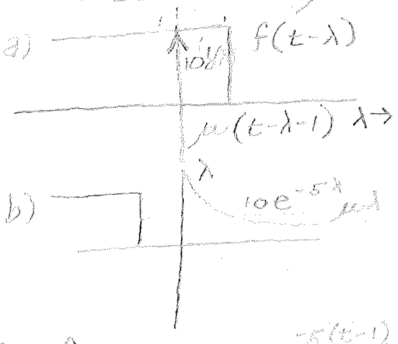






$$f_1 * f_2 = \int_{-\infty}^{\infty} \left[5\delta(\lambda) - 5e^{-5\lambda}\mu(t)\right] 2\mu(t-\lambda-1)\,d\lambda$$
$$= \int_{-\infty}^{\infty} 10\delta(\lambda)\mu(t-\lambda-1)\,d\lambda - \int_{-\infty}^{\infty} 10e^{-5\lambda}\mu(\lambda)\mu(t-\lambda-1)\,d\lambda$$

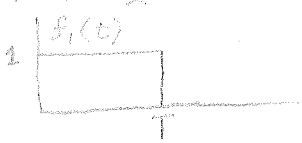
a) $t \geq 1 \Rightarrow \int_{-\infty}^{\infty} 10\delta(\lambda)\mu(t-\lambda-1)\,d\lambda = 10$

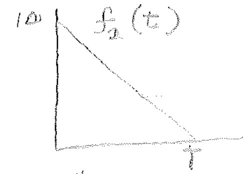


b) $t \geq 1 \Rightarrow \int_0^{t-1} 10e^{-5\lambda}\,d\lambda$
$$= 10\left[-2e^{-5\lambda}\right]_0^{t-1}$$
$$= -2e^{-5(t-1)} + 2$$

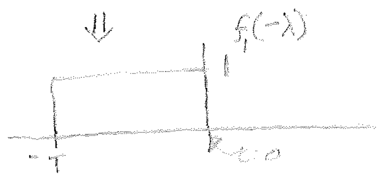
$$\therefore f_1 * f_2 = 10 + 20e^{-5(t-1)} - 2 = \left(8 + 20e^{-5(t-1)}\right)\mu(t-1)$$

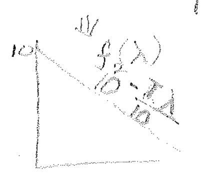
4-7) b)





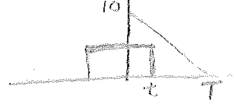
$T = K$





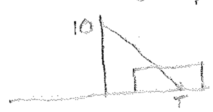
1) $t < 0 \Rightarrow f_1 * f_2 = 0$

2)



$0 \leq t \leq T \Rightarrow f_1 * f_2 = \int_0^t \left(10 - \frac{T\lambda}{10}\right)d\lambda = \left|10\lambda - \frac{T\lambda^2}{20}\right|_0^t$
$$= 10t - \frac{Kt^2}{20}$$

3)

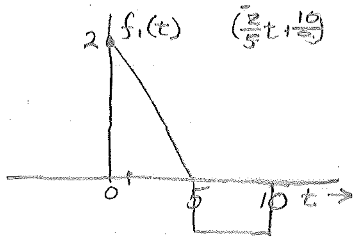


$T \leq t \leq 2T \Rightarrow f_1 * f_2 = \int_{t-T}^{T}\left(10 - \frac{T\lambda}{10}\right)d\lambda = \left|10\lambda - \frac{T\lambda^2}{10}\right|_{t-T}^{T}$
$$= \left[10T - \frac{T^3}{10}\right] - \left[10(t-T) - \frac{T(t-T)^2}{10}\right]$$
$$= 10T - \cancel{\frac{T^3}{10}} - 10t + 10T + \frac{Tt^2}{10} - \frac{2tT^2}{10} + \cancel{\frac{T^3}{10}}$$
$$= \frac{Tt^2}{10} - t\left(10 + \frac{2T}{10}\right) + 20T$$

4) $t \geq 2T \Rightarrow f_1 * f_2 = 0$

ETC.

4-9)



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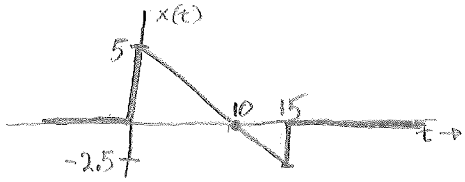
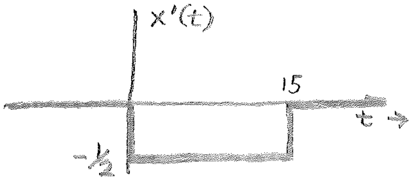
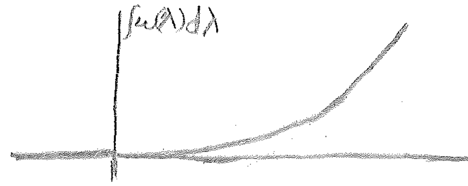
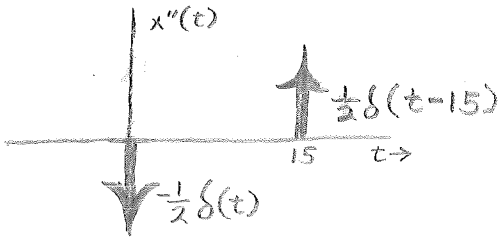


$$w(t) = \left[1 - e^{-\frac{t}{RC}}\right]\mu(t)$$

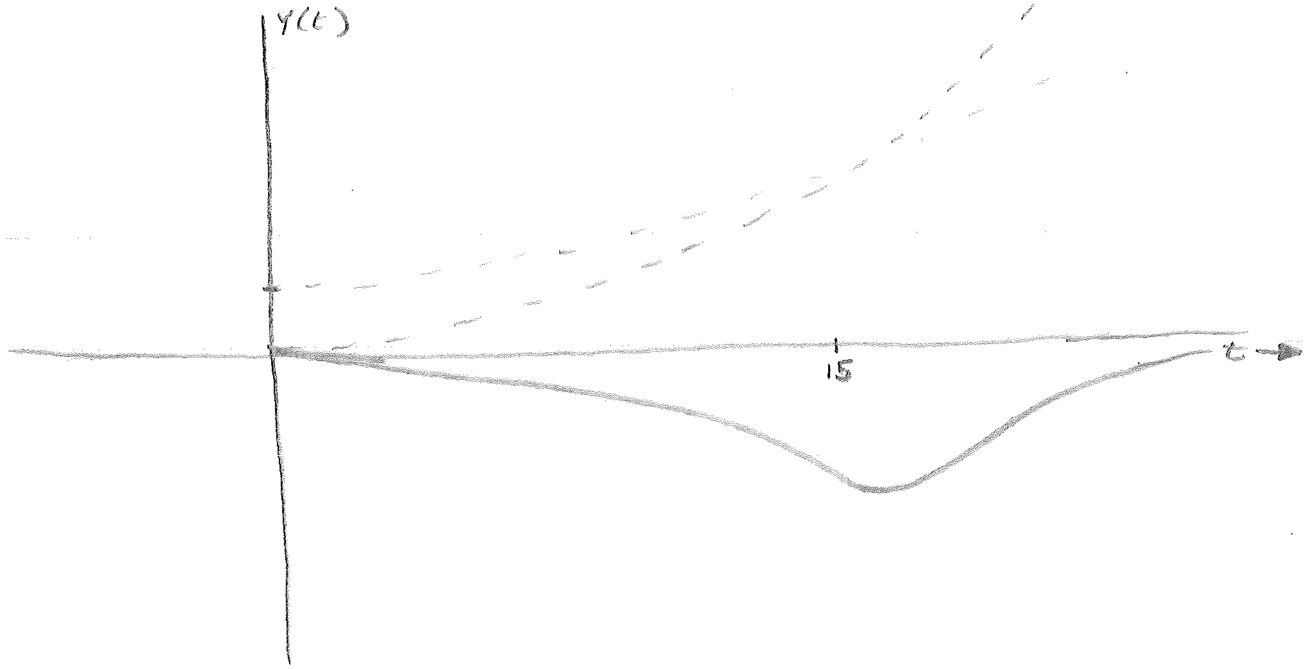


$$\int_{-\infty}^{t} w(\lambda)\,d\lambda$$
$$= \int_{0}^{t}\left[1 - e^{-\frac{\lambda}{RC}}\right]d\lambda$$
$$= \lambda + RCe^{-\frac{\lambda}{RC}}\Big|_{0}^{t}$$
$$= \left[t + RCe^{-\frac{t}{RC}} - RC\right]\mu(t)$$





$$Y = x''(t) * \int_{-\infty}^{t} w(\lambda)\,d\lambda$$
$$= -\frac{1}{2}\left[t + RCe^{-\frac{t}{RC}} - RC\right]\mu(t) + \frac{1}{2}\left[(t-15) + RCe^{-\frac{(t-15)}{RC}} - RC\right]\mu(t-15)$$
$$= \frac{1}{2}\left[t + RCe^{-\frac{(t-15)}{RC}} - (RC+15)\right]\mu(t-15) - \frac{1}{2}\left[t + RCe^{-t/RC} - RC\right]\mu(t)$$



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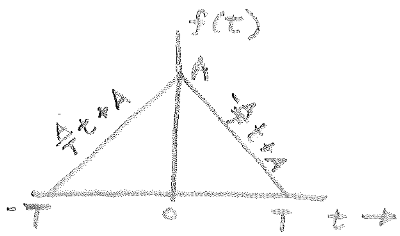
5-3)



$\mathcal{F}[f(t)] = F(j\omega)$
$= \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$
$= \int_{-T}^{0} (\frac{A}{T}t + A) e^{-j\omega t} dt$
$+ \int_0^T (\frac{-A}{T}t + A) e^{-j\omega t} dt$

$F(j\omega) = \int_{-T}^0 \frac{A}{T} t e^{-j\omega t} dt + \int_{-T}^0 A e^{-j\omega t} dt + \int_0^T \frac{-At}{T} e^{-j\omega t} dt + \int_0^T A e^{-j\omega t} dt$

$u = \frac{A}{T}t \quad dv = e^{-j\omega t}dt$

$du = \frac{A}{T}dt \quad V = \frac{-e^{-j\omega t}}{j\omega}$

$\frac{-A}{Tj\omega} t e^{-j\omega t}\Big]_{-T}^0 + \int_{-T}^0 \frac{A}{Tj\omega} e^{-j\omega t} dt$

" $- \frac{A}{T(j\omega)^2} e^{-j\omega t}\Big]_{-T}^0$

$\frac{-A}{Tj\omega}[0 + Te^{j\omega T}] + \left[\frac{A}{T\omega^2} e^{-j\omega t}\right]_{-T}^0$

$\frac{-AT}{Tj\omega} e^{j\omega T} + \frac{A}{T\omega^2}[1 - e^{j\omega T}]$

$\Rightarrow \frac{-A}{j\omega}e^{j\omega T} - \frac{A}{T\omega^2}e^{j\omega t} + \frac{A}{T\omega^2}$

$\int_0^T -\frac{At}{T} e^{-j\omega t} dt$

$u = \frac{-At}{T} \quad dv = e^{-j\omega t}dt$

$du = \frac{-A}{T}dt \quad V = \frac{-e^{-j\omega t}}{j\omega}$

$\frac{+A}{Tj\omega} t e^{-j\omega t}\Big]_0^T - \int_0^T \frac{A}{Tj\omega} e^{-j\omega t}$

" $- \frac{A}{T(j\omega)^2} e^{-j\omega t}\Big]_0^T$

$\frac{AT}{Tj\omega} e^{-j\omega T} - \frac{A}{T\omega^2}[e^{-j\omega T} - 1]$

$\Rightarrow \frac{A}{j\omega}e^{-j\omega T} - \frac{A}{T\omega^2}e^{-j\omega T} + \frac{A}{T\omega^2}$

$\int_{-T}^0 A e^{-j\omega t}dt = \frac{-A}{j\omega}e^{-j\omega t}\Big]_{-T}^0 = \frac{-A}{j\omega}[1 - e^{j\omega T}]$

$\Rightarrow \frac{-A}{j\omega} + \frac{A}{j\omega}e^{j\omega T}$

$\int_0^T A e^{-j\omega t}dt = \frac{-A}{j\omega}e^{-j\omega t}\Big]_0^T = \frac{-A}{j\omega}[e^{-j\omega T} - 1]$

$\Rightarrow \frac{A}{j\omega} - \frac{A}{j\omega}e^{-j\omega T}$

$\therefore F(j\omega) = \frac{-A}{j\omega}e^{j\omega T} - \frac{A}{T\omega^2}e^{j\omega T} + \frac{A}{T\omega^2} + \frac{A}{j\omega}e^{-j\omega T} - \frac{A}{T\omega^2}e^{-j\omega T} + \frac{A}{T\omega^2} - \frac{A}{j\omega} + \frac{A}{j\omega}e^{j\omega T} + \frac{A}{j\omega} - \frac{A}{j\omega}e^{-j\omega T}$

$= \left[\frac{-A}{j\omega} - \frac{A}{T\omega^2} + \frac{A}{j\omega}\right]e^{j\omega T} + \left[\frac{A}{j\omega} - \frac{A}{T\omega^2} - \frac{A}{j\omega}\right]e^{-j\omega T} + \left[\frac{A}{T\omega^2} + \frac{A}{T\omega^2}\right]$

$= \frac{-A}{T\omega^2}\left[e^{j\omega T} + e^{-j\omega T}\right] + \frac{2A}{T\omega^2}$

$= \frac{-2A}{T\omega^2}\cos \omega t + \frac{2A}{T\omega^2}$

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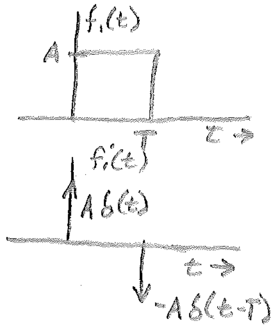
5-6) a)



$$\mathcal{F}\{f'(t)\} = A - Ae^{-j\omega T}$$

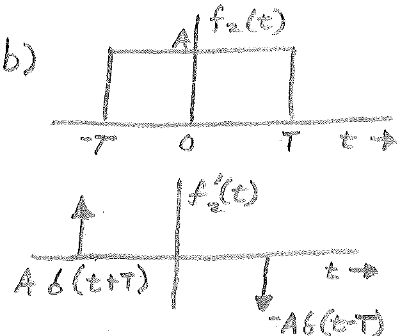
$$\Rightarrow \mathcal{F}\{f(t)\} = \frac{A(1-e^{-j\omega T})}{j\omega}$$

$$F(j\omega) = \frac{2A}{\omega} e^{-j\omega T/2} \frac{(e^{j\omega T/2} - e^{-j\omega T/2})}{2j}$$

$$= \frac{2A}{\omega} e^{-j\omega T/2} \sin\frac{\omega T}{2}$$

$$= AT e^{-j\omega T/2} \sin\frac{\omega T}{2} \Big/ \frac{\omega T}{2}$$

b)



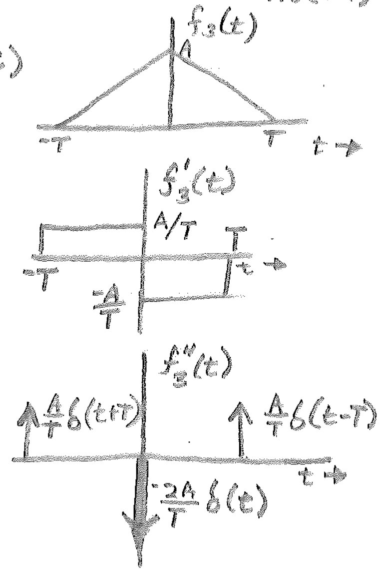
$$\mathcal{F}\{f'(t)\} = Ae^{-j\omega T} - Ae^{j\omega T}$$

$$F(j\omega) = \frac{-2A}{\omega} \frac{-e^{-j\omega T} + e^{j\omega T}}{2j}$$

$$= \frac{-2A}{\omega} \sin \omega T$$

$$= -2AT \frac{\sin \omega T}{\omega T}$$

c)



$$\cancel{\mathcal{F}\{f''(t)\} = -\frac{2A}{T}}$$

$$\cancel{\mathcal{F}\{f(t)\} = \frac{-2A}{T(j\omega)^2} = \frac{2A}{T\omega^2}}$$

$$\mathcal{F}\{f''(t)\} = \frac{A}{T} e^{+j\omega T} - \frac{2A}{T} + \frac{A}{T} e^{-j\omega T}$$

$$F(j\omega) = \frac{Ae^{j\omega T}}{T(j\omega)^2} - \frac{2A}{T(j\omega)^2} + \frac{A}{T(j\omega)^2} e^{-j\omega T}$$

$$= \frac{-Ae^{j\omega T}}{T\omega^2} + \frac{2A}{T\omega^2} - \frac{Ae^{-j\omega T}}{T\omega^2}$$

$$= \frac{-A[-2 + e^{j\omega T} + e^{-j\omega T}]}{T\omega^2}$$

$$= +\frac{4A}{T\omega^2} \frac{[e^{j\omega T/2} - e^{-j\omega T/2}]^2}{-4}$$

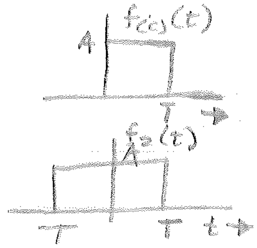
$$= \frac{4A}{T\omega^2} \sin^2 \frac{\omega T}{2}$$

$$= AT \left[\frac{\sin \frac{\omega T}{2}}{\frac{\omega T}{2}}\right]^2$$

5-8) $F_2(j\omega) = 2\,\mathrm{Re}\left[F_c(j\omega)\right]$

$F_3(j\omega) = 2\,\mathrm{Im}\left[F_c(j\omega)\right]$

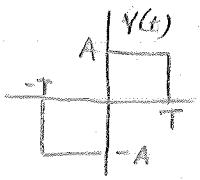
a) EVEN FUNCTION



$$F_c(j\omega) = AT\, e^{-j\frac{\omega T}{2}} \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}$$

$$F_2(j\omega) = 2TA \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}$$

b) ODD FUNC.

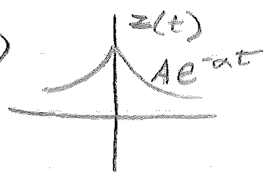


$$F_c(j\omega) = AT\, e^{-j\frac{\omega T}{2}} \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}$$

$$= AT\left(\cos\frac{\omega T}{2} - j\sin\frac{\omega T}{2}\right] \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}$$

$$Y(j\omega) = -2jAT \frac{\sin^2\frac{\omega T}{2}}{\frac{\omega T}{2}}$$

c)



$$F(j\omega) = \frac{A}{(\alpha + j\omega)} \frac{(\alpha - j\omega)}{(\alpha - j\omega)}$$

$$= \frac{(\alpha - j\omega)A}{(\alpha^2 + \omega^2)}$$

$$Z(j\omega) = \frac{2A\alpha}{\alpha^2 + \omega^2}$$

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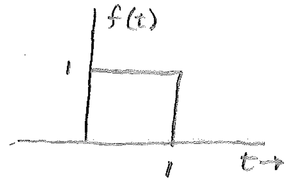
5-2)



$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_0^1 e^{-j\omega t}\, dt$$

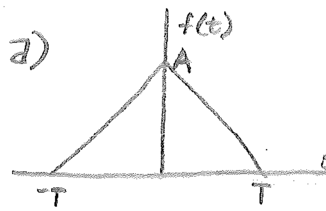
$$= \left. \frac{e^{-j\omega t}}{-j\omega} \right|_0^1$$

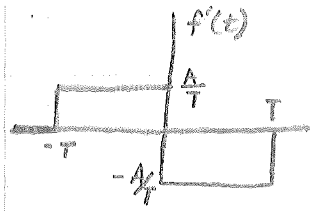
$$= \frac{1}{-j\omega}\left[e^{-j\omega} - 1\right]$$

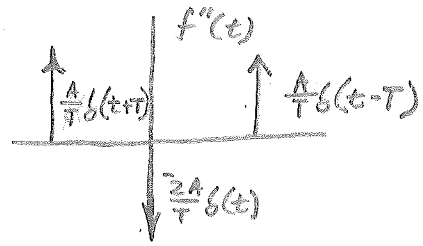
$$= \frac{e^{j\omega/2}\left[e^{j\omega/2} - e^{-j\omega/2}\right]}{-j\omega}$$

$$= \frac{1}{2} e^{-\frac{j\omega}{2}} \frac{\sin \frac{\omega}{2}}{\omega/2}$$

5-3) a)







$$\mathcal{F}[f''(t)]\, F_2(j\omega) = \frac{A}{T}\left[e^{j\omega T} - 2 + e^{-j\omega T}\right]$$

$$= \frac{A}{T}\left[e^{-\frac{j\omega T}{2}} - e^{\frac{j\omega T}{2}}\right]^2$$

$$= \frac{-4A}{T}\,\frac{\left[e^{-\frac{j\omega T}{2}} - e^{\frac{j\omega T}{2}}\right]^2}{-4}$$

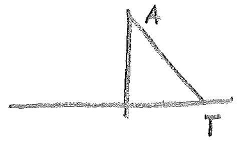
$$= \frac{-4A}{T} \sin^2 \frac{\omega T}{2}$$

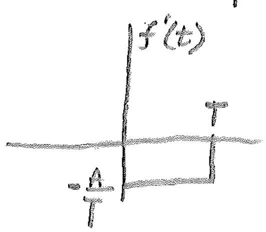
$$= \cancel{\frac{T(-4A\omega^2)}{T\,4}}\; \frac{\sin^2 \frac{\omega T}{2}}{\frac{\omega^2 T^2}{4}}$$

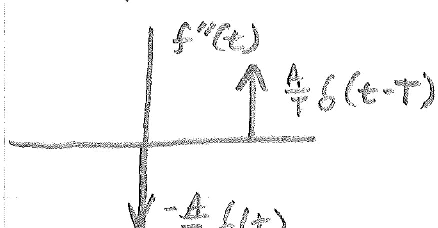
$$= -TA\omega^2 \left[\frac{\sin \frac{\omega T}{2}}{\frac{\omega T}{2}}\right]^2$$

$$F(j\omega) = TA\left[\frac{\sin \frac{\omega T}{2}}{\frac{\omega T}{2}}\right]^2$$

~~b)~~ EVEN FUNC







$$\mathcal{F}[f''(t)] = \frac{A}{T}\left[-1 + e^{-j\omega T}\right]$$

$$= \frac{A}{T}\left[e^{-j\omega T} - 1\right]$$

$$\mathcal{F}[f(t)] = \frac{A}{-\omega^2 T} e^{-\frac{j\omega T}{2}}\left[e^{-\frac{j\omega T}{2}} - e^{\frac{j\omega T}{2}}\right]$$

$=$

~~3.1~~ a) $h(t) = \left[e^{-t} - e^{-3t}\right] \mu(t)$

$$\mathcal{F}^{-1}[h(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} h(t) e^{j\omega t}\, dt$$

$$= \frac{1}{2\pi}\int_{0}^{\infty} \left[e^{-t} - e^{-3t}\right] e^{j\omega t}\, dt$$

$$= \frac{1}{2\pi}\int_{0}^{\infty} \left[e^{t(j\omega-1)} - e^{t(j\omega-3)}\right] dt$$

$$= \frac{1}{2\pi} \frac{1}{[j\omega-1]} e^{t(j\omega-1)} - \frac{1}{2\pi} \frac{1}{}$$

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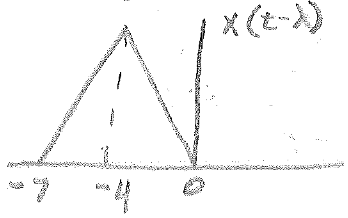
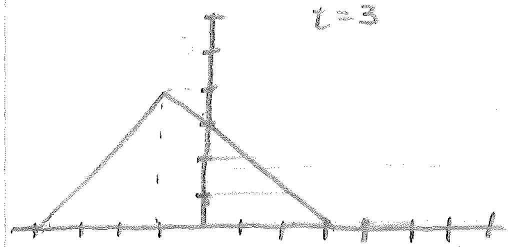
4-10) $h(t) * x(t)$ $y(3) \sim 7$





b) $\frac{1}{2}(4.5 + 2(12.5) + 2(24) + 2(8.5) + 2)$

$\frac{1}{2}(4.5 + 25.0 + 48.0 + 17.0 + 2.0)$

$\frac{1}{2}(96.5) = 48.25$

Pg 139

5-4) $f(t) = \frac{A}{T}t + A \qquad -T \le t \le 0$

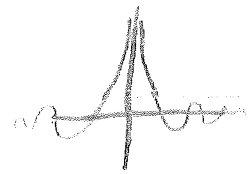
$= -\frac{A}{T}t + A \qquad 0 \le t \le T$

$= 0$ ELSEWHERE

$F(j\omega) = AT\left[\frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}\right]^2$

$\int_{-\infty}^{\infty} f(t) = AT \qquad f(0) = A$

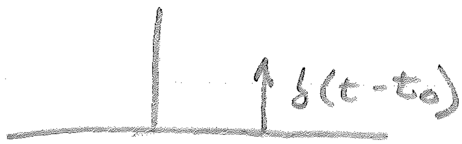
$\Rightarrow$ DURATION $= T$



$\int_{-\infty}^{\infty} \frac{\sin^2\frac{\omega T}{2}}{\frac{\omega T}{2}} \Rightarrow 2\int_{-\infty}^{\infty} \frac{1-\cos\omega t}{\omega T}$

$= 2\int_{-\infty}^{\infty} \frac{1}{\omega t} d\omega \; 2\int_{-\infty}^{\infty} \frac{\cos\omega t}{\omega t} d\omega$

$F(0) = AT$

5) a) 

$$F(j\omega) = \int_{-\infty}^{\infty} \delta(t-t_0) e^{-j\omega t} dt$$

$$U = e^{-j\omega t} \quad dV = \delta(t-t_0)\,dt$$

$$dU = \frac{e^{-j\omega t}}{-j\omega} dt \quad V = \mu(t-t_0)$$

$$F(j\omega) = e^{-j\omega t}\mu(t-t_0)\Big]_{-\infty}^{\infty} + \frac{1}{j\omega}\int_{-\infty}^{\infty} e^{-j\omega t}\mu(t-t_0)\,dt$$

$$= e^{-j\omega t}\Big]_{t_0}^{\infty} + \frac{1}{j\omega}\int_{t_0}^{\infty} e^{-j\omega t} dt$$

$$= 0 - e^{-j\omega t_0} + \frac{1}{j\omega}\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{t_0}^{\infty}$$

$$= -e^{-j\omega t_0} + \frac{1}{\omega^2} e^{-j\omega t}\Big]_{t_0}^{\infty}$$

$$= -e^{-j\omega t_0} + \frac{1}{\omega^2}\left[0 - e^{-j\omega t_0}\right]$$

$$= -e^{-j\omega t_0}\left[\frac{1}{\omega^2} + 1\right]$$

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13) $F(j\omega) = \frac{8}{4+j\omega}$

$|F(j\omega)| = \frac{8}{\sqrt{16+\omega^2}}$

$|F(j\omega)|^2 = \frac{64}{(16+\omega^2)}$

$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} [F(j\omega)]^2 d\omega$

$= \frac{64}{2\pi}\int_{-\infty}^{\infty} \frac{1}{16+\omega^2} d\omega$

$\omega = 4\tan x \Rightarrow d\omega = 4\sec^2 x\, dx$

$E = \frac{32}{\pi}\int_{-\infty}^{\infty} \frac{4}{16\sec^2 x} \sec^2 x\, dx$

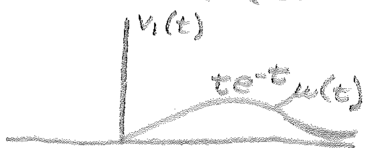
$= \frac{8}{\pi}\int_{-\infty}^{\infty} dx$

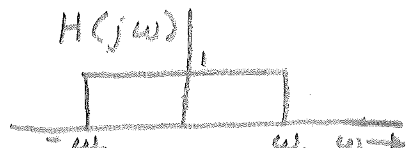
$= \frac{8}{\pi}\left[x\right]_{-\infty}^{\infty}$

$= \frac{8}{\pi}\left[\tan^{-1}\frac{\omega}{4}\right]_{-\infty}^{\infty}$

$= \frac{8}{\pi}\left(\tan^{-1}\infty - \tan^{-1}-\infty\right)$

$= \frac{8}{\pi}\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = 8$

14) 



$|Y(j\omega)|^2 = |V(j\omega)|^2 |H(j\omega)|^2$

$V(j\omega) = j\frac{d}{d\omega}\frac{1}{(j\omega+1)} = j\frac{d}{d\omega}(j\omega+1)^{-1}$

$= j\left[-j(j\omega+1)^{-2}\right] = \frac{1}{(j\omega+1)^2}$

$|V(j\omega)| = \frac{1}{\omega^2+1} \Rightarrow |V(j\omega)|^2 = \frac{1}{(\omega^2+1)^2}$

$|Y(j\omega)|^2 = \frac{1}{(\omega^2+1)^2}|H(j\omega)|^2$

$\therefore \int_0^{\infty}\frac{1}{(\omega^2+1)^2}d\omega = \int_{-2\pi W}^{2\pi W}\frac{1}{(\omega^2+1)^2}d\omega$

$\int\frac{1}{(\omega^2+1)^2}d\omega$

let $\omega = \tan x \Rightarrow d\omega = \sec^2 x\, dx$

$\int\frac{1}{\sec^4 x}\sec^2 x\, dx = \int\cos^2 x\, dx = \frac{1}{2}\int(1+\cos 2x)dx$

$= \frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right]$

$\omega_0 = 2\pi W$ $\qquad = \frac{1}{2}\left[\tan^{-1}\omega + \frac{1}{2}\sin(\tan^{-1}\omega)\right]$

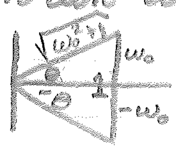
$\left[\tan^{-1}\omega + \frac{1}{2}\sin(\tan^{-1}\omega)\right]_0^{\infty} = \left[\tan^{-1}\omega + \frac{1}{2}\sin(\tan^{-1}\omega)\right]_{-2\pi W}^{2\pi W}$

$\tan^{-1}\infty + \frac{1}{2}\sin(\tan^{-1}\infty) - \tan^{-1}0 + \frac{1}{2}\sin\tan^{-1}0 = \frac{\pi}{2} + \frac{1}{2} = \frac{\pi+1}{2}$

$\tan^{-1}\omega_0 + \frac{1}{2}\sin(\tan^{-1}\omega_0) - \tan^{-1}-\omega_0 - \sin\tan^{-1}-\omega_0 = \frac{\pi+1}{2}$

$$\frac{\pi+1}{2}=\tan^{-1}\omega_0+\tfrac{1}{2}\sin\tan^{-1}\omega_0-\tan^{-1}-\omega_0-\tfrac{1}{2}\sin\tan^{-1}-\omega_0$$

$$\theta=\tan^{-1}\omega_0$$



$$\frac{\pi+1}{2}=\theta+\frac{1}{2}\frac{\omega_0}{\sqrt{\omega_0^2+1}}+\theta+\frac{1}{2}\left[\frac{\omega_0}{\sqrt{\omega_0^2+1}}\right]$$

$$\frac{\pi+1}{2}=2\tan^{-1}\omega_0+\frac{\omega_0}{\sqrt{\omega_0^2+1}}$$

$$\frac{\pi+1}{2}=2X+\sin X$$ GOOD LORD!

$$Y=2X+\sin X-\frac{\pi+1}{2}$$

```
      PI= 3.1416...
      X= PI/6.
      DO  15    J= 1,20
      Y= 2.*X+ SIN(X)- (PI+1)/2.
      DYDX= 2.+ COS(X)
      X= X- Y/DYDX
15    CONTINUE
      W= ATAN (X)
      F= W/ (2.*PI)
      WRITE (5,70) F
70    FORMAT (F10.8)
```

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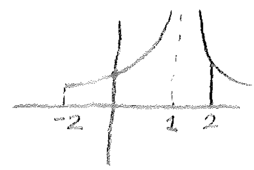
5-1) a) NOT - NOT ABSOLUTELY INTEGREBLE

$$\int_{-\infty}^{\infty} f(t)\,dt = \int_{-\infty}^{0} t\,dt = \tfrac{1}{2}t^2\Big]_{-\infty}^{0} = \infty$$

b) NOT - HAS INFINITE MAX & MIN ON [-1, 1]

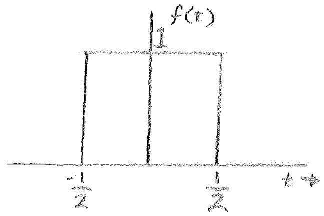
c) 

NOT ABSOLUTELY INTEGREBLE

$$\int_{-2}^{2} \frac{1}{t-1}\,dt = \ln[t-1]\Big|_{-2}^{2} = \ln 1 - \ln{-1} = \infty$$

d) NOT ABSOLUTELY INTEGRATABLE

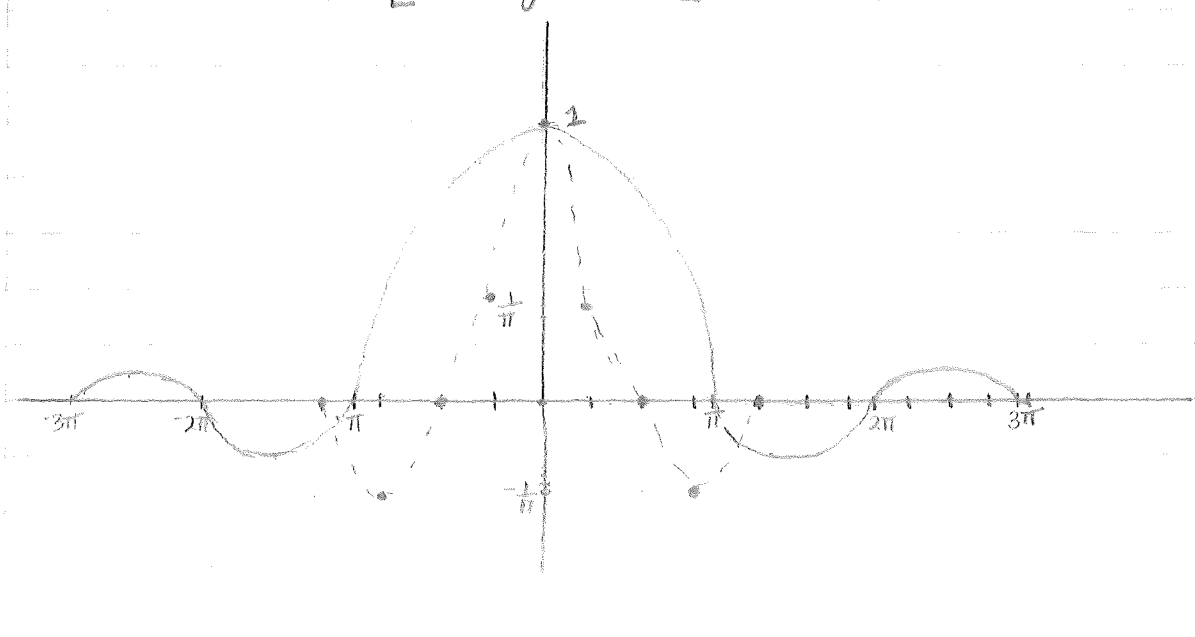
$$\int_{-\infty}^{\infty} e^{-\alpha t}\,dt = \frac{-1}{\alpha} e^{-\alpha t}\Big]_{-\infty}^{\infty} = 0-(-\infty) = \infty$$

5-2) 

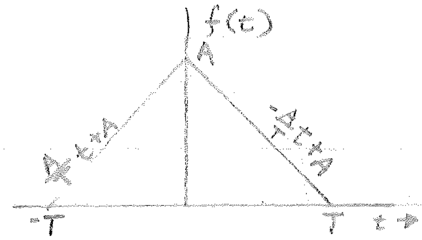
$$\mathcal{F}\{f(t)\} = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j\omega t}\,dt$$
$$= \frac{e^{-j\omega t}}{-j\omega}\Big]_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{e^{-\frac{j\omega}{2}} - e^{\frac{j\omega}{2}}}{-j\omega}$$
$$= \frac{2}{\omega}\frac{e^{\frac{j\omega}{2}} - e^{-\frac{j\omega}{2}}}{2j} = \frac{2}{\omega}\sin\frac{\omega}{2}$$
$$\frac{\sin \omega/2}{\omega/2}$$

$$\cancel{\alpha_n = \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-jn\omega_0 t}\,dt = \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j\pi t n}\,dt}$$
$$\cancel{= \frac{1}{2}\frac{e^{-j\pi n t}}{-j\pi n}\Big]_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{-1}{2j\pi n}\left[e^{-\frac{j\pi n}{2}} - e^{\frac{j\pi n}{2}}\right]}$$
$$\cancel{= \frac{1}{2j\pi n}\left[e^{\frac{j\pi n}{2}} - e^{-\frac{j\pi n}{2}}\right] = \frac{\sin \pi n/2}{}}$$

$$\alpha_n = \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-jn\omega_0 t}\,dt = \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-jn\pi t}\,dt$$
$$= \frac{-1}{2}\int_{\frac{1}{2}}^{-\frac{1}{2}} e^{-jn\omega_0 t}\,dt = \frac{1}{2jn\pi}\left[e^{-jn\pi t}\right]_{\frac{1}{2}}^{-\frac{1}{2}}$$
$$= \frac{1}{2jn\pi}\left[e^{\frac{jn\pi}{2}} - e^{-\frac{jn\pi}{2}}\right] =$$
$$= \frac{1}{n\pi}\left[\frac{e^{\frac{jn\pi}{2}} - e^{-\frac{jn\pi}{2}}}{2j}\right] = \frac{\sin\frac{n\pi}{2}}{n\pi}$$



5-3)



$$\mathfrak{F}\{f(t)\} = F(j\omega) = \int_{-T}^{0}\left(\frac{A}{T}t+A\right)e^{-j\omega t}dt + \int_{0}^{T}\left(\frac{-A}{T}t+A\right)e^{-j\omega t}dt$$

$$\frac{F(j\omega)}{A} = \int_{-T}^{0}\frac{t}{T}e^{-j\omega t}dt + \int_{-T}^{0}e^{-j\omega t}dt + \int_{0}^{T}e^{-j\omega t}dt - \int_{0}^{T}\frac{t}{T}e^{-j\omega t}dt$$

$$\int \frac{t}{T}e^{-j\omega t}dt \Rightarrow u = \frac{t}{T}\ ;\ dv = e^{-j\omega t}dt$$

$$du = \frac{1}{T}dt\ ;\ v = \frac{-e^{-j\omega t}}{j\omega}$$

$$\int \frac{t}{T}e^{-j\omega t}dt = \left[\frac{-t}{jT\omega}e^{-j\omega t}\right] + \int \frac{1}{j\omega T}e^{-j\omega t}dt$$

$$= \left[\frac{-t}{j\omega T}e^{-j\omega t}\right] + \left[\frac{1}{\omega^2 T}e^{-j\omega t}\right]$$

$$\Rightarrow \int_{-T}^{0}\frac{t}{T}e^{-j\omega t}dt = \left[0+\frac{1}{\omega^2 T}\right] - \left[\frac{1}{j\omega}e^{+j\omega T} + \frac{1}{\omega^2 T}e^{+j\omega T}\right]$$

$$= \frac{1}{\omega^2 T} - \frac{1}{j\omega}e^{+j\omega T} - \frac{1}{\omega^2 T}e^{+j\omega T}$$

$$\Rightarrow \int_{0}^{T}\frac{t}{T}e^{-j\omega t}dt = \left[\frac{-1}{j\omega}e^{-j\omega T} + \frac{1}{\omega^2 T}e^{-j\omega T}\right] - \left[0+\frac{1}{\omega^2 T}\right]$$

$$= -\frac{1}{\omega^2 T} - \frac{1}{j\omega}e^{-j\omega T} + \frac{1}{\omega^2 T}e^{-j\omega T}$$

$$\int e^{-j\omega t}dt = \left[-\frac{e^{-j\omega t}}{j\omega}\right]$$

$$\Rightarrow \int_{-T}^{0}e^{-j\omega t}dt = \left[-\frac{1}{j\omega} - \frac{e^{j\omega T}}{j\omega}\right]$$

$$\Rightarrow \int_{0}^{T}e^{-j\omega t}dt = \left[-\frac{e^{-j\omega T}}{j\omega} + \frac{1}{j\omega}\right]$$

$$\therefore \frac{F(j\omega)}{A} = \frac{1}{\omega^2 T} - \frac{1}{j\omega}e^{+j\omega T} - \frac{1}{\omega^2 T}e^{j\omega T} \cancel{- \frac{1}{j\omega}} - \frac{e^{j\omega T}}{j\omega}$$

$$- \frac{e^{-j\omega T}}{j\omega} \cancel{+ \frac{1}{j\omega}} + \frac{1}{\omega^2 T} + \frac{1}{j\omega}e^{-j\omega T} - \frac{1}{\omega^2 T}e^{-j\omega T}$$

$$= \frac{2}{\omega^2 T}$$ ETC (ARG!)

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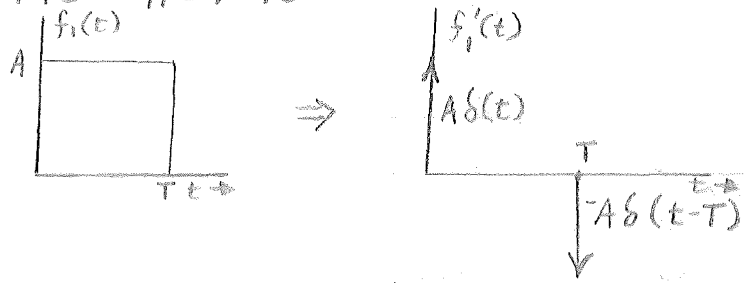
5-6) a)



$\mathcal{F}[f'(t)] = A + Ae^{-j\omega T}$

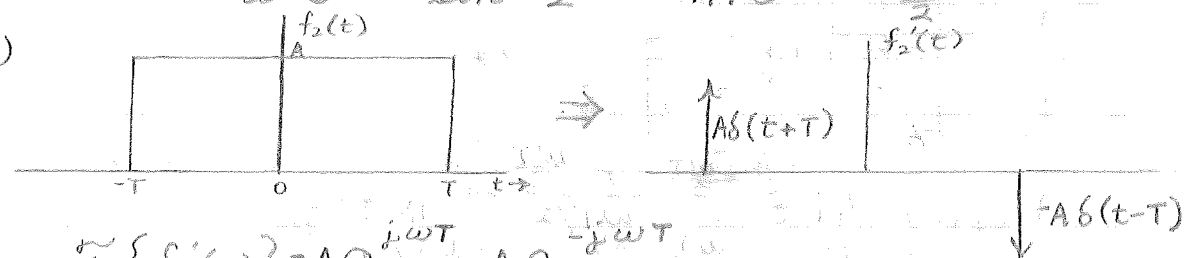
$\mathcal{F}[\frac{d}{dt}f(t)] = A - Ae^{-j\omega T} \Leftrightarrow \mathcal{F}[f(t)] = \frac{A - Ae^{-j\omega T}}{j\omega}$

$F(j\omega) = \frac{A}{j\omega}(1 - e^{-j\omega T}) = \frac{A}{j\omega}e^{-\frac{j\omega T}{2}}(e^{\frac{j\omega T}{2}} - e^{-\frac{j\omega T}{2}})$

$= \frac{2A}{\omega}e^{-\frac{j\omega T}{2}}\left(\frac{e^{\frac{j\omega T}{2}} - e^{-\frac{j\omega T}{2}}}{2j}\right)$

$= \frac{2A}{\omega}e^{-\frac{j\omega T}{2}}\sin\frac{\omega T}{2} = ATe^{-\frac{j\omega T}{2}}\frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}$

b)

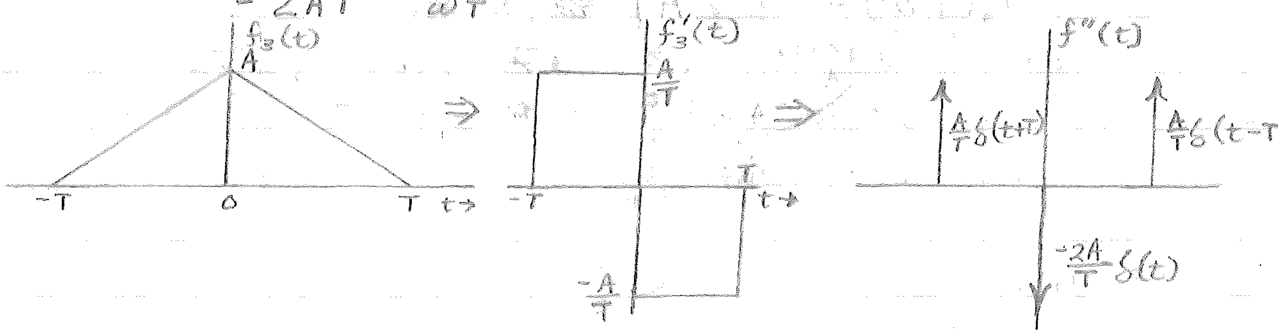


$\mathcal{F}\{f_2'(t)\} = Ae^{j\omega T} - Ae^{-j\omega T}$

$\Rightarrow \mathcal{F}\{f_2(t)\} = \frac{2A}{\omega}\frac{(e^{j\omega T} - e^{-j\omega T})}{2j} = \frac{2A}{\omega}\sin\omega T$

$= 2AT\frac{\sin\omega T}{\omega T}$

c)



$\mathcal{F}\{f''(t)\} = \frac{A}{T}e^{j\omega T} - \frac{2A}{T} + \frac{A}{T}e^{-j\omega T} = \frac{A}{T}(e^{j\omega T} - 2 + e^{-j\omega T})$

$\Rightarrow \mathcal{F}\{f(t)\} = \frac{-A}{T\omega^2}[e^{j\omega T} + e^{-j\omega T} - 2]$

$= \frac{-2A}{T\omega^2}\left[\frac{e^{j\omega T} + e^{-j\omega T}}{2} - 1\right]$

$= \frac{-2A}{T\omega^2}[\cos\omega t - 1]$

$= \frac{-2A}{T\omega^2}[(2\cos^2\frac{\omega T}{2} - 1) - 1]$

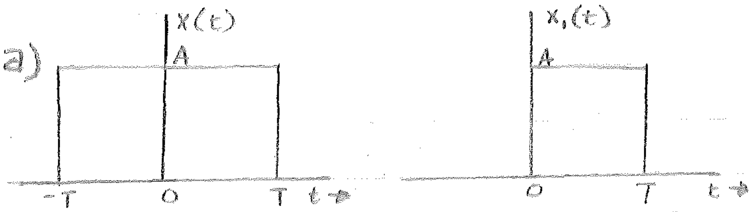
$= \frac{4A}{T\omega^2}(1 - \cos^2\frac{\omega T}{2})$

$= \frac{4A}{T\omega^2}\sin^2\frac{\omega T}{2}$

$= AT\frac{\sin^2\frac{\omega T}{2}}{\frac{\omega^2T^2}{4}} = AT\left[\frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}\right]^2$

$\boxed{\begin{array}{l}\cos^2\frac{\omega T}{2} = \frac{1}{2} + \frac{1}{2}\cos\omega t \\ \Rightarrow \cos\omega t = 2\cos^2\frac{\omega T}{2} - 1\end{array}}$

5-8) a)

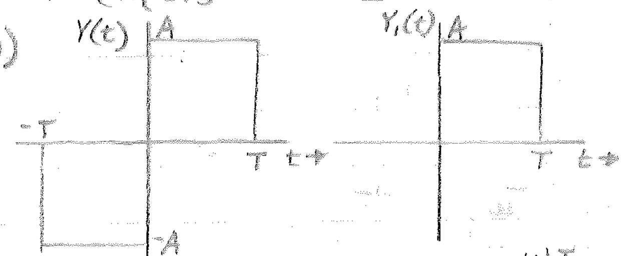


$$\mathcal{F}\{x_1(t)\} = AT\, e^{-\frac{j\omega T}{2}} \frac{\sin \frac{\omega T}{2}}{\omega T/2}$$

$$RE\left[\mathcal{F}\{x_1(t)\}\right] = AT \frac{\sin \frac{\omega T}{2}}{\frac{\omega T}{2}}$$

$$\Rightarrow \mathcal{F}\{x(t)\} = 2 \cdot RE\left[\mathcal{F}\{x_1(t)\}\right] = 2AT \frac{\sin \frac{\omega T}{2}}{\frac{\omega T}{2}}$$

b)



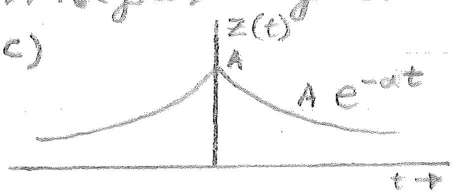
$$Y_1(j\omega) = AT\, e^{-\frac{j\omega T}{2}} \frac{\sin \frac{\omega T}{2}}{\omega T/2}$$

$$= \left(\cos \frac{\omega T}{2} - j \sin \frac{\omega T}{2}\right) AT \frac{\sin \frac{\omega T}{2}}{\omega T/2}$$

$$I_m Y_1(j\omega) = -j \sin \frac{\omega T}{2} AT \frac{\sin \frac{\omega T}{2}}{\omega T/2}$$

$$\therefore Y(j\omega) = -j\, 2AT \frac{\sin \frac{\omega T}{2}}{\omega T/2}$$

c)



$$Z_1(j\omega) = \frac{1}{j\omega + \alpha} \cdot \frac{j\omega - \alpha}{j\omega - \alpha}$$

$$= \frac{j\omega - \alpha}{-\omega^2 - \alpha^2}$$

$$= \frac{\alpha}{\omega^2 + \alpha^2}$$

$$\Rightarrow Z(j\omega) = \frac{2\alpha}{\alpha^2 + \omega^2}$$

5-9) a) $h(t) = \left[e^{-t} - e^{-3t}\right] \mu(t)$

$$\Rightarrow H(j\omega) = \frac{1}{j\omega + 1} - \frac{1}{j\omega + 3} = \frac{j\omega + 3 - j\omega - 1}{(j\omega + 1)(j\omega + 3)}$$

$$= \frac{2}{(j\omega + 1)(j\omega + 3)}$$

b) $h(t) = \delta(t) - e^{-2t} \mu(t)$

$$= 1 - \frac{1}{j\omega + 2} = \frac{j\omega + 1}{j\omega + 2}$$

c) $h(t) = e^{-t} \cos t\, \mu(t)$

$$= e^{-t}\left[\frac{e^{jt} + e^{-jt}}{2}\right] \mu(t)$$

$$= \frac{1}{2}\left[e^{t(j-1)} + e^{-t(j+1)}\right] \mu(t)$$

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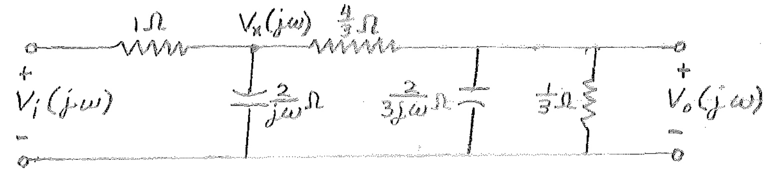
5-10)



AT $V_x \Rightarrow \frac{3(V_x - V_o)}{4} + \frac{j\, V_x \omega}{2} + V_x - V_i = 0$

$$3V_x - 3V_o - j\,2V_x\omega + 4V_x - V_i = 0$$

$$V_x(7 - j2\omega) = 3V_o + V_i \Rightarrow V_x = \frac{3V_o + V_i}{7 - j2\omega}$$

AT $V_o \Rightarrow \frac{3(V_o - V_x)}{4} + V_o\left(\frac{2}{j3\omega} + \frac{1}{3}\right) = 0$

$$3V_o - 3V_x + 4V_o\left(\frac{2}{j3\omega} + \frac{1}{3}\right) = 0$$

$$V_o\left(\frac{-j2}{3\omega} + \frac{10}{3}\right) = 3V_x \Rightarrow V_x = V_o\left(\frac{10}{9} - \frac{j2}{9\omega}\right)$$

$$\therefore \quad V_o\left(\frac{10}{9} - \frac{j2}{9\omega}\right) = V_o\left(\frac{3}{7 - j2\omega}\right) + \frac{V_i}{7 - j2\omega}$$

$$V_o\left[\frac{10}{9} - \frac{j2}{9\omega} + \frac{3}{j2\omega - 7}\right] = \frac{V_i}{7 - j2\omega}$$

$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{\left(\frac{10}{9} - \frac{j2}{9\omega} + \frac{3}{j2\omega - 7}\right)(7 - j2\omega)}$$

I QUIT!

5-13) $F(j\omega) = \dfrac{8}{4+j\omega}$

$|F(j\omega)| = \dfrac{8}{\sqrt{\omega^2+16}}$

$|F(j\omega)|^2 = \dfrac{64}{\omega^2+16}$

$E = \dfrac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$

$= \dfrac{64}{2\pi}\int_{-\infty}^{\infty} \dfrac{1}{\omega^2+16}\, d\omega$

$\omega = 4\tan x \Rightarrow d\omega = 4\sec^2 x\, dx$

$\Rightarrow E = \dfrac{32}{\pi}\int_{-\infty}^{\infty} \dfrac{1}{4\sec^2 x} \sec^2 x\, dx$

$= \dfrac{8}{\pi}\left[x\right]_{-\infty}^{\infty}$

$= \dfrac{8}{\pi}\left[\tan^{-1}\omega\right]_{-\infty}^{\infty}$

$= \dfrac{8}{\pi}\left[\dfrac{3\pi}{2} - \dfrac{\pi}{2}\right]$

$= 8$

5-14) $v_i(t) = t e^{-t} \Rightarrow j \dfrac{d}{d\omega} \dfrac{1}{j\omega+1} = j \cdot j \dfrac{1}{(j\omega+1)^2} = V(j\omega)$

$= \dfrac{-1}{(j\omega+1)^2}$

$|V(j\omega)|^2 = \dfrac{1}{(\omega^2+1)^2}$

$\dfrac{1}{2}\int_{-\infty}^{\infty} \dfrac{1}{(\omega^2+1)^2} d\omega = \int_{-\omega_0}^{\omega} \dfrac{1}{(\omega^2+1)^2} d\omega \quad \forall \ \omega_0 = 2\pi W$

CHUG & PLUG & CHUG

SOLVING FOR W (FUNSIES!)

5-16) $x(t) \rightarrow \boxed{h(t)} \rightarrow x(t-t_0)$

$\rightarrow H(j\omega) = e^{-j\omega t_0}$

5-17) $F(j\omega) = \dfrac{1+j\omega}{8-\omega^2+j6\omega}$

a) $\mathcal{F}\{f(3t)\} = \dfrac{1}{a}\left[\dfrac{1+j\omega/a}{8-\omega^2/a^2+j6\omega/a}\right]$

$= \dfrac{1}{a}\left[\dfrac{a^2+ja\omega}{8a^2-\omega^2+ja6\omega}\right]$

$= \dfrac{3+j\omega}{72+\omega^2+18j\omega}$

b) $\mathcal{F}\{f(t-2)\} = e^{-j\omega 2}\left[\dfrac{1+j\omega}{8-\omega^2+j6\omega}\right]$

c) $\mathcal{F}\{f(3t-2)\} = e^{-j2\omega}\left[\dfrac{3+j\omega}{72-\omega^2+\omega j18}\right]$

d) $\mathcal{F}\{5f(\tfrac{1}{2}t)\} = 5\left[\dfrac{1+2j\omega}{8-4\omega^2+j12\omega}\right]$

$= \dfrac{5+10j\omega}{8-4\omega^2+j12\omega}$

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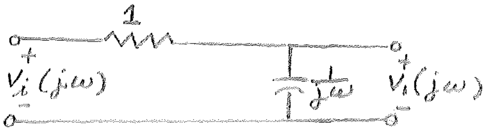
5-18)



$$\frac{V_1}{V_i} = \frac{\frac{1}{j\omega}}{1+\frac{1}{j\omega}} = \frac{1}{j\omega+1}$$

$$\Rightarrow V_1 = \frac{V_i}{j\omega+1}$$

$$V_2(t) = V_i(t-1) \Rightarrow V_2(j\omega) = e^{-j\omega} V_1(j\omega)$$

$$V_o(j\omega) = V_1(j\omega) - V_2(j\omega)$$

$$= V_i(j\omega)\left(\frac{1}{j\omega+1} - e^{-j\omega}\right)$$

$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{j\omega+1} - e^{-j\omega} = \frac{1-(j\omega+1)e^{-j\omega}}{j\omega+1}$$

5-19) $F(j\omega) = \int_0^\infty e^{-j\omega t} f(t)\,dt$

$$\int_0^\infty e^{-j\omega t} e^{-\alpha t} f(t)\,dt = \int_0^\infty e^{-t(j\omega+\alpha)} f(t)\,dt = F(j\omega+\alpha)$$

5-20) $F(j\omega) = \frac{1+j\omega}{8-\omega^2+j6\omega}$

a) $\mathfrak{F}\{f(t-2)\} = e^{-j2\omega}\left[\frac{1+j\omega}{8-\omega^2+j6\omega}\right]$

$\mathfrak{F}\{e^{j6t} f(t-2)\} = e^{-j2(\omega-6)}\left[\frac{1+j(\omega-6)}{8-(\omega-6)^2+j6(\omega-6)}\right]$

b) $\mathfrak{F}\{6\frac{d}{dt}f(t)\} = 6j\omega\left[\frac{1+j\omega}{8-\omega^2+j6\omega}\right]$

c) $\mathfrak{F}\{f(2t)\} = \frac{1}{2}\left[\frac{1+j\omega/2}{8-\omega^2/4+j3\omega}\right]$

$= \frac{1}{2}\left[\frac{4+j\omega 2}{32-\omega^2+j12\omega}\right] = \frac{2+j\omega}{32-\omega^2+j12\omega}$

d) $\mathfrak{F}\{f(3t)\} = \frac{1}{3}\left[\frac{1+j\omega/3}{8-\omega^2/9+j2\omega}\right] = \frac{1}{3}\left[\frac{9+j\omega 3}{72-\omega^2+j18\omega}\right]$

$= \frac{3+j\omega}{72-\omega^2+j18\omega}$

$= \frac{3}{72-\omega^2+j18\omega} + \frac{j\omega}{72-\omega^2+j18\omega}$

$\frac{d}{d\omega}\mathfrak{F}\{f(3t)\} = \mathfrak{F}\{t\,f(3t)\}$

$= \frac{3(-2\omega+j18)}{(72-\omega^2+j18\omega)^2} + \frac{j\omega(-2\omega+j18)}{(72-\omega^2+j18\omega)^2} + \frac{j}{72-\omega^2+j18\omega}$

$\mathfrak{F}\{5t\,f(3t)\} = 5$ TIMES THE ABOVE MESS

e) $\mathfrak{F}\{f(-t)\} = \frac{3-j\omega}{72-\omega^2-j18\omega}$

$\mathfrak{F}\{f(2-t)\} = e^{-j2\omega}\left[\frac{3-j\omega}{72-\omega^2-j18\omega}\right]$

f) ?

5-21) $F(j\omega) = \frac{1}{j\omega+1} \frac{(j\omega-1)}{(j\omega-1)} = \frac{j\omega-1}{-\omega^2-1} = \frac{1-j\omega}{\omega^2+1}$

a) $F_a(j\omega) = 2 \cdot \mathrm{Re}\, F(j\omega) = \frac{2}{\omega^2+1}$

b) $F_b(j\omega) = 2 \cdot \mathrm{Im}\, F(j\omega) = \frac{-2j\omega}{\omega^2+1}$

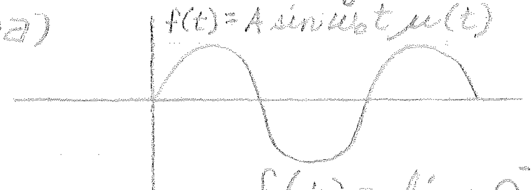
c) $F_c(j\omega) = j \frac{d}{d\omega} \frac{1-j\omega}{\omega^2+1}$

$= j\left[\frac{-2j}{\omega^2+1} - \frac{4j\,\omega^2}{(\omega^2+1)^2}\right]$

$= \frac{2\omega^2+2+4\omega^2}{(\omega^2+1)^2}$

$= \frac{1}{(j\omega+1)^2}$

5-27) a)



$f(t) = \lim_{\alpha\to 0} e^{-\alpha t} \sin\omega_0 t\ \mu(t)$

$\mathcal{F}\{f(t)\} = A \lim_{\alpha\to 0} \int_0^\infty e^{-\alpha t} \sin\omega_0 t\, e^{-j\omega t} dt$

$= A \lim_{\alpha\to 0} \frac{\omega_0}{(\alpha+j\omega)^2+\omega_0^2}$

$F(j\omega) = \frac{A\omega_0}{-\omega^2+\omega_0^2} \qquad \pm\omega \neq \omega_0$

$= \delta(\omega \mp \omega_0) \qquad \pm\omega = \omega_0$

b) $f(t) = (1 + m\cos\omega_1 t)\cos\omega_0 t$

$= \cos\omega_0 t + m\cos\omega_1 t \cos\omega_0 t$

$= \cos\omega_0 t + m\cos(\omega_1 - \omega_0) + m\cos(\omega_1+\omega_0)$

$F(j\omega) = \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)] + m\pi[\delta(\omega-\omega_1+\omega_0)$
$+ \delta(\omega+\omega_1-\omega_0) + \delta(\omega-\omega_1-\omega_0) + \delta(\omega+\omega_1+\omega_0)]$

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5-28) $\mathfrak{F}\left\{\sum_{n=-\infty}^{\infty}\alpha_n e^{\frac{j2\pi n t}{T}}\right\} = 2\pi\sum_{n=-\infty}^{\infty}\alpha_n\delta\left(\omega-\frac{2\pi n}{T}\right)$

$$\alpha_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,e^{-jn\omega_0 t}\,dt$$

$$\alpha_n = \frac{1}{T}\int_{-T/2}^{0}(e^{t}-3)\,e^{-jn\omega_0 t}\,dt + \frac{1}{T}\int_0^{\frac{T}{2}}(2e^{-t})\,e^{-jn\omega_0 t}\,dt$$

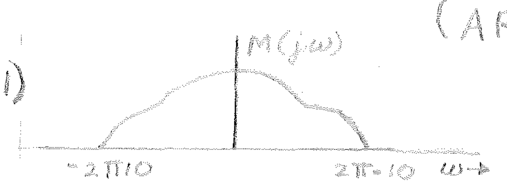
$$= \frac{1}{T}\int_{-T/2}^{0}e^{t(1-jn\omega_0)}\,dt - \frac{3}{T}\int_{-T/2}^{0}e^{-jn\omega_0 t}\,dt + \frac{2}{T}\int_0^{T/2}e^{-t(1+jn\omega_0)}\,dt$$

$$= \frac{1}{T(1-jn\omega_0)}\left[e^{t(1-jn\omega_0)}\right]_{-T/2}^{0} + \frac{3}{jTn\omega_0}\left[e^{-jn\omega_0 t}\right]_{-T/2}^{0} - \frac{1}{T(1+jn\omega_0)}\left[e^{-t(1+jn\omega_0)}\right]_0^{T/2}$$

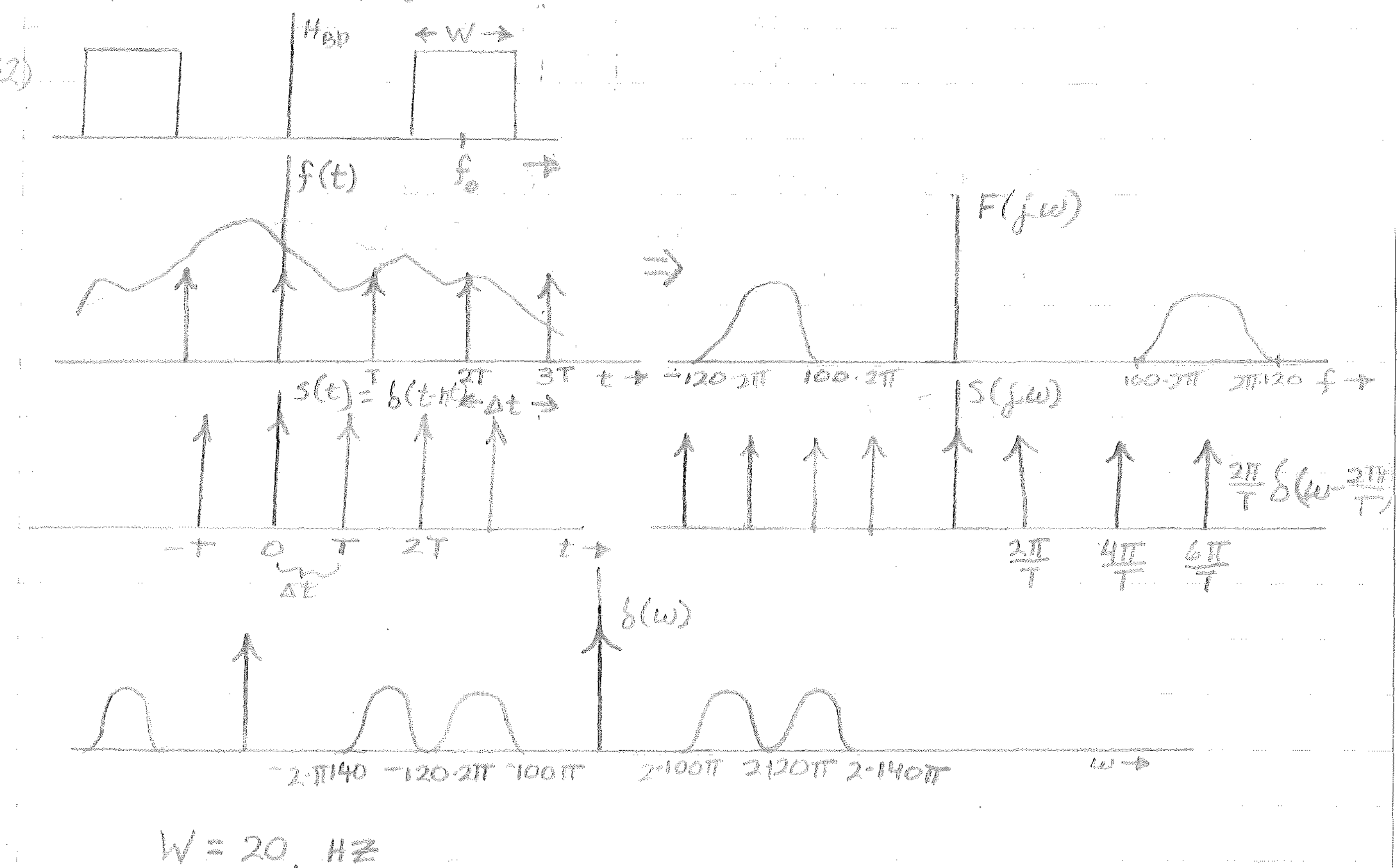
$$= \frac{1}{T(1-jn\omega_0)}\left[1-e^{-\frac{T(1-jn\omega_0)}{2}}\right] + \frac{3}{jTn\omega_0}\left[1-e^{\frac{jn\omega_0 T}{2}}\right] - \frac{1}{T(1+jn\omega_0)}\left[e^{-\frac{T(1+jn\omega_0)}{2}}-1\right]$$

(ARG!)

5-31)



5-32)



$W = 20$ HZ

$\frac{1}{2W} = \frac{1}{2\cdot 20} = \frac{1}{40}$ SEC

SAMPLING MUST BE AT $< \frac{1}{40}$ SEC OR > 40 HZ

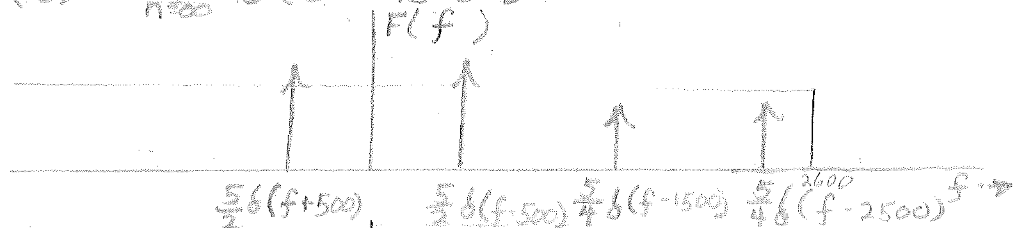
5-33) $f(t) = 5\cos(2\pi \cdot 500t)\cos^2(2\pi \cdot 1000t)$

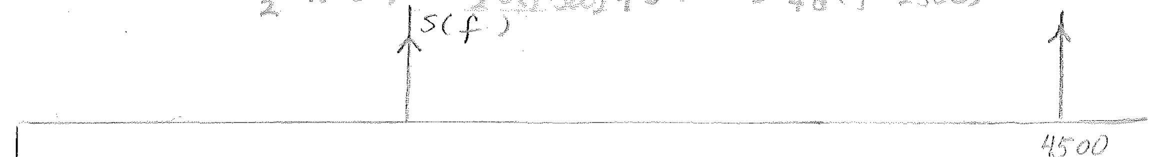
$$= 5\cos(2\pi \cdot 500t)\left[\tfrac{1}{2} + \tfrac{1}{2}\cos 2\pi \cdot 2000t\right]$$

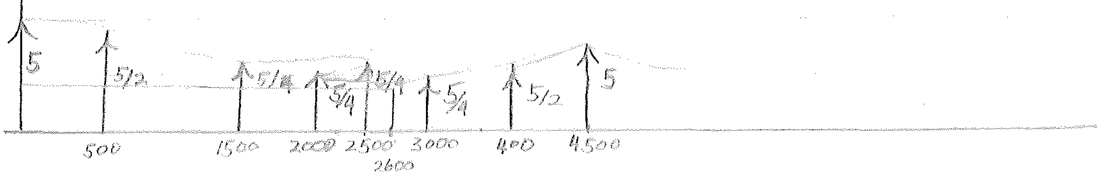
$$= \tfrac{5}{2}\cos 2\pi \cdot 500t + \tfrac{5}{2}\cos 2\pi \cdot 500t \cos 2\pi \cdot 2000t$$

$$= \tfrac{5}{2}\cos 2\pi \cdot 500t + \tfrac{5}{4}\cos 2\pi \cdot 1500t + \tfrac{5}{4}\cos 2\pi \cdot 2500t$$

$$s(t) = \sum_{n=-\infty}^{\infty} \delta\left(t - \frac{n}{4500}\right)$$







$$f_o(t) = 5 + \tfrac{5}{2}\cos 2\pi \cdot 500t + \tfrac{5}{4}\cos 2\pi \cdot 1500t + \underbrace{\tfrac{5}{4}\cos 2\pi \cdot 2000t}_{\text{DISTORTION}} + \tfrac{5}{4}\cos 2\pi\, 2500$$

FOR NO DISTORTION, SAMPLING MUST OCCUR > 5000 HZ

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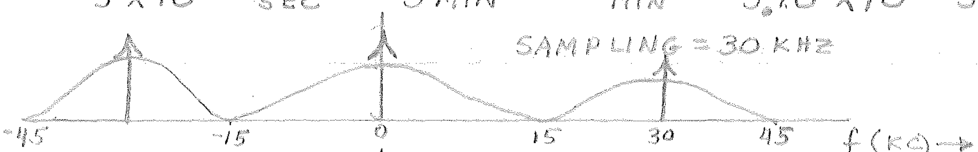
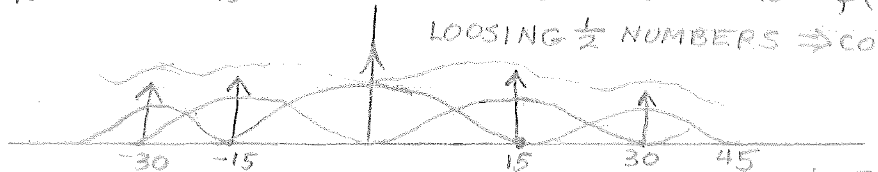
5-34) ASSUME SINGER'S RANGE IS $0 < R \leq 15$ KC.

$\Rightarrow$ SAMPLING RATE $> 30 \times 10^3$ HZ OR $< \frac{1}{30} \times 10^{-3}$ SEC

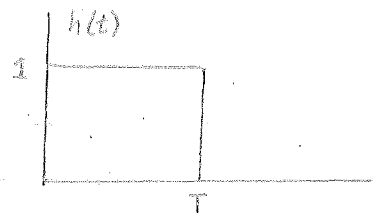
$3 \times 10^4 \frac{\text{SAMPLES}}{\text{SEC}} \cdot 3 \text{ MIN} \cdot \frac{60 \cdot \text{SEC}}{\text{MIN}} = 540 \times 10^6$ SAMPLES



LOOSING ½ NUMBERS ⇒ COMPLETE DISTORTION

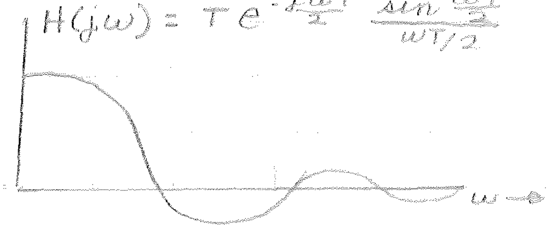


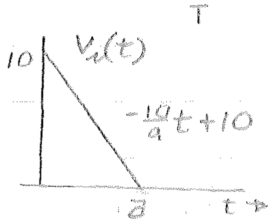
5-35)



$\Rightarrow$

$H(j\omega) = T e^{-\frac{j\omega T}{2}} \frac{\sin \frac{\omega T}{2}}{\omega T/2}$





$$V(j\omega) = \int_0^a \left(-\frac{10}{a}t + 10\right) e^{-j\omega t}\,dt$$
$$= \int_0^a -\frac{10}{a} t e^{-j\omega t}\,dt + \int_0^a 10 e^{-j\omega t}\,dt$$

$U = \frac{-10}{a}t \quad dV = e^{-j\omega t}dt$

$dU = \frac{-10}{a}dt \quad V = \frac{-e^{-j\omega t}}{j\omega}$

$$\frac{10t}{ja\omega} e^{-j\omega t}\Big]_0^a - \int \frac{10}{aj\omega} e^{-j\omega t}\,dt$$
$$\frac{10}{j\omega} e^{-jat} + \frac{10}{aj\omega}\left[\frac{+1}{j\omega} e^{-j\omega t}\right]_0^a$$
$$\frac{10}{j\omega} e^{-jat} - \frac{10}{a\omega^2}\left[e^{-j\omega a} - 1\right]$$
$$\frac{10}{j\omega} e^{-jat} - \frac{10}{a\omega^2} e^{-j\omega a} + \frac{10}{a\omega^2}$$

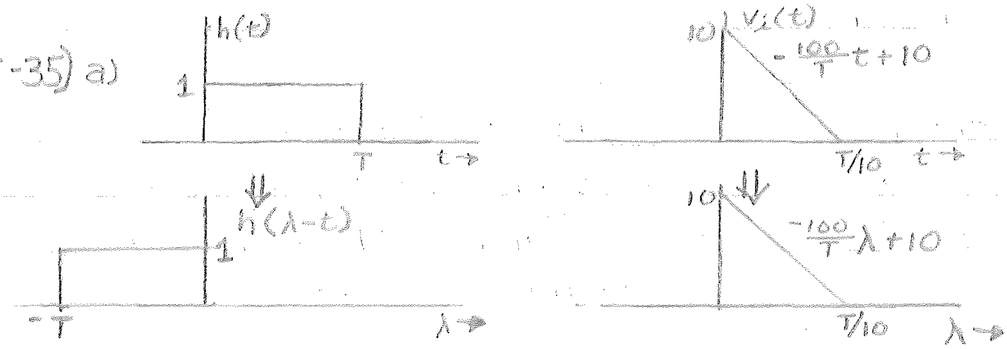
$$\int_0^a 10 e^{-j\omega t}\,dt = \frac{-10}{j\omega}\left(e^{-j\omega a} - 1\right) = \frac{-10}{j\omega} e^{-j\omega a} + \frac{10}{j\omega}$$

$$\Rightarrow V(j\omega) = \cancel{\frac{10}{j\omega} e^{-jat}} - \frac{10}{a\omega^2} e^{-j\omega a} + \frac{10}{a\omega^2} - \cancel{\frac{10}{j\omega} e^{-j\omega a}} + \frac{10}{j\omega}$$
$$= 10\left(\frac{1}{j\omega} - \frac{10}{a\omega^2}\right) - \frac{10}{a\omega^2}$$

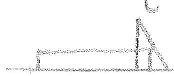
NUTZ!

(OVER!)

5-35) a)



$t \leq 0 \Rightarrow h * v = 0$



$0 \leq t \leq \frac{T}{10}$

$$h * v = \int_0^t \left(\frac{-100}{T}\lambda + 10\right) d\lambda$$

$$= \left[\frac{-50}{T}\lambda^2 + 10\lambda\right]_0^t = \frac{-50}{T}t^2 + 10t$$

$\frac{T}{10} \leq t \leq T$

$$h * v = \int_0^{T/10} \left(\frac{-100}{T}\lambda + 10\right) d\lambda$$

$$= \left[\frac{-50}{T}\lambda^2 + 10\lambda\right]_0^{T/2} = \frac{-50T^2}{4T} + \frac{10T}{2}$$

$$= \frac{-50}{4}T + \frac{20T}{4} = \frac{30T}{4}$$

$T \leq t \leq \frac{3T}{2}$

$$h * v = \int_{t-T}^{T/2} \left(\frac{-100}{T}\lambda + 10\right) d\lambda$$

$$= \left[\frac{-50}{T}\lambda^2 + 10\lambda\right]_{t-T}^{T/2} = \frac{30T}{4} + \frac{50}{T}(t-T)^2 - 10(t-T)$$

$$= \frac{30T}{4} + \frac{50}{T}t^2 - \frac{100}{T}tT + \frac{50}{T}T^2 - 10t + 10T$$

$$= \frac{30T}{4} + \frac{50}{T}t^2 - 100t + 50T - 10t + 10T$$

$t \geq \frac{3T}{2}$

SO DAMN WHAT?!

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5-37) a) $V_1(t)\,V_2(t) = 10\,e^{-.01t}\cos^2(2\pi\times10^6 t)\,\mu(t)$

$$= 10\,e^{-.01t}\left[\tfrac{1}{2} + \cos(4\pi\times10^6 t)\right]\mu(t)$$

$$= \left[5\,e^{-.01t} + 10\,e^{-.01(t)}\cos(4\pi\times10^6 t)\right]\mu(t)$$

$$\mathcal{F}\{V_1(t)V_2(t)\} = \frac{5}{j\omega+.01} + \frac{.01+j\omega}{(.01+j\omega)^2+16\pi^2\times10^{12}}$$

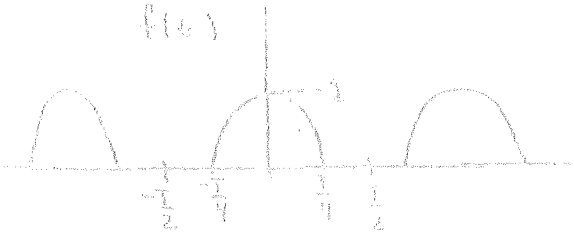
$$\Rightarrow \mathcal{F}\{V_0(t)\} \simeq \frac{5}{j\omega+.01}$$

b) $V_1(t)V_2(t) = 10\,e^{-.01}\cos 2\pi\cdot10^6 t\,\sin 2\pi\cdot10^6 t\,\mu(t)$

$$= 5\,e^{-.01}\sin 4\pi\cdot10^6 t\,\mu(t)$$

$$\mathcal{F}\{V_1(t)V_2(t)\} = \frac{5\,4\pi\cdot10^6}{(.01+j\omega)^2+16\pi^2\cdot10^{12}}$$

$$\Rightarrow \mathcal{F}\{V_0(t)\} \simeq 0$$



$T = \frac{2\pi}{\omega}$

$b_n = 0 \quad \forall n$ since wave is even function

$$a_n = \frac{4}{T}\int_0^{T/4} \cos\omega_0 t \cos n\omega_0 t \, dt = \frac{2}{T}\int_0^{T/4} \{\cos(n+1)\omega_0 t + \cos(n-1)\omega_0 t\} \, dt$$

$$= \frac{2}{T}\left\{ \frac{\sin(n+1)\omega_0 t\Big]_0^{T/4}}{(n+1)\omega_0} + \frac{\sin(n-1)\omega_0 t\Big]_0^{T/4}}{(n-1)\omega_0} \right\}$$

$$= \frac{1}{\pi}\left\{ \frac{1}{n+1}\left[\sin(n+1)\frac{\pi}{2} - 0\right] + \frac{1}{n-1}\left[\sin(n-1)\frac{\pi}{2} - 0\right] \right\}$$

$$= \boxed{\frac{1}{\pi}\left[\frac{\sin(n+1)\frac{\pi}{2}}{n+1} + \frac{\sin(n-1)\frac{\pi}{2}}{n-1} \right]}$$

$$= \boxed{\frac{1}{2}\left[\frac{\sin(n+1)\frac{\pi}{2}}{(n+1)\frac{\pi}{2}} + \frac{\sin(n-1)\frac{\pi}{2}}{(n-1)\frac{\pi}{2}} \right]} \quad \text{since } \frac{\sin x}{x} \text{ form}$$

$a_0 = \boxed{\frac{2}{\pi}} \implies$ d-c value $= \frac{a_0}{2} = \frac{1}{\pi}$ ✓ (correct for 1/2 wave)

$$a_1 = \frac{1}{2}\left[\frac{\sin\pi}{\pi} + 1\right] = \boxed{\frac{1}{2}}$$

$$a_2 = \frac{1}{2}\left[\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}} + \frac{\sin\frac{\pi}{2}}{\frac{\pi}{2}}\right] = \frac{1}{2}\left[\frac{-2}{3\pi} + \frac{2}{\pi}\right] = \frac{1}{2}\left[\frac{-2}{3\pi} + \frac{6}{3\pi}\right]$$

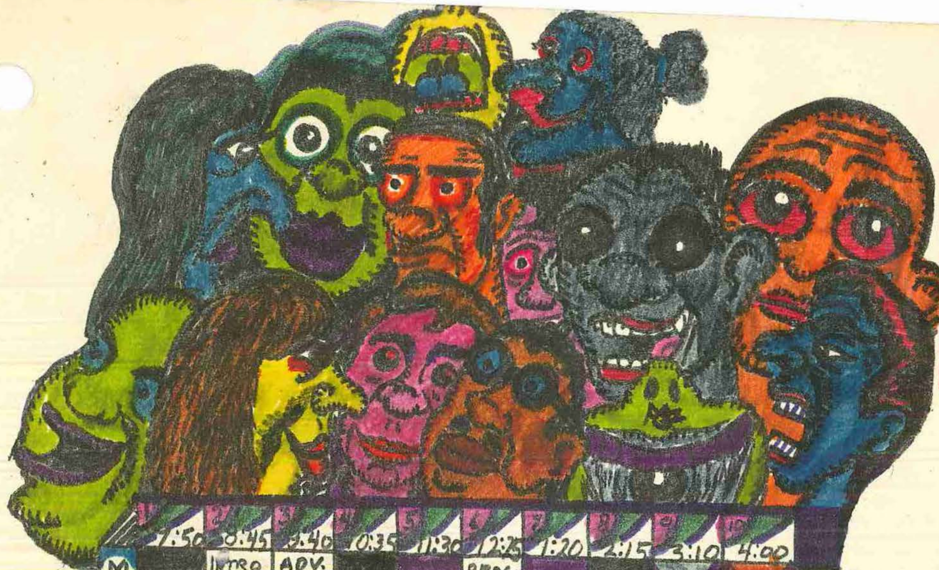
$$= \frac{1}{2}\left[\frac{4}{3\pi}\right] = \boxed{\frac{2}{3\pi}}$$

$n = \text{ODD} \Rightarrow \sin(n \pm 1)\frac{\pi}{2} = 0$ EXCEPT $n = 1$

$n = \text{EVEN} \Rightarrow \sin(n+1)\frac{\pi}{2} = (-1)^{\frac{n}{2}}$

$\sin(n-1)\frac{\pi}{2} = (-1)^{\frac{n}{2}+1} \Rightarrow a_{n_{ODD}} = \frac{1}{\pi}\left[\frac{(-1)^{\frac{n}{2}}}{n+1} + \frac{(-1)^{\frac{n}{2}+1}}{n-1}\right]$

E. SCI.



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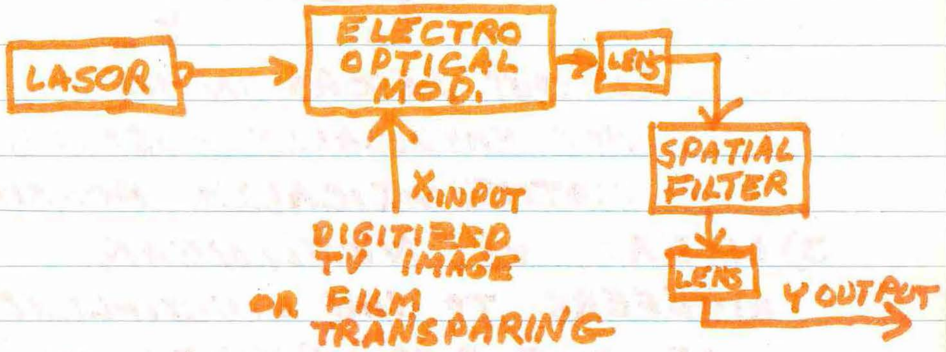
MONDAY		INTRO TO PROB / 001 AVS	ADV. CALC. / F208 DO			PROG PRINC / A241 TFK				
TUESDAY		INTRO TO PROB / 001 AVS	ADV. CALC. / ~~F208~~ A212 DO					E.SCI III TFK		
WEDNESDAY						PROG PRINC / A241 TFK		E SCI III TKF		
THURSDAY		INTRO TO PROB / 001 AVS	ADV. CALC / A212 ~~F208~~ DO			PROG PRINC / A241 TFK		E SCI III TFR		
FRIDAY		INTRO TO PROB / 001 AVS	ADV CALC / F208 D.O.			PROG PRINC / A241 TFK		E SCI III TFK		

METHOD OF SIGNAL AND SYSTEMS ANALYSIS
COOPER; McGILLEM

9-11-70

FIRST SEVEN CHAPTERS

9-15-70



MUST HAVE MATHEMATICAL MODEL TO FIT EACH COMPONENT, FOR INPUT/OUTPUT

FOR BLACK BOX:

$$a_n Y^n(t) + a_{n-1} Y^{n-1}(t) + \dots + a_0 Y(t)$$

$$= b_n X^m(t) + b_{m-1} X^{m-1}(t) + \dots + b_0 X^0(t)$$

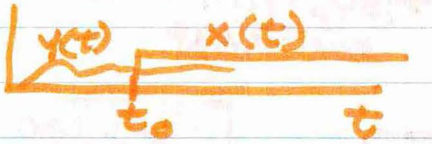
⇒ n^{TH} ORDER D.E
LINEAR D.E.
CONSTANT COEFFICIENT D.E.

CLASSIFICATION OF SYSTEMS

1) ORDER: n^{TH} ORDER OF HIGHEST ORDER DEPENDENT VARIABLE

2) CAUSAL, NON-CAUSAL:

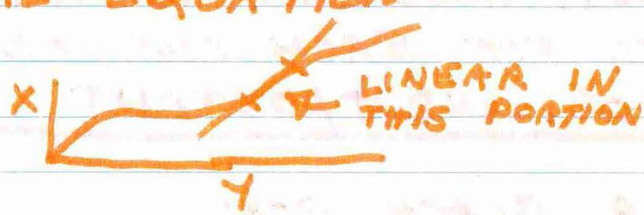
a) NON CAUSAL



(OUTPUT BEFORE INPUT NOT PHYSICALLY POSSIBLE, YET MATHEMATICALLY POSSIBLE)

3) LINEAR VS NONLINEAR

a) REFERS TO THE MULTIPLICITY OF THE DERIVATIVES IN THE EQUATION

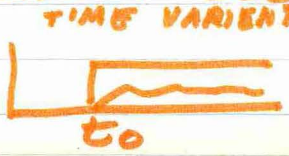


b) LINEARITY WILL IMPLY SUPERPOSITION (IF X_1 RESULTS IN OUTPUT Y_1, & X_2 YEILDS Y_2, THEN X_1+X_2 INPUT YIELDS Y_1+Y_2 OUTPUT)

4) FIXED TIME VS. TIME VARIENT

CONSTANT COEFF VS. TIME VARIENT COEFF. IN D.E.

TIME VARIENT



5) LUMPED PARAMETER VS.
DISTRIBUTED PARAMETER
a) LUMPED ⇒ ORDINARY D.E. \$ D.E.
b) DIST. PARA ⇒ PARTIAL D.E.

6) CONTINUOUS OR DISCRETE
D.E. DIFFERENCE EQUATION

7) INSTANTANEOUS VS. DYNAMIC
COMBANATIONAL LOGIC | MEMORY (ENERGY
RESISTIVE STORAGE)

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NORMAL FORM ON EQNS. n 1ST ORDER
WITH n VARIABLES ("STATE VARIABLES")

DEFINE STATES: $q_1(t), q_2(t) \ldots, q_n(t)$
WRITE SYSTEM OF EQNS:

$$q_1'(t) = a_{11} q_1(t) + a_{12} q_2(t) + \ldots + a_{1n} q_n(t) + b_1 x(t)$$

$$q_2'(t) = a_{12} q_1(t) + a_{22}(t) q_2(t) + \ldots + a_{2n} q_n(t) + b_2 x(t)$$

$$\vdots$$

$$q_n'(t) = a_{n1} q_1(t) + \ldots a_{nn} q_n(t) + b_n x(t)$$

OUTPUT

$$y(t) = c_1 q_1(t) + c_2 q_2(t) + \ldots + c_n q_n(t)$$

$$\underset{\sim}{Q}^{(1)}(t) = \begin{bmatrix} q_1'(t) \\ q_2'(t) \\ \vdots \\ q_n'(t) \end{bmatrix} \qquad C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

$$\underset{\sim}{Q}(t) = \begin{bmatrix} q_1(t) \\ \vdots \\ q_n(t) \end{bmatrix}$$

$$\underset{\sim}{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix}$$

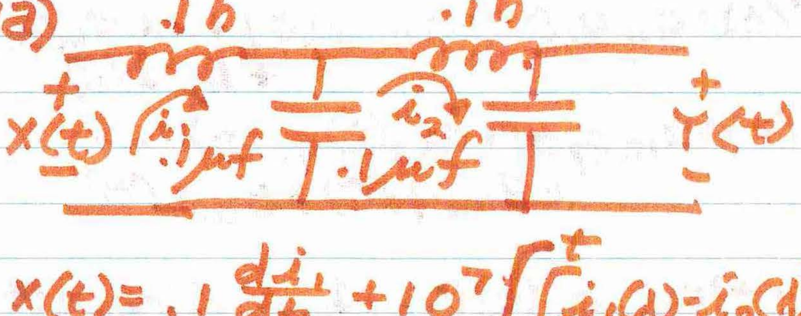
$$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

x(t) IS A SCALAR

$$\underset{\sim}{Q}'(t) = \underset{\sim}{A}\,\underset{\sim}{Q}(t) + \underset{\sim}{b}\,x(t)$$

$$Y(t) = \underset{\sim}{C}^T \underset{\sim}{Q}(t)$$

PROBLEM EXAMPLE 2.7a)



$$x(t) = .1\frac{di_1}{dt} + 10^7 \int_{-\infty}^{t} [i_1(\lambda) - i_2(\lambda)]\,d\lambda$$

$$0 = 10^7 \int_{-\infty}^{t} [i_2(\lambda) - i_1(\lambda)]\,d\lambda + .1\frac{di_2}{dt} + 10^7 \int_{-\infty}^{t} i_2(\lambda)\,d\lambda$$

(CONT PG. FOLL.)

TO SOLVE:

WHAP ON LAPLACE $\qquad \int \to \frac{1}{s}$

$$\frac{I_1(s)}{s} \Leftrightarrow \int_0^t i_1(\lambda)\,d\lambda$$

$$sI(s) - i(0) \Leftrightarrow di/dt$$

$$a_{11} I_1(s) + a_{12} I_2(s) = \frac{x(s)}{s}$$

$$a_{21} I_1(s) + a_{22} I_2(s) = 0$$

$$\underset{\sim}{I} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad V_s = \begin{bmatrix} \frac{V_s}{s} \\ 0 \end{bmatrix}$$

$$\underset{\sim}{A}\underset{\sim}{I} = \underset{\sim}{V_s} \Rightarrow I = A^{-1} V_s$$

KRAMER:

$$I_1 = \frac{\begin{vmatrix} \frac{x}{s} & a_{12} \\ 0 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(CONT FROM 2 PGS AGO)

$$Y(t) = 10^7 \int_{-\infty}^{t} i_2(\lambda)\,d\lambda$$

GET RID OF INTEGRALS

$$Y'(t) = 10^7\, i_2(t) \Rightarrow Y^{iv}(t) = 10^7 \frac{d^3 i_2}{d t^3}$$

FROM BEFORE

$$\frac{di_2}{dt} = 10^8 \int i_1(\lambda)\,d\lambda - 10^8 \int i_2(\lambda)\,d\lambda$$
$$- 10^8 \int i_2(\lambda)\,d\lambda$$

$$\frac{d^2 i_2}{dt^2} = 10^8 i_1 - 2\times 10^8 i_2$$

$$\frac{d^3 i_2}{dt^3} = 10^8 \frac{di_1}{dt} - 2\times 10^8 \frac{di_2}{dt}$$

PLUG IN FROM ABOVE

FROM THE PREVIOUS

$$10^8 \int i_1 = 10^{-8} Y''(t) + 20\,Y(t)$$
$$10^8\, di_1/dt = 10^{-7} Y^{4}(t) + 2\times 10^{15}\, Y^{(2)}(t)$$

PLOP THESE QUANTITIES INTO ORIGINAL MESH EQNS.

$$x(t) = 10^{-16}\, Y^{4}(t) + 2\times 10^{-6}\, Y^{2}(t)$$
$$+ 10^{-9}\, Y^{3}(t) + 2\,Y(t) - Y(t)$$

HOWEVER, SOMEONE SKREWED UP THE PROBLEM, SO 'DE ANSWER AIN'T RIGHT

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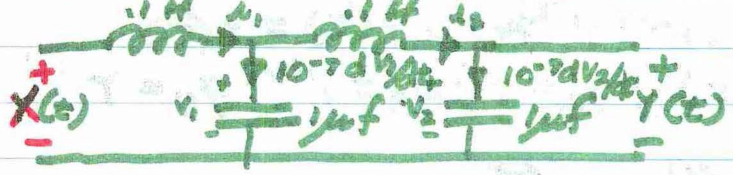

(H CONTROLS MAG. FIELD)

LET $q_1 = i_1$, $q_2 = i_2$, $q_3 = V_1$, $q_4 = V_2 \ (= Y(t))$

$$\left.\begin{aligned} x(t) - V_1 &= .1 \frac{di_1}{dt} \\ V_1 - V_2 &= .1\, di_2/dt \end{aligned}\right\} \begin{aligned} x(t) - q_3 &= .1 \frac{dq_1}{dt} \\ q_3 - q_4 &= .1 \frac{dq_2}{dt} \end{aligned}$$

$$q_1' = -10q_3 + 10x(t)$$

$$q_3' = 10q_3 - 10q_4$$

$$(i = C\, dV/dt)$$

$$\left.\begin{aligned} i_1 &= i_2 + 10^{-7}\, dV_1/dt \\ i_2 &= 10^{-7}\, dV_2/dt \end{aligned}\right\} \begin{aligned} q_1 &= q_2 + 10^{-7} q_3' \\ q_2 &= 10^{-7}\, dq_4/dt \end{aligned}$$

NORMAL FORM:

$$q_1' = -10q_3 + 10\,x(t)$$

$$q_2' = 10q_3 - 10q_4$$

$$q_3' = 10^7 q_1 - 10 q_2$$

$$q_4' = 10^7 q_2$$

$$Y = q_4$$

NORMAL FORM EQUATIONS NOT UNIQUE

(FROM SAME SYSTEM)

$10^{-16}\, Y^{(4)}_{(t)} + 3\times10^{-8}\, Y^{(2)}(t) + Y(t) = X(t)$

LET $q_1 = Y$; $q_2 = Y'$; $q_3 = Y''$; $q_4 = Y'''$

$$q_1^{(1)} = q_2$$
$$q_2^{(1)} = q_3$$
$$q_3^{(1)} = q_4$$
$$q_4^{(1)} = -3\times10^{8} - 10^{16} q_1 + 10^{16} X$$
$$Y = q_1$$

SAYS SAME THING AS PREVIOUS NORMAL FORM, IF INITIAL CONDITIONS ARE KNOWN

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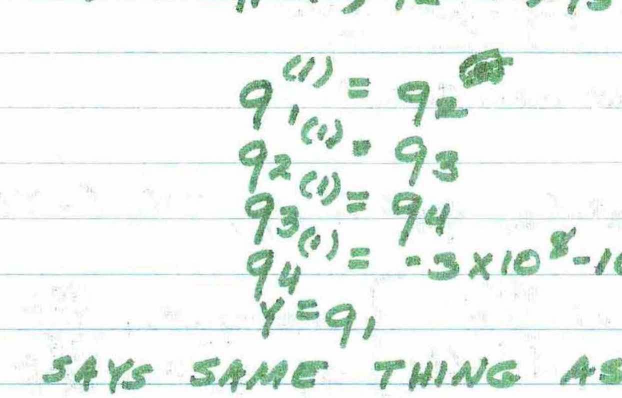
2-10)



$Y = i_2$

$$X = i_1 + i_1' - i_2' - i_3$$
$$0 = i_2' - i_1' + i_2 - i_3 + i_2$$
$$0 = i_3 - i_1 + i_3 - i_2 + \int i_3$$

$$X = i_1 + i_1' - Y' - i_3$$
$$0 = Y' - i_1' + 2Y - i_3$$
$$0 = 2i_3 - i_1 \mp Y + \int i_3$$

$x(t) - Y(t) = \int i_3(t) \Rightarrow X'(t) - Y'(t) = i_3$

$X = i_1 + i_1' - Y' - X' + Y'$

a) $0 = Y' - i_1' + 2Y - X' + Y'$

$0 = 2X' - 2Y' - i_1 + Y + X - Y$

$i_1 = 2X' - 2Y' + X$

$i_1' = 2X'' + 2Y'' + X'$

PLUG INTO a

$0 = Y' - 2X'' + 2Y'' - X' + 2Y - X' + Y'$

ALGEBRA $\Rightarrow 2Y'' + 4Y' + 2Y = 2X'' + 2X'$

$Y'' + 2Y' + Y = X' + X''$

WHAP ON LAPLACE

$(s^2 + 2s + 1)Y(s) = (s^2 + s)X(x)$

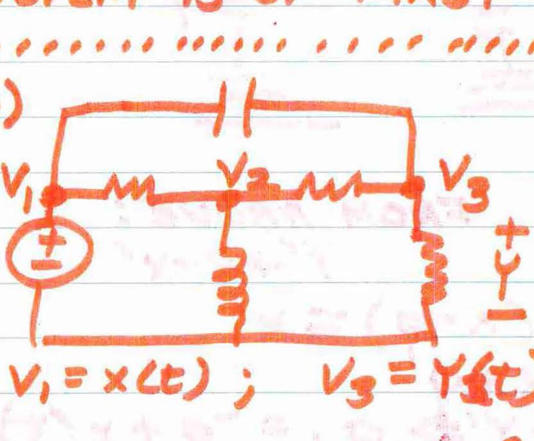
$(s+1)^2 Y(s) = s(s+1)X(s)$

OR $Y' + Y = X'$

$\therefore$ SYSTEM IS OF FIRST ORDER

. . . .

2-10)



$V_1 = x(t)$; $V_3 = Y(t)$

ON NODE 2: $V_1 - X + \int V_2 + V_2 - Y = 0$

ON NODE 3: $Y - V_2 + Y + Y' - X' = 0$

(CONT.)

SOLVE FOR V_2:

$V_2 = 2Y + Y' - X'$

PLUG INTO TOP EQ:

$2Y+Y'+-X'$

$-X + \int(2Y+Y'-X') + 2Y + Y' - X' - Y = 0$

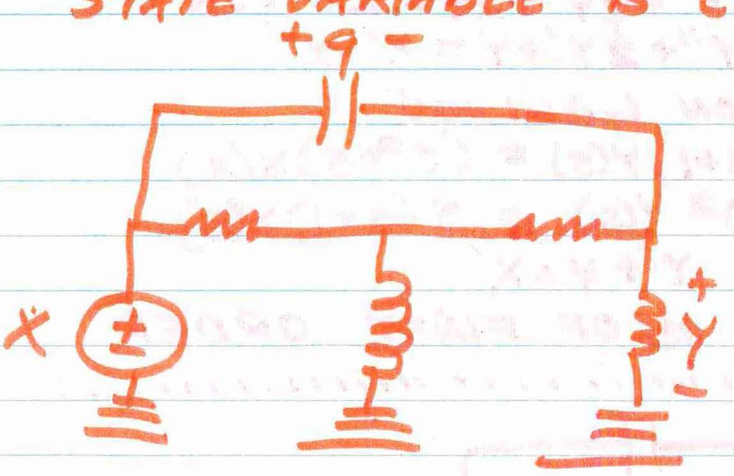
DIFFERINTIATE

$2Y' + Y'' + X'' - X' + 2Y + Y' - X' + 2Y' + Y'' - X'' - Y' = 0$

$\Rightarrow Y'' + 2Y' + Y = X'' + X$

PUT IN NORMAL FORM

STATE VARIABLE IS CAP. V



$q = X - Y$ FROM ABOVE:

$q' = X' - Y'$ $Y' + Y = X'$

$\therefore Y' + (X - q) = X'$

$\Rightarrow Y' = X' - X + q$

$q' = X' - Y' \Rightarrow q_1 = -q + x(t)$

ALSO $Y = X - q$

CHAPTER III

CLASS OF SIGNALS

1. PERIODIC VS NON-PERIODIC (APERIODIC)
2. RANDOM VS. DETERMINISTIC
3. ENERGY VS POWER

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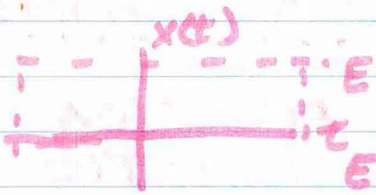
3. ENERGY VS POWER

A) ENERGY: $x(t)$ WHICH SATISFIES $E = \int_{-\infty}^{\infty} x^2(t)\,dt < \infty$

$$\left(E = \int p(t)\,dt\right)$$

$$P = i^2 R = V^2/R$$



BOUNDED IN ENER. & TIME

B) POWER SIGNALS.

a) PERIODIC SIGNALS

TALK OF AVER PWR.

$$P_{AVE} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} x^2(t)\,dt$$

IF $x(t)$ IS A PWR SIGNAL:

$$0 < P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\,dt < \infty$$

(CONT.)

EX) $x(t) = e^{-at}$; $-\infty < t < \infty$

$E = \int_{-\infty}^{\infty} e^{-2at} dt$

$= \frac{-1}{2a} e^{-2at} \Big|_{-\infty}^{\infty} \rightarrow \infty$

$\therefore$ NOT ENERGY SIGNAL

$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} e^{-2at} dt$

$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{-1}{2a} e^{-2at} \right]_{-T}^{T}$

$= \lim_{T\to\infty} -\frac{1}{4aT} e^{-2aT} + \frac{1}{4aT} e^{2aT}$

MUST APPLY LAHOPITAL'S RULE

. . . .

REPRESENTING FUNCTIONS FROM BASE FUNCTION

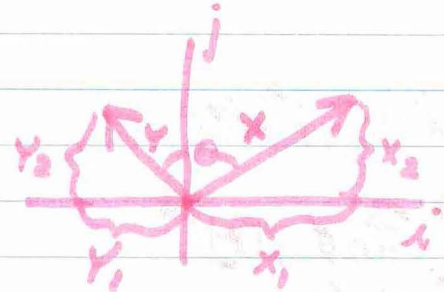
$x(t) = \sum_{n=0}^{\infty} a_n \phi_n(t)$

1) SHOULD BE ABLE TO FIND a_n WITHOUT KNOWING ANY OTHERS (CALLED FINALITY OF COEFF)

- FINALITY OF COEFFICIENTS: OBTAINED BY INSISTING ON ORTHOGONAL BASIC FUNC. $\phi_n(t)$

ORTH: $\underbrace{\int_{t_1}^{t_2} \phi_n(t)\phi_m(t)\,dt}_{\text{INNER PROD OR DOT PROD}} = \begin{cases} 0 & \text{IF } n \neq m \\ \lambda_n & \text{IF } n = m \end{cases}$

$\lambda_n = 1$ IF BASES ARE ORTH.



$$\hat{x} = x_1 \hat{i} + x_2 \hat{j}$$
$$\hat{y} = y_1 \hat{i} + y_2 \hat{j}$$

$$\hat{x}\cdot\hat{y} = |\hat{x}||\hat{y}|\cos\theta = \sum_{\ell=1}^{2} x_\ell y_\ell$$

ANALOGOUS TO BASIC FUNCTION

$$= x_1 y_1 + x_2 y_2$$

$$\int_{t_1}^{t_2} \phi_n(t)\, x(t) = \int_{t_1}^{t_2} \phi_n(t) \sum_{n=0}^{\infty} a_n \phi_n(t)\, dt$$

$$= \sum_{n=0}^{\infty} a_n \underbrace{\int_{t_1}^{t_2} \phi_j(t)\phi_n(t)\, dt}_{\lambda_j \Rightarrow (n=j)}$$

$$= a_j \lambda_j = \int_{t_1}^{t_2} \phi_j(t)\, x(t)\, dt$$

$$a_j = \frac{1}{\lambda_j} \int_{t_1}^{t_2} \phi_j(t)\, x(t)\, dt$$

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$$I = \int_{t_1}^{t_2} [x(t) - \hat{x}(t)]^2 \, dt$$

$$= \int_{t_1}^{t_2} \left[x(t) - \sum_{n=0}^{M} \hat{a}_n \phi_n(t)\right]^2$$

$$\hat{a}_n = a_n = \frac{1}{\lambda_n} \int_{t_1}^{t_2} x(t) \phi_n(t) \, dt$$

$$I = \int_{t_1}^{t_2} \Big[x(t)^2 - 2x(t) \sum \hat{a}_n \phi_n(t)$$

$$+ \sum_{i=0}^{M} \sum_{j=0}^{M} a_i a_j \phi_i(t) \phi_j(t)\Big] dt$$

$$I = \int_{t_1}^{t_2} x^2(t) \, dt - 2 a_n \sum \int_{t_1}^{t_2} x(t) \phi_n t \, dt$$

$$+ \sum \sum a_i a_j \int_{t_1}^{t_2} \phi_i(t) \phi_j(t) \, dt$$

WANT TO MINIMIZE, → ALL STATEMENTS MAY BE TREATED AS A CONSTANT WITH $\hat{a}_n$ IN THEM. THUS, STATEMENT REDUCES TO:

~~[illegible]~~

$$I = K - 2 \sum_{n=0}^{M} \hat{a}_n a_n \lambda_n$$

$$+ \sum_{n=0}^{M} \hat{a}_n^2 \lambda_n$$

$$I = K + \sum_{n=0}^{M} \lambda_n (a_n - \hat{a}_n)^2 - \sum_{n=0}^{M} a_n^2 \lambda_n$$

MINIMUM OCCURS AT $a_n = \hat{a}_n$

$$I = K - \sum_{n=0}^{M} a_n^2 \lambda_n = \text{MINIMUM M.S. ERROR}$$

$\rightarrow$ = ENERGY OF ERROR

. . . .

$$\frac{\text{ERROR EN.}}{\text{SIGNAL EN}} = \frac{I}{K} = 1 - \frac{\sum_{n=0}^{M} a_n^2 \lambda_n}{K}$$

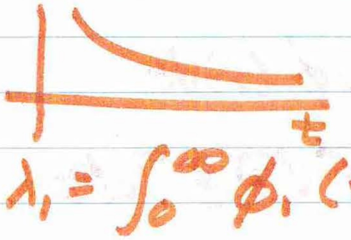
RATIO BETWEEN 0 & 1

$$\int_{t_1}^{t_2} x^2(t)\,dt = \sum_{n=0}^{\infty} a_n^2 \lambda_n$$

PARSEVAL'S THER.

3-11 $\phi_1(t) = \sqrt{2}\,\epsilon^{-t}$

$\phi_2(t) = A\epsilon^{-2t} + B\epsilon^{-t}$



$$\lambda_1 = \int_0^{\infty} \phi_1(t)\,dt = +1$$

$$0 = \int_0^{\infty} \phi_1(t)\,\phi_2(t)\,dt$$

$$= \int_0^{\infty} (\sqrt{2}A\epsilon^{-3t} + \sqrt{2}B\epsilon^{-2t})\,dt$$

$$= \frac{-\sqrt{2}A}{3}\epsilon^{-3t} - \frac{\sqrt{2}B}{2}\epsilon^{-2t}\Big|_0^\infty$$

$$\Rightarrow A = -\frac{3}{2}B$$

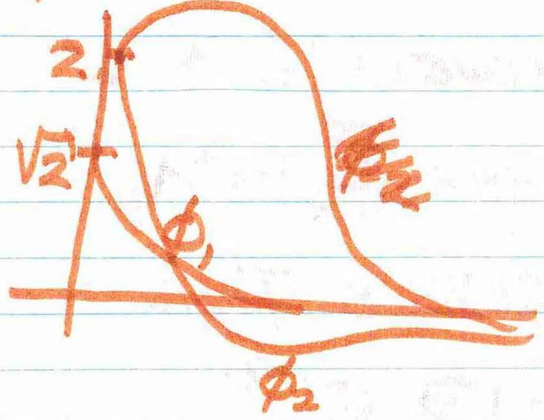
$$\int_0^\infty \left(A^2\epsilon^{-4t} + 2AB\epsilon^{-3t} + B^2\epsilon^{-2t}\right)dt = 1$$

$$= \frac{A^2}{4} + \frac{2}{3}AB + \frac{B^2}{2} = 1$$

PLUG IN $A = -\frac{3}{2}B$

$$\Rightarrow B = \pm 4 \Rightarrow A = \mp 6$$

LET: $\phi_2(t) = 6\epsilon^{-3t} \; 4\epsilon^{-t}$



TRYING TO REPRESENT $x(t)$

$$a_1 = \frac{1}{\lambda_1}\int_0^\infty x(t)\phi_1(t)\,dt$$

$$= \int_0^\infty 10\epsilon^{-\frac{1}{2}t}\sqrt{2}\,\epsilon^{-t}dt$$

$$= 4\sqrt{2}$$

$$\therefore \hat{x} = \sum_{n=1} a_n\phi_n(t) = a_1\phi_1(t) + a_2\phi_2 t$$

(CONT.)

$= (4\sqrt{2})(\sqrt{2}\,\varepsilon^{-t}) + a_2[6\varepsilon^{-2t} - 4\varepsilon^{-t}]$

COMPUTE a_2,

THEN $\frac{I}{K} = 1 - \frac{\sum_{n=1}^{2} a_n \lambda_n}{\int_0^\infty x^2(t)dt}$

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$$\text{ERROR} = 1 - \frac{\sum \lambda_n a_n^2}{\int x^2(t)dt}$$

WALSH (4 UNITS)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ +1 & -1 & 1 & +1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 4 \\ 2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 8 \\ 4 \\ 0 \\ 4 \end{bmatrix} = \overset{W}{\begin{bmatrix} 4 \\ -2 \\ 0 \\ 2 \end{bmatrix}}$$

$$w_j = \sum_{k=1}^{4} c_{jk} x_k$$

FOURIER BASIS FUNCTIONS (EXP.)

BASIS FUNC → $e^{jn\omega_0 t}$ $n = 0, \pm 1, \pm 2 \ldots$

$\omega_0 = \frac{2\pi}{T} = 2\pi f_0$

SHOW $\phi_n(t)$ ARE ORTHOGONAL &
FIND λ TO MAKE THEM ORTHONORMAL

$$\int_{t_0}^{t_0+T} \phi_n(t)\, \phi_k^*(t)dt$$

$$= \int_{t_0}^{t_0+T} e^{jn\omega_0 t} e^{-jk\omega_0 t} dt$$

(CONT. OF COURSE)

$$= \int_{t_0}^{t_0+T} e^{j(n-k)\omega_0 t}\,dt$$

$$= \frac{1}{j(n-k)\omega_0} e^{j(n-k)\omega_0 t} \Big|_{t_0}^{t_0+T}$$

$$\omega_0 = \frac{2\pi}{T}$$

$$= \frac{1}{j(n-k)\omega_0} e^{j(n-k)\pi 2t_0/T_0} e^{j(n-k)\frac{2\pi T}{T}}$$

$$- \frac{1}{j(n-k)\omega_0} e^{j(n-k)\frac{2\pi t_0}{T}}$$

$$= \frac{1}{j(n-k)\omega_0} e^{j(n-k)\frac{2\pi t_0}{T}} \left(e^{j(n-k)2\pi} - 1 \right)$$

$$e^{j(\text{INT})2\pi}$$

$$= \cos 2m\pi + j\sin 2\pi m - 1$$

$\therefore\ \lambda_n = 0$ FOR ALL $n \neq k$

MUST USE LAHOPITAL'S RULE

$$\lim_{(n-k)\to 0} \frac{j\,2\pi\, e^{j(n-k)2\pi}}{j\,\omega_0} = T = \lambda$$

EXP. ORTHOGONAL, BUT NOT ORTHONORMAL

TO BE ORTHONORMAL:

$$\phi_n = \frac{1}{\sqrt{T}} e^{jn\omega_0 t}$$

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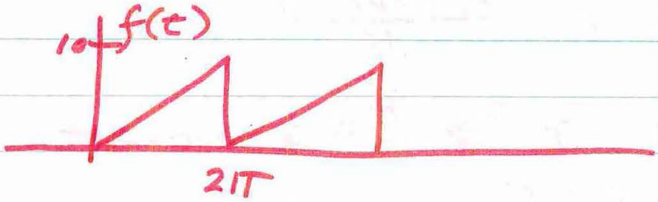


$$f(t) = \frac{5}{\pi} t \quad \begin{cases} 0 \le t \le 2\pi \\ 0 \text{ ELSEWHERE} \end{cases}$$

$$T = 2\pi \Rightarrow \frac{2\pi}{T} = 1$$

$$C_n = \frac{1}{T}\int_0^{2\pi} f(t)\,\phi^*(t) \qquad (\phi_n(t) = e^{jn\omega_0 t})$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \underbrace{\frac{5t}{\pi}}_{U}\, \underbrace{e^{-jnt}\,dt}_{dV}$$

$$dU = \frac{5}{\pi}dt \quad V = \frac{-1}{jn}e^{-jnt} = V$$

$$C_n = \frac{-1}{2\pi}\, \frac{5t}{\pi}\,\frac{1}{jn}e^{-jnt}\Big|_0^{2\pi} + \int_0^{2\pi}\frac{1}{jn}e^{-jnt}\frac{5}{\pi}\,dt$$

$$= -\frac{5(1)}{2\pi^2 jn} - 0 + \left[-\frac{1}{(jn)^2}e^{-jnt}\right]_0^{2\pi}$$

$$= \frac{5}{2\pi^2 jn} \;;\; n = 0, \pm 1, \pm 2 \ldots$$

BUT $n \neq 0$

$$\Rightarrow C_n = \frac{-5}{2\pi^2 jn} \quad n = \pm 1, \pm 2, \pm 3 \ldots$$

~~A~~ C_0 = D.C. VALUE = 5

$C_0 = 5$

$C_n = \frac{j5}{2\pi^2 n}$; $n = \pm 1, \pm 2 \pm 3 \ldots$

$$f(t) = 5 \pm \frac{j5}{2\pi^2}e^{\mp jt} \pm \frac{j5}{4\pi^2}e^{\mp j2t} + \ldots$$

(CONT.)

$$F.S.E. = 1 - \frac{\Sigma |a_n|^2 \lambda_n}{K}$$

$$(f(t))^2 = \frac{3}{6}t^2 \Rightarrow E = \frac{1}{3}\cdot 100 \cdot 2\pi$$

$C_0 = a_0 = 5$

$C_1 = |a_1| = \frac{5}{2\pi^2}$

$C_{-1} = |a_2| = \frac{5}{2\pi^2}$ $\quad \lambda_n = \frac{1}{T}$

CHUG & PLUG & CHUG

PARCEVAL'S THEOREM:

SUM OF INFINITE COEFF EQUAL TO TOTAL OF ENERGY

TEST OVER;

~~OVER~~ FOURIER-WALSH BASIS FUNCTIONS

9-29-70

UNIT IMPULSE

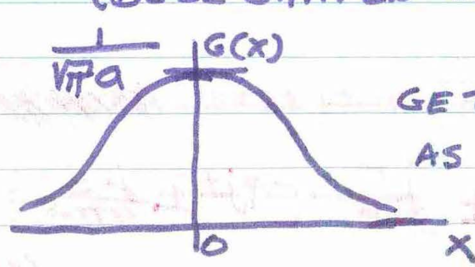
$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ 1, & t = 0 \end{cases}$$

REFERS TO AREA, NOT MAGN.

GENERATE $\delta(t)$ VIA GAUSSIAN OPERATOR

$$G = \frac{1}{\sqrt{\pi}a} e^{\frac{-x^2}{2a^2}}$$

(BELL SHAPED CURVE)



GET NARROWER AS a GETS SMALLER

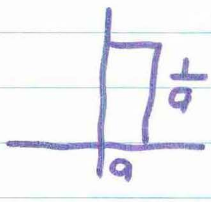
(CONT)

$\int G = 1$

$\lim_{a \to 0} G(x) = \delta(x)$ (CENTERED AROUND ORIGIN)

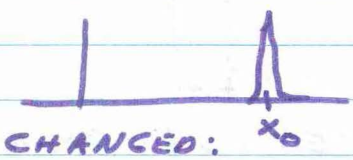
DIRAC DELTA FUNCTION

CAN ALSO DERIVE $\delta(x)$ AS FOLLOWS:



$\delta = \lim a \to 0$

$\delta(x - x_0) \Rightarrow$



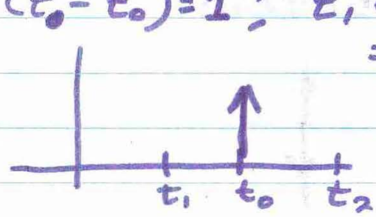
IF SCALE IS CHANGED:

$b\,\delta(bx)$ TO KEEP UNIT AREA

. . . . . IE) $\int f(t)\delta(t) = \int f(bt)\delta(bt)dt$

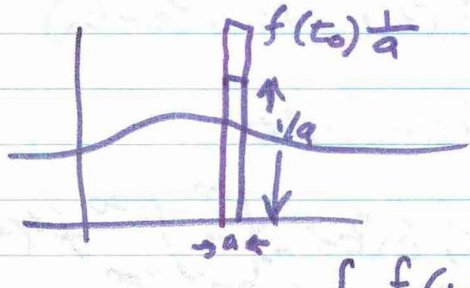
$\int_{t_1}^{t_2} \delta(t_0 - t_0) = 1$; $t_1 < t_0 < t_2$

$= 0$ IF NOT



$\int_{-\infty}^{\infty} f(t)\,\delta(t - t_0)\,dt = f(t_0)$

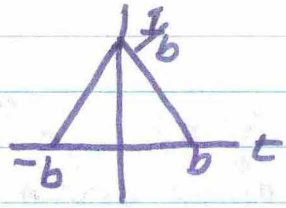
(SIFTING PROP. OF DEL. FUNCTION)



$\int f(t) \frac{1}{a} \cdot a$

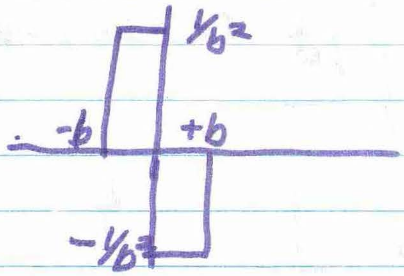
BIG?

DOUBLET:



$\sim \delta(t)$ AS $b \to 0$

TAKING DERIVITIVE

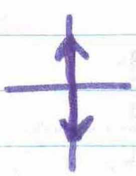


CALLED UNIT DOUBLET

$\sim \delta'(t)$ AS $b \to 0$

$$\int_{-\infty}^{\infty} \delta'(t)\,dt = 0$$

REPRESENTED:



$$\int_{-\infty}^{\infty} \underset{u}{f(t)}\, \underset{v}{\delta'(t-t_0)}\,dt$$

INTEGRATING BY PARTS

$du = f'(t) \quad V = \delta(t-t_0)$

$$\Rightarrow \int_{-\infty}^{\infty} f(t)\,\delta'(t-t_0)\,dt$$

$$= \underbrace{f(t)\,\delta(t-t_0)\Big|_{-\infty}^{\infty}}_{=0} - \int_{-\infty}^{\infty} f'(t)\,\delta(t-t_0)\,dt$$

(CONT.)

$$\Rightarrow \int_{-\infty}^{\infty} f(t)\,\delta'(t-t_0)\,dt = -f'(t_0)$$

SINGULARITIES

RAMP $\quad$ UNIT STEP $\quad$ IMPULSE $\quad$ DOUBLET

$r(t) \Leftrightarrow \mu(t) \Leftrightarrow \delta(t) \Leftrightarrow \delta'(t$

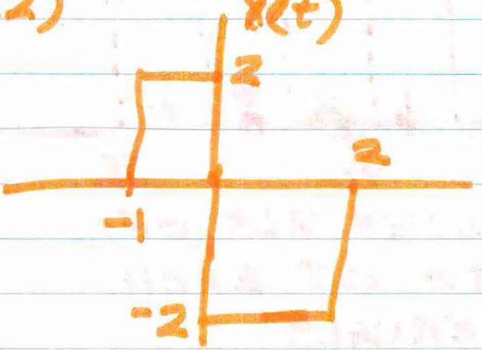
$\Rightarrow$ TAKE DERIV.

$\Leftarrow$ TAKE INTEGRALS

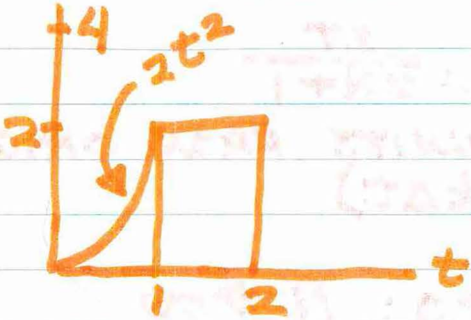
9-30-70

3-22)

b)



d)



$2t^2\mu(t) - 2t^2\mu(t-1) + 2\mu(t-1)$
$\quad - 2\mu(t-2)$

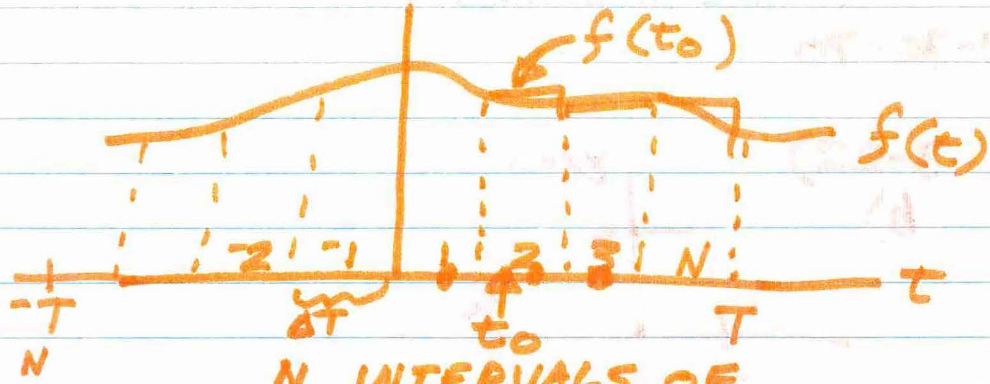
(CONT.)

INTEGRAL TRANSFORMS:

$$\int f(t)e^{-st}dt = \mathcal{L}\{f(t)\}$$

$$y' = Ax + B$$

$$\int e^{-st}y'dt = \int Axe^{-st}dt + \int Be^{-st}dt$$



N INTERVALS OF WIDTH ΔT EACH

$2N+1$ INTERVALS

$$\Delta T = \frac{2T}{2N+1}$$

$t_0 = k\Delta t$ UNITS FROM ORIGIN

$f(t_0) = f(k\Delta t)$

AS $\Delta T \Rightarrow 0$; $N \rightarrow \infty$

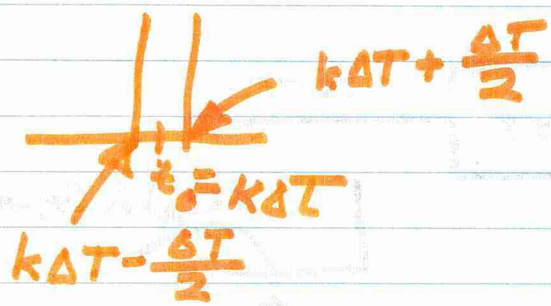
$N\Delta T = 2T$

$k\Delta T \Rightarrow \lambda$ A CONT. VARIABLE

(BEFORE IT WAS DISCRETE)

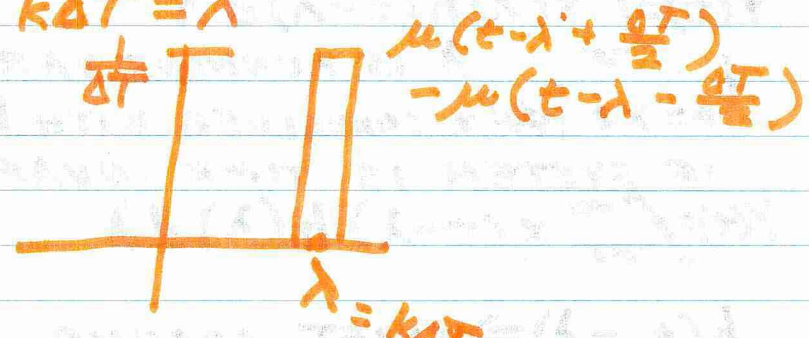
$$f(t) \simeq \sum_{k=-N}^{N} f(k\Delta t)\mu(t - k\Delta T$$

$$+ \frac{\Delta T}{2}) - \mu\left(t - k\Delta T - \frac{\Delta T}{2}\right)$$



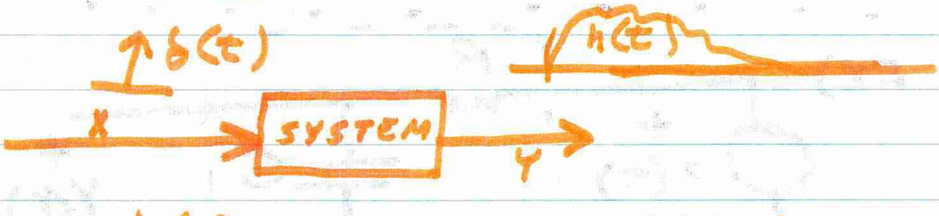
$\lim \Delta T \to 0 = d\lambda$

$k\Delta T = \lambda$

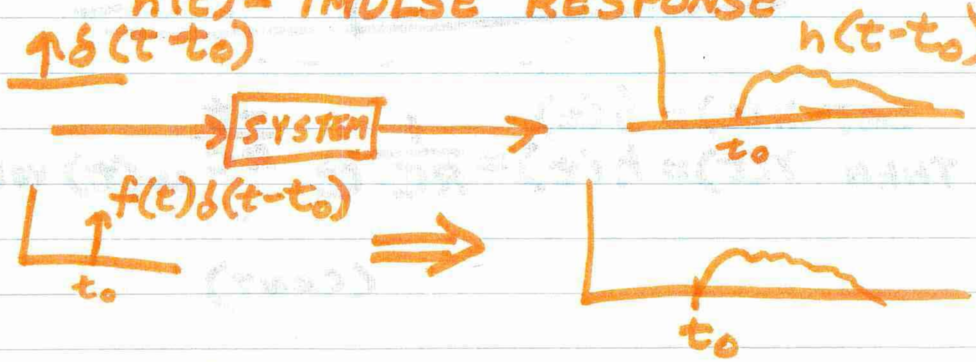


AS $\Delta T \to 0$; FUNC. APROACHES $\delta(t-\lambda)$

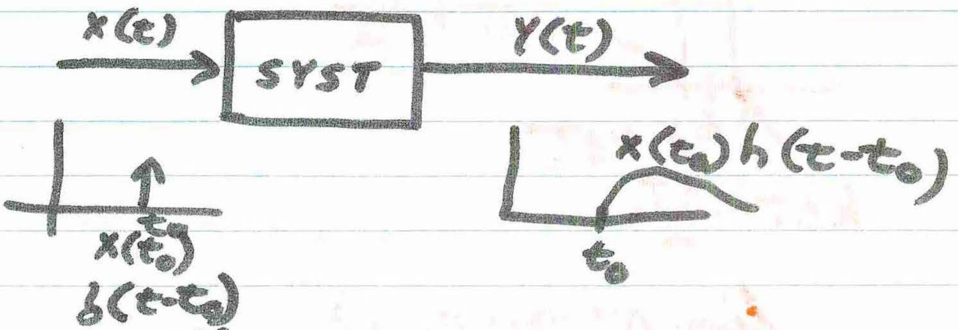
$$f(t) = \int_{-\infty}^{\infty} f(\lambda)\,\delta(t-\lambda)\,d\lambda$$



$h(t)$ = IMULSE RESPONSE



10-1-70



$$Y(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$

λ IS RUNNING VARIABLE

$= x * h \Rightarrow$ (x CONVOLVED WITH h)

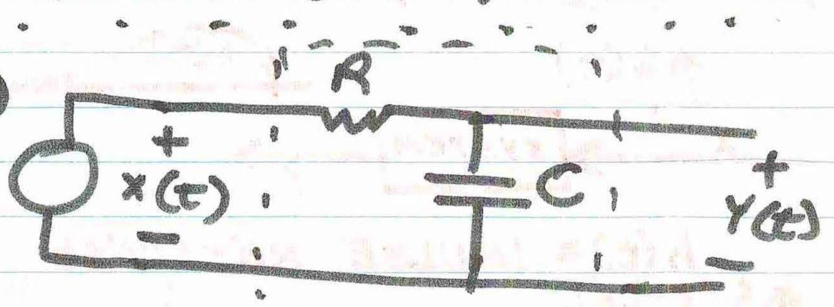
IF SYSTEM IS TIME INVARIENT

$$Y(t) = \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda$$

$h(t_0-\lambda) \Rightarrow$ SHIFT AROUND Y AXIS (ROTATED)

$$\delta(\lambda - t) = \delta(t-\lambda)$$

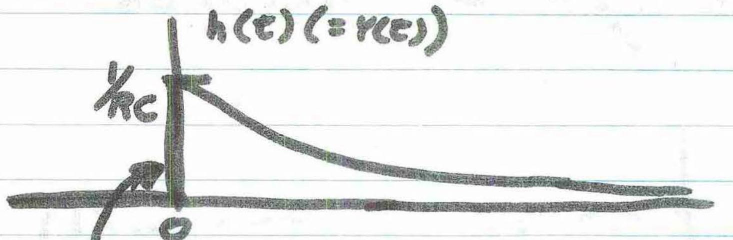
EX)



LET $x(t) = \delta(t)$

THEN $Y(t) = h(t) = \frac{1}{RC} e^{\frac{-t}{RC}} \mu(t)$ VOLTS

(CONT)



CAN JUMP, FOR δ HAS INFINITE EN.

$$i(0+) = \frac{\delta(t)}{R}$$

$$V_C = \frac{1}{C}\int i\,dt$$

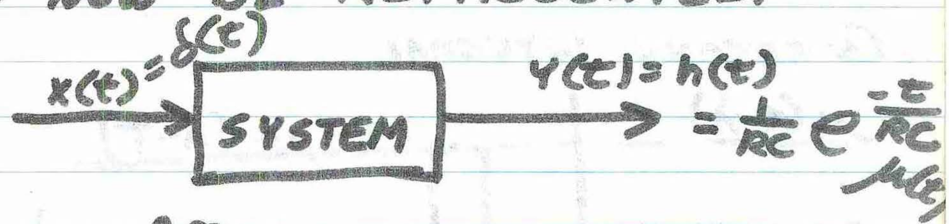
$$= \frac{1}{C}\int_0^\infty \frac{\delta(t)}{R}\,dt$$

$$= \frac{1}{RC} \text{ VOLT}$$

$$\tau = \frac{1}{RC}$$

$$\therefore\ Y(t) = h(t) = \frac{1}{RC}\, e^{-\frac{t}{RC}}$$

MAY NOW BE REPRESENTED:



$$Y(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)$$

LET $x(t) \Rightarrow$

$$x(t) = A\mu(t) - A\mu(t-T)$$

$$\therefore\ Y(t) = \int_{-\infty}^{\infty} \left[A\mu(\lambda) - A\mu(\lambda - T)\right] \cdot \frac{1}{RC}\, e^{-\frac{(t-\lambda)}{RC}} \mu(t-\lambda)\, d\lambda$$

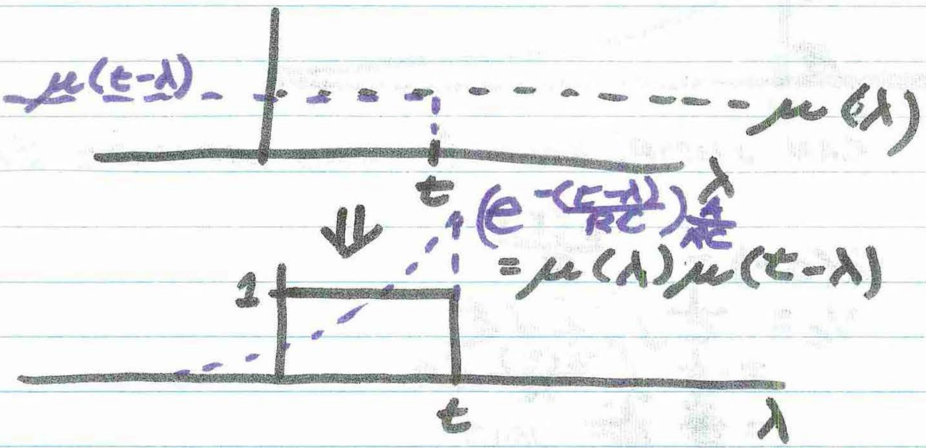
$$= \int_{-\infty}^{\infty} A\mu(\lambda)\, e^{-\frac{(t-\lambda)}{RC}} \mu(t-\lambda)$$

$$- \int_{-\infty}^{\infty} \frac{A}{RC}\mu(\lambda - T)\, e^{-\frac{(t-\lambda)}{RC}} \mu(t-\lambda)\, d\lambda$$

(CONT)

GOOD TO LOOK AT PICTURES:

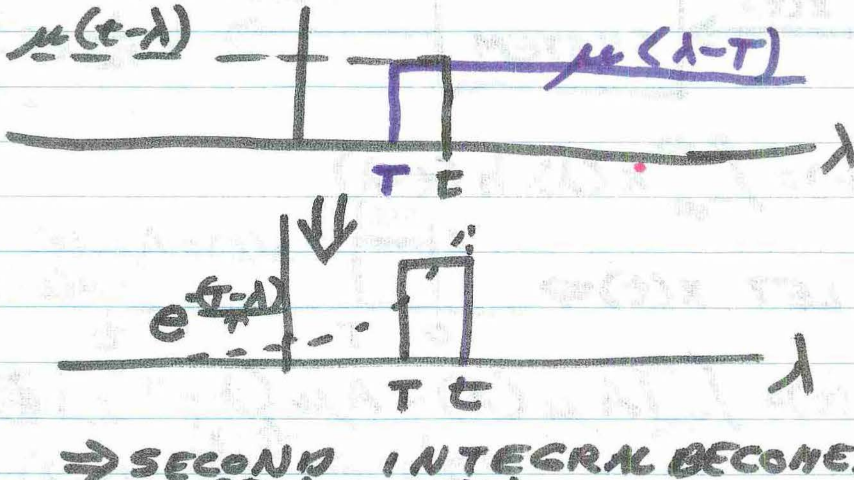
① FIRST $\int$



$\Rightarrow$ FIRST INTEGRAL BECOMES

$$\int_0^t \frac{A}{RC} e^{-\frac{(t-\lambda)}{RC}} d\lambda$$

② SECOND INTEGRAL



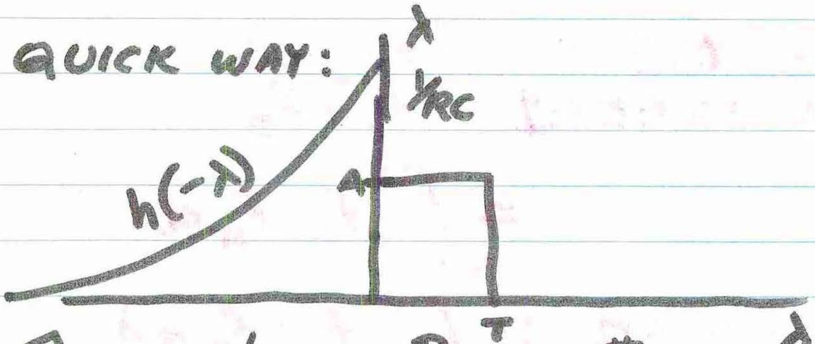
$\Rightarrow$ SECOND INTEGRAL BECOMES

$$-\int_T^t \frac{A}{RC} e^{-\frac{t-\lambda}{RC}} d\lambda$$

WHOLE SHABANG:

$$Y(t) = \underbrace{\int_0^t \frac{A}{RC} e^{-\frac{(t-\lambda)}{RC}} d\lambda}_{t>0} - \underbrace{\int_T^t \frac{A}{RC} e^{-\frac{(t-\lambda)}{RC}} d\lambda}_{\text{FOR } t>T}$$

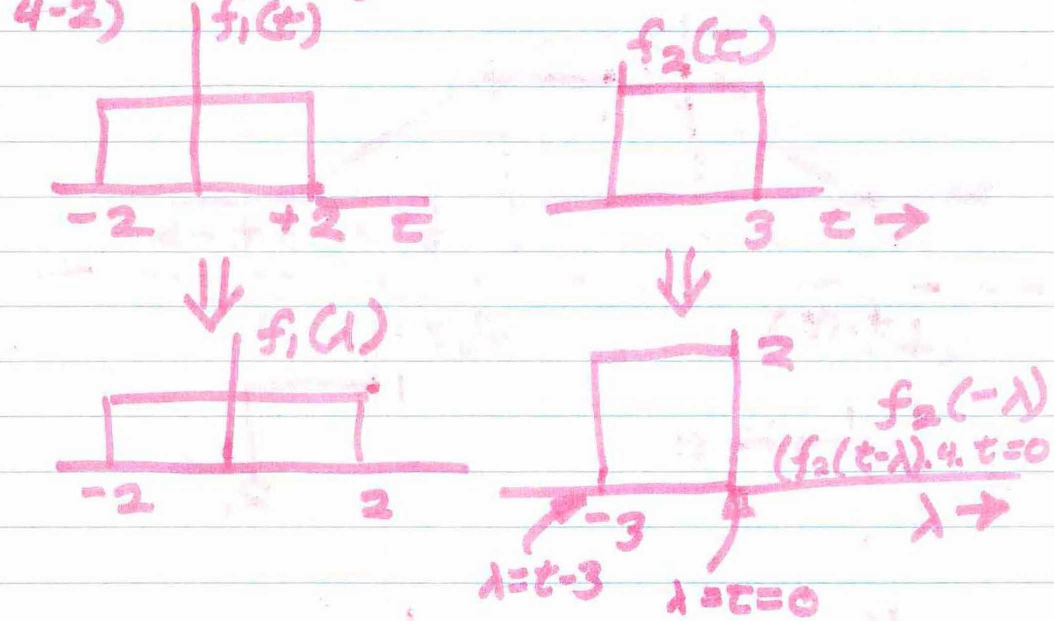
QUICK WAY:



SHIFT THIS WAY AND THAT (←, →)

10-2-70 Pg 80

4-2)



MUST FIRST MULTIPLY
2 FUNCTIONS TOGETHER

(CONT.)

FOR $t \leq -2$, $f_1 * f_2 = 0$

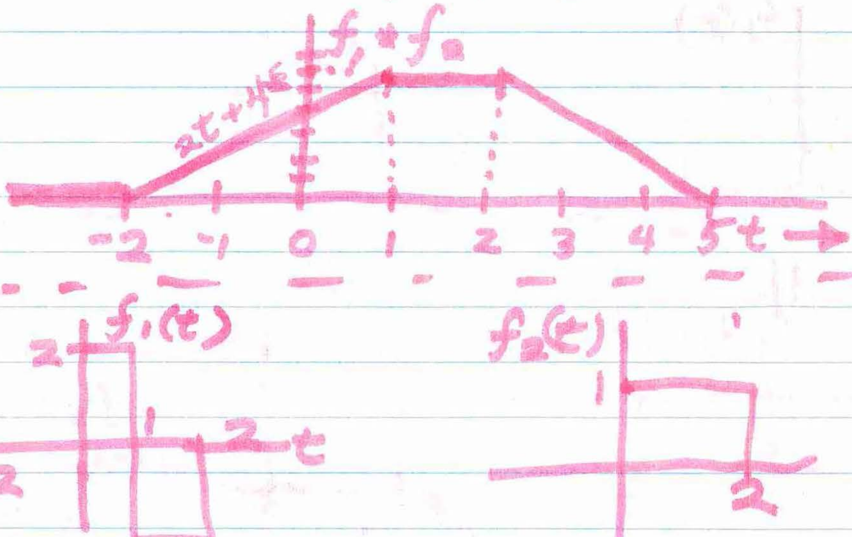
FOR $-2 \leq t \leq 1$, $f_1 * f_2$

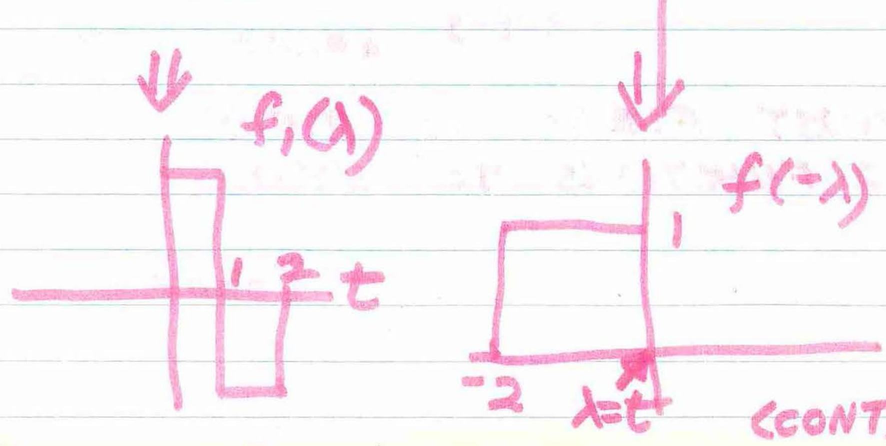
$$= \int_{-2}^{t} f_1 \cdot f_2 \, d\lambda$$

FOR $1 \leq t \leq 2$; $\int_{t-3}^{t} f_1 \cdot f_2 \, d\lambda$

FOR $2 \leq t \leq 5$; $\int_{t-3}^{2} f_1 \cdot f_2 \, d\lambda$

FOR $t \geq 5$; $f_1 * f_2 = 0$





(CONT)

$-\infty < t < 0 \Rightarrow f_1 * f_2 = 0$

$0 < t < 1 \Rightarrow \int_0^t f_1 \cdot f_2 \, d\lambda$

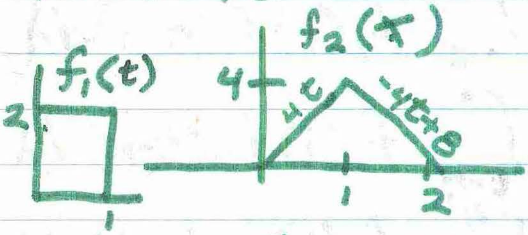
$1 < t < 2 \Rightarrow \int_0^1 2\,d\lambda + \int_1^t -2\,d\lambda$

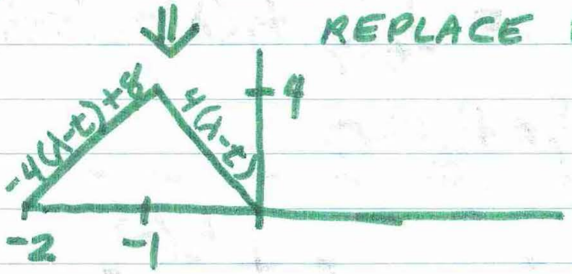
$2 < t < 3 = \int_{t-2}^1 2\,d\lambda' + \int_1^2 -2\,d\lambda$

$3 < t < 4 \Rightarrow \int_{t-2}^2 (-2)\,d\lambda$

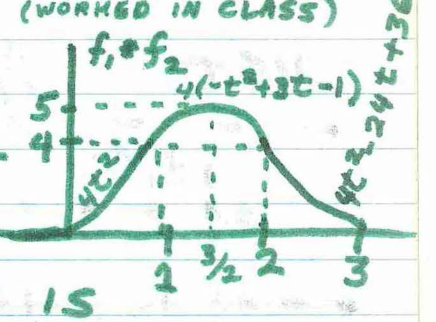
$t < 4 \quad f_1 * f_2 = 0$

10-6-70



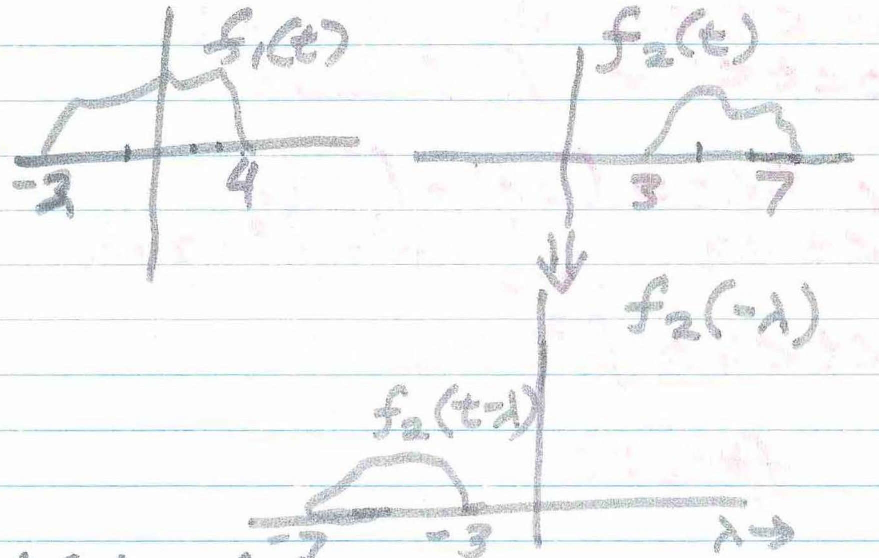


REPLACE t BY $(\lambda - t)$
(WORKED IN CLASS)



TIME OF CONVOLUTION IS SUM OF BASES OF TWO FUNTIONS
(FOR ABOVE FUNC., TIME FOR CONV = 3)

10-7-70



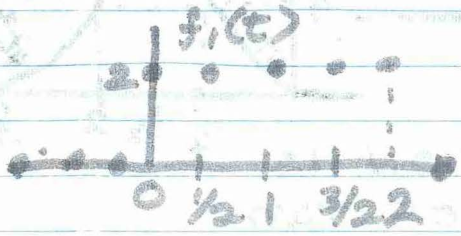
$t \leq 1 \quad f_1 * f_2 = 0$

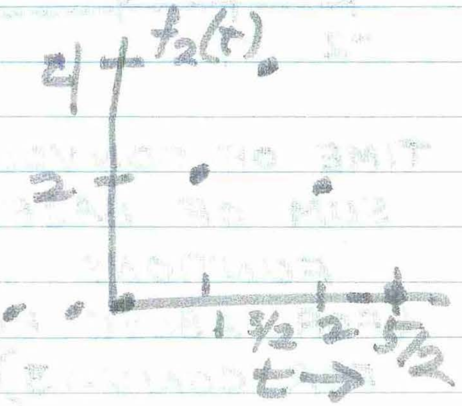
$1 \leq t \leq 5 \quad f_1 * f_2 = \int_{-2}^{t-3} f_1(\lambda) f_2(t-\lambda)\, d\lambda$

$5 \leq t \leq 7 \quad = \int_{t-7}^{t-3} f_1(\lambda) f_2(t-\lambda)\, d\lambda$

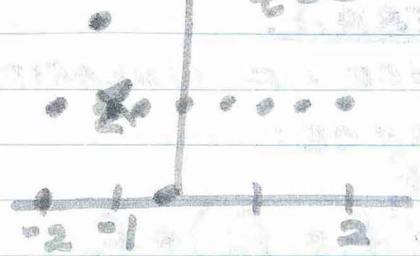
$7 \leq t \leq 11 \quad = \int_{t-7}^{4} f_1(\lambda) f_2(t-\lambda)\, d\lambda$

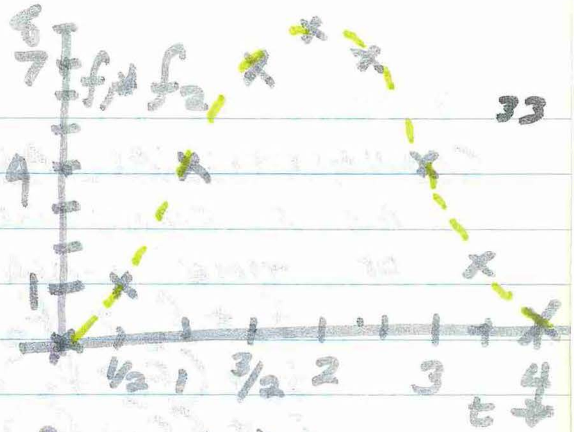
$11 \leq t \quad f_1 * f_2 = 0$

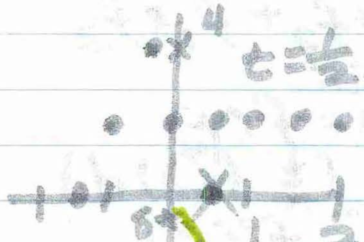




NUMERICAL CONVO.







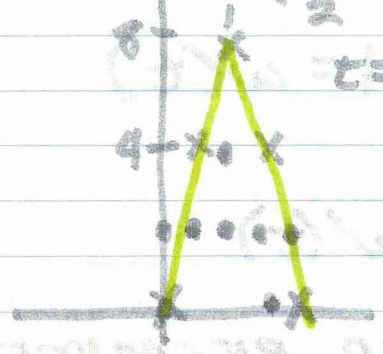
$t=\frac{1}{2} \Rightarrow \int = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(4+0)$

USE:

$\int = \frac{\Delta t}{2}\left(x_0 + 2x_1 + 2x_2 + \ldots + x_n\right)$

$t=1 \quad \frac{1}{2}\frac{1}{2}\left(8+2(4)+0\right) = 4$

$t=3/2 \quad \Rightarrow \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(4+2(8)+2(4)+0\right) = 7$



$t=2 \quad \frac{1}{2}\frac{1}{2}\left(0+2(4)+2(8)+2(4)+0\right) = 8$

GOING FURTHER WILL GIVE SAME POINT i.e.) FUNCTION IS SYMETRIC

10-9-70

CONVOLUTIONAL ALGEBRA:

AS A CONSEQUENCE OF LINEARITY OF TIME INVARIANCE:

$$f_1 * f_2 = f_2 * f_1$$

$$f_1 * (f_2 * f_3) = (f_1 * f_2) * f_3$$

$$f_1 * (f_2 + f_3) = f_1 * f_2 + f_1 * f_3$$

OTHER PROPERTIES

$$f(t) * \delta(t) = f(t)$$

$$f(t) * \mu(t) = \int_{-\infty}^{t} f(\lambda)\, d\lambda$$

$$f(t) * \delta'(t) = f'(t)$$

THE SUPERPOSITION INTEGRAL (DUHAMEL'S INTEGRAL)

$$\omega(t) = \mu(t) * h(t)$$

$$= \int_{-\infty}^{\infty} \mu(\lambda)\, h(t-\lambda)\, d\lambda$$

$$= \int_0^t h(t-\lambda)\, d\lambda$$

$$= \int_0^t h(\lambda)\, d\lambda = \omega(t)$$

$$\cancel{\mu(t)}$$

$$\Downarrow$$

$$h(t) = \omega'(t)$$

$\omega(t)$ = UNIT STEP RESPONSE

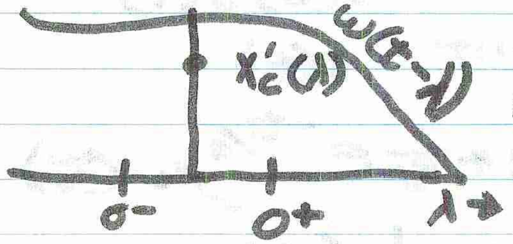
$$x'(t) * w(t) = x(t) * w'(t) = y(t)$$

FOR CAUSAL $x(t) = x_c(t) = 0$ FOR $t<0$

$$y(t) = x_c(t) * w'(t) = x_c'(t) * w(t)$$

$$= \int_{0^-}^{\infty} x_c'(\lambda)\, w(t-\lambda)\, d\lambda$$

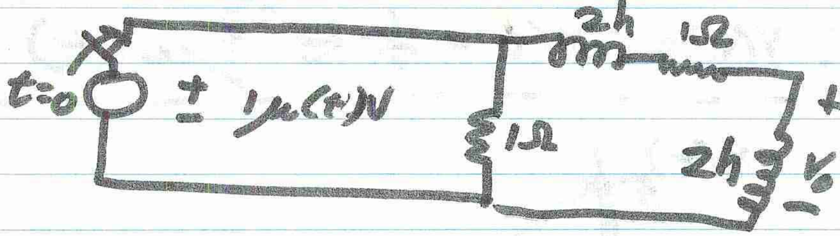
$$= \int_{0^-}^{0^+} x_c'(t)\, w(t-\lambda)\, d\lambda + \int_{0^+}^{\infty} x_c'(\lambda)\, w(t-\lambda)\, d\lambda$$



$$\Rightarrow \int_{0^-}^{0^+} x_c'(t)\, w(t-\lambda)\, d\lambda = x_c'(0)\, w(t)$$

$$\therefore\ y(t) = x_c'(0)\, w(t) + \int_{0^+}^{\infty} x_c'(\lambda)\, w(t-\lambda)\, d\lambda$$

EX) 4-11)



$x(t) = 1\mu(t)$; $y(t) = V_o$

$$V_o(t) = V_o(\infty) + V_{o_n}(t)$$
$$= 0 + V_{o_n}(t)$$
$$V_{o_n}(t) = A e^{-t/\tau} \mu(t)$$

(CONT.)

(CONT)

$\tau = \frac{L_{EQ}}{R_{EQ}} = \frac{4}{1}$

$A = V_0(0) = \frac{di}{dt} = \frac{1}{2}$

$\therefore V_n(t) = \frac{1}{2} e^{-t/4} \mu(t) = \omega(t)$

$h(t) = \omega'(t) = -\frac{1}{8} e^{-t/4} \mu(t) + \frac{1}{2} e^{-t/4} \delta(t)$

$\Rightarrow -\frac{1}{8} e^{-t/4} \mu(t) + \frac{1}{2} \delta(t)$

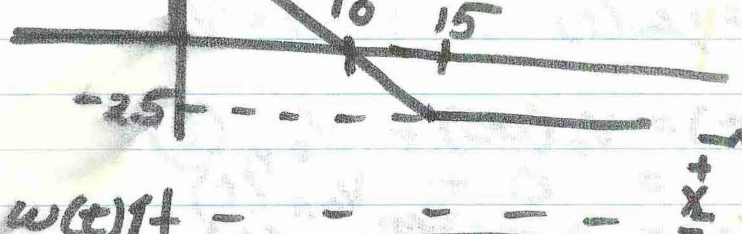
$= f(0)\,\delta(t)$

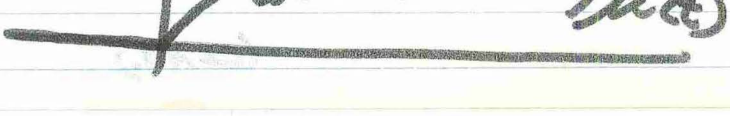
GIVEN ANY INPUT $x(t)$

$Y(t) = x * \omega' = -x(t) * \frac{1}{8} e^{-t/4} \mu(t) + x(t) * \frac{1}{2}\delta(t)$

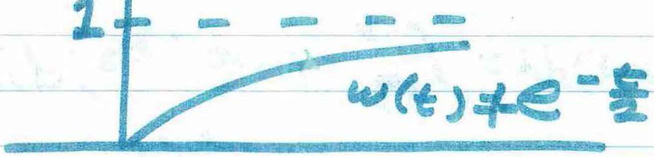
$\Rightarrow \frac{1}{2} x(t)$

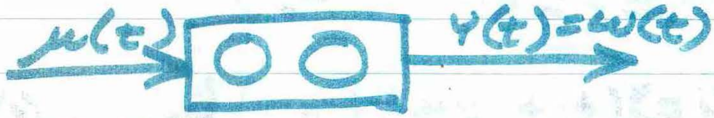
$\therefore Y(t) = -x(t) * \frac{1}{8} e^{-t/4} \mu(t)$

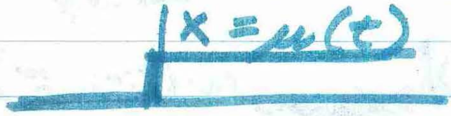




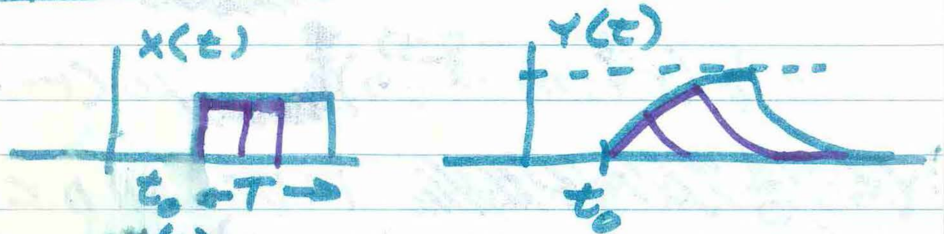
4-13)



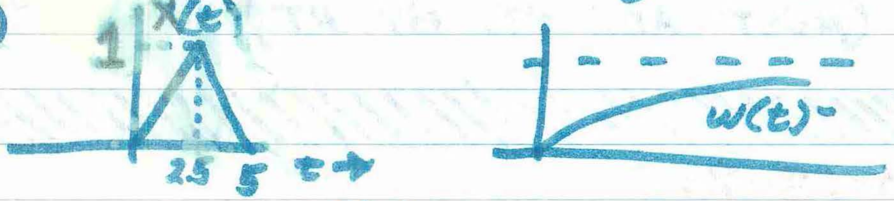




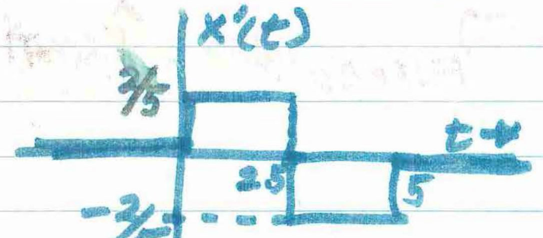
a)



b)

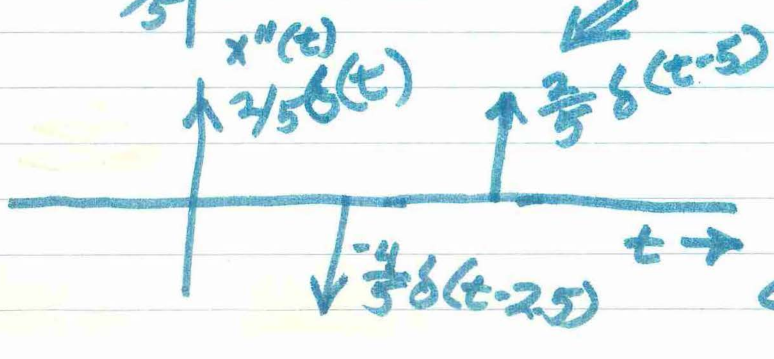


$Y(t) = x'(t)\, w(t)$



TO GET

δ IN



CONT.

(CONT.) 38

$$Y = x'' * \int_{-\infty}^{t} \omega(\lambda)\,d\lambda$$

$$\int_{-\infty}^{t} \omega(\lambda)\,d\lambda = \int_{-\infty}^{t} (1-e^{-t/.2})\,d\lambda$$

$$= t + .2e^{-t/.2} - .2$$

$$Y = x'' * \int \omega(\lambda)\,d\lambda = \tfrac{2}{5}(t + .2e^{-\frac{t}{.2}} - .2)\,\mu(t)$$

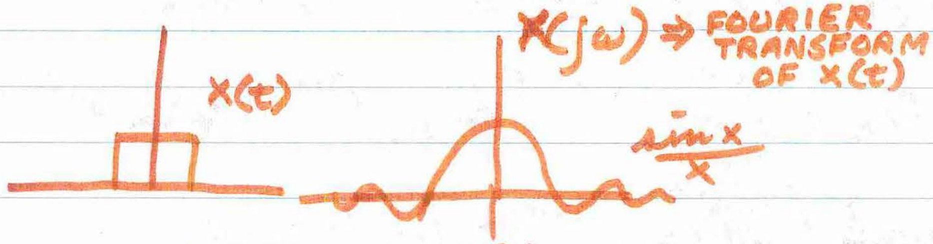
$$- \tfrac{4}{5}(t-2.5 + .2e^{-\frac{t-2.5}{.2}} - .2)\,\mu(t-2.5)$$

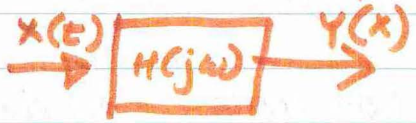
$$+ \tfrac{2}{5}(t-5 + .2e^{-\frac{(t-5)}{.2}} - .2)\,\mu(t-5)$$

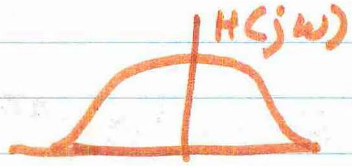
PLUG AND CHUG
(MUST ADD PT. BY PT)

10-13-70

FOURIER TRANSFORMS AND SUCH







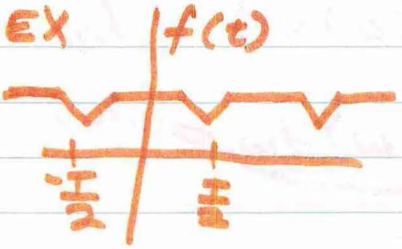
$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

AND $C_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\omega_0 t} dt$

AND $\omega_0 = 2\pi / T$

$$\Rightarrow f(t) = \sum_{n=-\infty}^{\infty} e^{jn\omega_0 t} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\omega_0 t} dt$$

EX



AS $T \to \infty$; $\omega_0 = \frac{2\pi}{T} \to d\omega$

$n\omega_0 \to \omega$ (ANOTHER CONT. VARIABLE)

(CONT.)

$\frac{1}{T} = \frac{\omega_0}{2\pi} \to d\omega/2\pi$

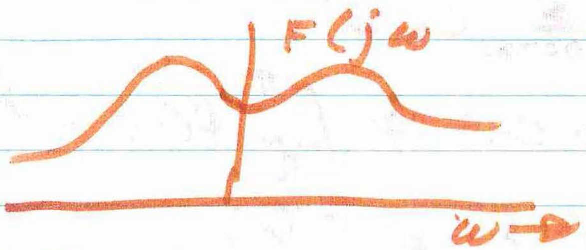
$f(t) = \int_{-\infty}^{\infty} e^{j\omega t}\left[\frac{1}{2\pi} d\omega \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt\right]$

$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f(x) e^{-j\omega t} dt\right] e^{j\omega t} d\omega$

$= \mathfrak{F}\{f(t)\}$ = FOURIER TRANSFORM OF $f(t)$

$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega) e^{jnt} d\omega$

(INVERSE FORIER TRANSFORM

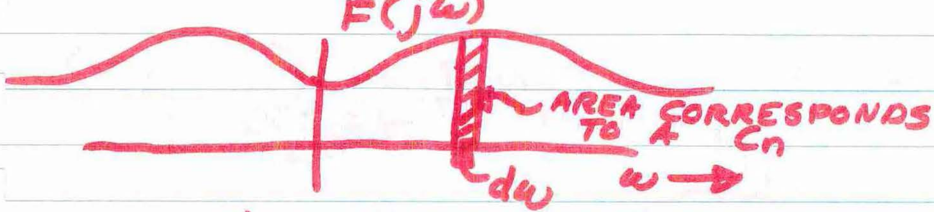


10-14-70

$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$

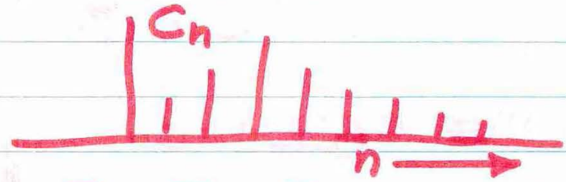
$= \int_{-\infty}^{\infty} \underbrace{\frac{1}{2\pi} F(j\omega) d\omega}_{C_n} e^{j\omega t}$

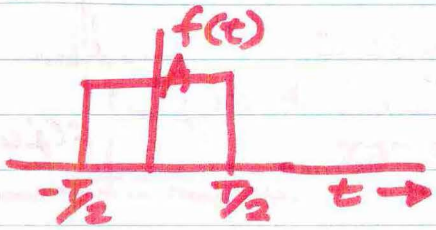


$$\frac{1}{2\pi} F(j\omega)\, d\omega \Rightarrow C_n$$

$$F(j\omega) \Rightarrow \frac{2\pi C_n}{d\omega}$$

AS OPPOSED TO





$$f(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$f(j\omega) = \int_{-\frac{T}{2}}^{\frac{T}{2}} A e^{-j\omega t} dt = \frac{-A}{j\omega} e^{-j\omega t} \Big|_{-\frac{T}{2}}^{\frac{T}{2}}$$

$$= \frac{A}{j\omega}\left[-e^{-\frac{j\omega T}{2}} + e^{\frac{j\omega T}{2}}\right]$$

$$\left(\frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin\theta\right)$$

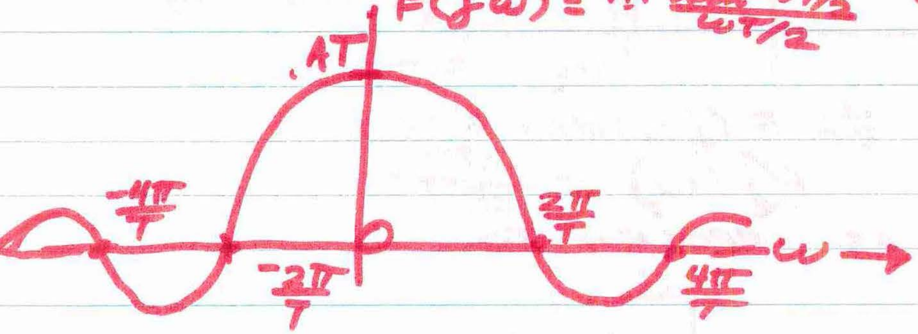
$$f(j\omega) = \frac{2A}{\omega} \sin\frac{\omega T}{2} \qquad \left(\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta\right)$$

$$= \frac{TA \sin\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega T}{2}\right)}$$

(CONT.)

A PURE REAL FUNCTION (NO j's)

$F(j\omega) = AT \frac{\sin \omega T/2}{\omega T/2}$

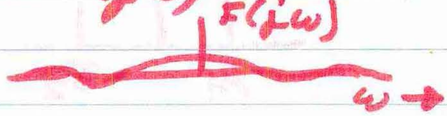


$\frac{\omega T}{2} = \pi \Rightarrow F(j\omega) = 0$

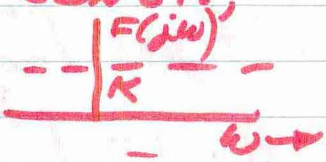
AS T INCREASES

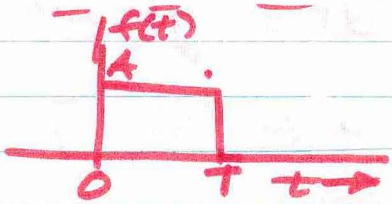
AS $T \rightarrow \infty$; $F(j\omega) \rightarrow \delta(j\omega)$

AT T DECREASES



AS $T \rightarrow 0$, AT KEPT A CONST.,
TRANSFORM OF $\delta(t) \Rightarrow$

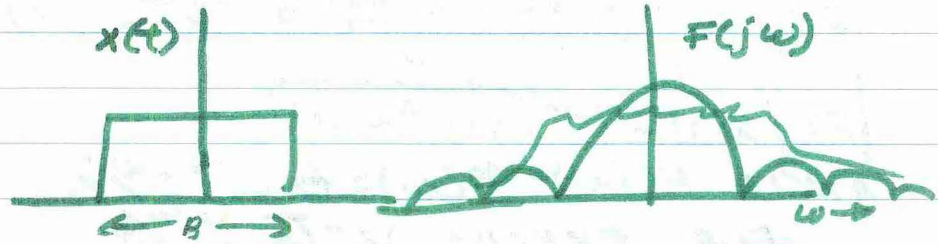




$\Rightarrow F(j\omega) = e^{-j\frac{\omega T}{2}} \cdot AT \frac{\sin \frac{\omega T}{2}}{\frac{\omega T}{2}}$

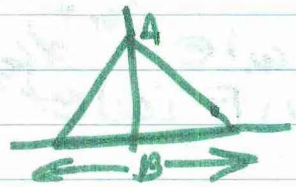
$e^{-j\frac{\omega T}{2}}$ JUST ADDS PHASE ANGLE
INFORMATION ($\angle \frac{\omega T}{2}$)

10-15-70



TIME WIDTH x ~~FREQ WIDTH~~ = CONST

$BW = \int f(t)dt \big/ f(0) = \text{AREA} \big/ \text{HT. AT ORIGIN}$



$BW = \frac{\frac{1}{2}BA}{A} = \frac{1}{2}B$

$\therefore \frac{\int_{-\infty}^{\infty} f(t)dt}{f(0)} \cdot \frac{\int_{-\infty}^{\infty} |F(j\omega)|}{F(0)} = ? = 2\pi$

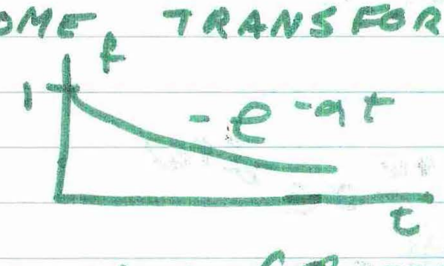
$f(t) = \frac{1}{2\pi}\int F(j\omega)\, e^{j\omega t} d\omega$

$f(0) = \frac{1}{2\pi}\int F(j\pi)\, d\omega$

$= \int F(j\omega)\, d\omega = 2\pi f(0)$

$\Rightarrow ? = 2\pi$

SOME TRANSFORMS



$F(j\omega) = \int_{-\infty}^{\infty} e^{-\alpha t} \mu(t) e^{-j\omega t} d\tau$

$= \int_0^{\infty} e^{-(\alpha + j\omega)\tau} d\tau$

$= \frac{1}{\alpha + j\omega}$ (CONT.)

$$F(j\omega) = \frac{\alpha - j\omega}{\alpha^2+\omega^2} = \frac{\alpha}{\alpha^2+\omega^2} - \frac{j\omega}{\alpha^2+\omega^2}$$

$$|F(j\omega)| = \frac{1}{\sqrt{\alpha^2+\omega^2}}$$

ANGLE $F(j\omega) = \theta(\omega) = \tan^{-1} -\omega/\alpha$

FOR EXPOD: $\angle = -\tan^{-1} \omega/\alpha$

$$F(j\omega) = \frac{1}{\alpha + j\omega}$$

2) $\mathfrak{F}\left\{\frac{d f(t)}{dt}\right\} = ?$

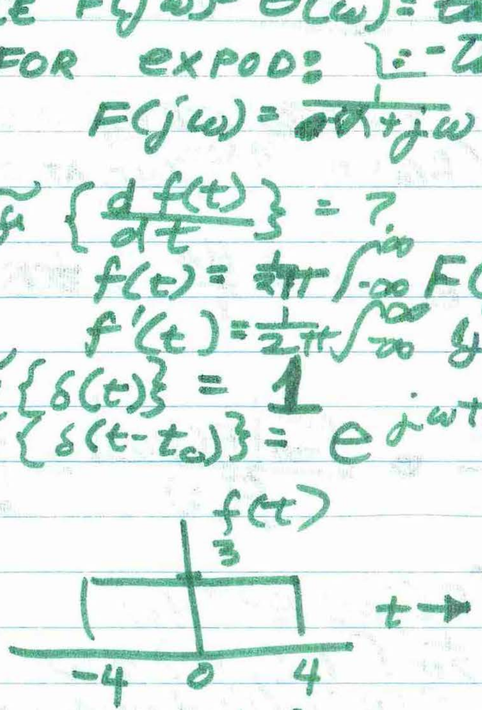
$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\, e^{j\omega t}\, d\omega$$

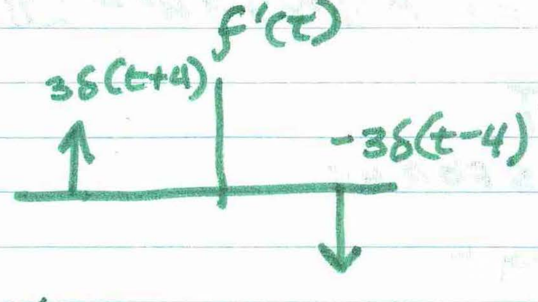
$$f'(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} (j\omega)\, F(j\omega)\, e^{j\omega t}\, d\omega$$

3) $\mathfrak{F}\{\delta(t)\} = 1$

4) $\mathfrak{F}\{\delta(t-t_0)\} = e^{j\omega t_0}$

EX)





$$\mathfrak{F}\{f'(t)\} = 3e^{j4\omega} - 3e^{-j4\omega}$$

$$\mathfrak{F}\{f(t)\} = \frac{\mathfrak{F}\{f'(t)\}}{j\omega} = F(j\omega)$$

$$= \frac{6}{\omega}\sin 4\omega = 24\,\frac{\sin 4\omega}{4\omega}$$

10-16-70

$$f(t) = f_e(t) + f_o(t)$$
(EVEN + ODD)

$$f_e = \frac{f(t) + f(-t)}{2}$$

$$f_o = \frac{f(t) - f(-t)}{2}$$

EVEN FUNC $\Rightarrow f(t) = f(-t)$
ODD FUNC $\Rightarrow f(t) = -f(-t)$

EX) $e^{jt} = \cos t + j \sin t$

$$f_e = \frac{e^{jt} + e^{-jt}}{2}$$

$$f_o = e^{jt} - e^{-jt}/2j = \sin t$$

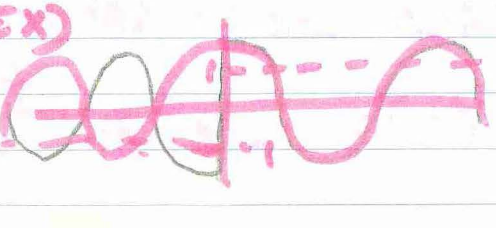
$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}$$

$$= \int_{-\infty}^{\infty} [f_e(t) + f_o(t)] \cdot [\cos \omega t - j \sin \omega t] dt$$

$$= \int_{-\infty}^{\infty} [f_e \cos \omega t - j f_o(t) \sin \omega t] dt$$

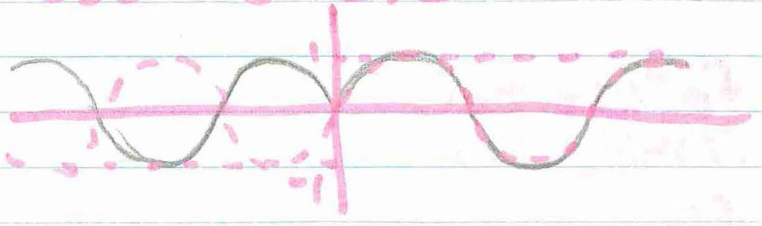
EVEN · ODD = ODD

EX)



$\int_{-T}^{T} \text{ODD} = 0$

ODD · ODD = EVEN



RETURNING TO PROB:

$$F(j\omega)=\int_{-\infty}^{\infty}\left[f_e(t)\cos\omega t - j f_o(t)\sin\omega t\right]dt$$

$$F(j\omega) = F_{REAL}(j\omega) + F_{IMAG}(j\omega)$$

$$F_{REAL} = \int_{-\infty}^{\infty} f_e(t)\cos\omega t\,dt$$

F_{REAL} IS EVEN

$$F_{IMAG} = -\int_{-\infty}^{\infty} f_o(t)\sin\omega t\,dt$$

F_{IMAG} IS ODD

1) $|F(j\omega)|$ IS EVEN

$\left(|F(j\omega)| = \sqrt{F_{REAL}^2 + F_{IMAG}^2}\right)$

2) ANGLE $F(j\omega)$ IS ODD

$\left(\text{ANGLE } F(j\omega) = \text{atan}\,\frac{F_{IMAG}}{F_{REAL}}\right)$

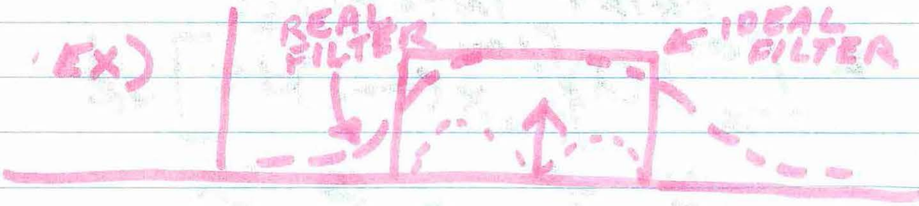
TALKING OF CAUSAL SYSTEM & SIGNALS (ie) $x(t)=0$ FOR $t<0$

$$f_e(t) = f_o(t) = \frac{f(t)}{2}$$

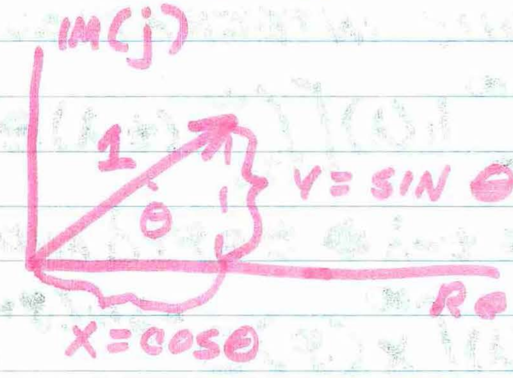
FOR PHYSICALLY REALIZABLE SYSTEMS (CAUSAL) WE FIND

$|F(j\omega)| = A(\omega)$ MUST NOT GO TO ZERO FOR ANY FINITE BAND WIDTH

EX)



$e^{j\theta} = 1 \angle \theta$



10-20-70

$$Y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$

$$\mathfrak{F}[Y(t)] = Y(j\omega) = \int_{-\infty}^{\infty} Y(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda\right] e^{-j\omega t}\, dt$$

SHIFT INTEGRALS

$$= \int_{-\infty}^{\infty} h(\lambda) \left[\int_{-\infty}^{\infty} x(t-\lambda)\, e^{-j\omega t}\, dt\right] d\lambda$$

$$\sum_{\lambda=-\infty}^{\infty} \sum_{t=-\infty}^{\infty} \Rightarrow \sum_{t=-\infty}^{\infty} \sum_{\lambda=-\infty}^{\infty}$$

$\Rightarrow$ FOR THIS INTEGRATION

$$\mathfrak{F}[Y(t)] = \int_{-\infty}^{\infty} h(\lambda) \left[\int_{-\infty}^{\infty} x(t-\lambda)\, e^{-j\omega t}\, dt\right] d\lambda$$

DEFINE $\rho = t - \lambda \Rightarrow t = \rho + \lambda$ AND $d\rho = dt$

$$\therefore Y(j\omega) = \int_{-\infty}^{\infty} h(\lambda) \int_{-\infty}^{\infty} x(\rho)\, e^{-j\omega(\rho+\lambda)}\, d\rho]\, d\lambda$$

$$= \int_{-\infty}^{\infty} h(\lambda)\, e^{-j\omega\lambda} \underbrace{\left[\int_{-\infty}^{\infty} x(\rho)\, e^{-j\omega\rho}\, d\rho\right]}_{X(j\omega)\ \text{a}\ f(\omega)} d\lambda$$

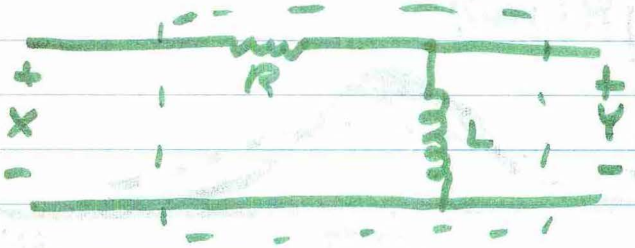
$$= X(j\omega) \int_{-\infty}^{\infty} h(\lambda)\, e^{-j\omega\lambda}\, d\lambda$$

(CONT.)

$$= H(j\omega)\, X(j\omega)$$

$\therefore\ Y(t) = x(t) * h(t) \Rightarrow Y(j\omega) = X(j\omega)\, H(j\omega)$

EX) 1



$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} \Rightarrow \text{TRANSFER FUNCTION}$$

USING IMPEDENCES:

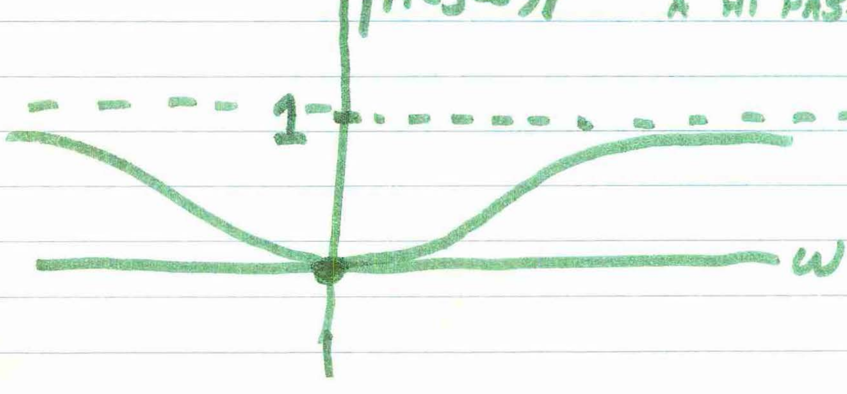
$R \rightarrow R \;;\; L \rightarrow j\omega L$

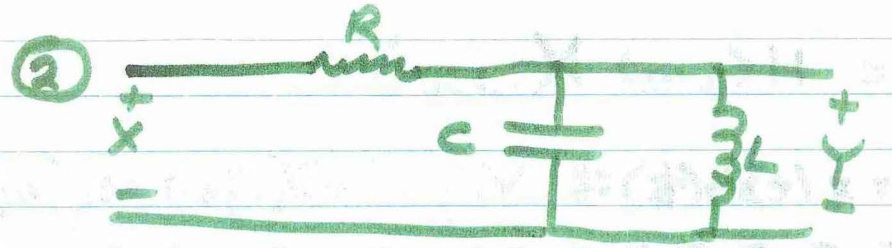
$$Y(j\omega) = \frac{x(j\omega)\, j\omega L}{[R + j\omega L]}$$

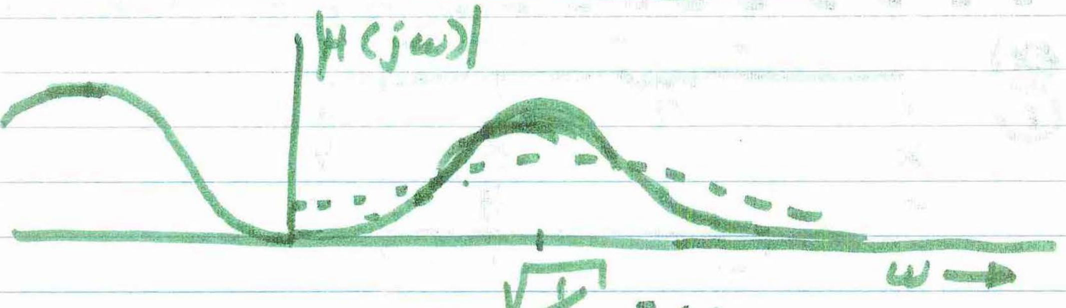
$$\Rightarrow H(j\omega) = \frac{j\omega L}{R + j\omega L} \Leftarrow \text{FOURIER TRANSFORM OF } h(t)$$

$$|H(j\omega)| = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}$$

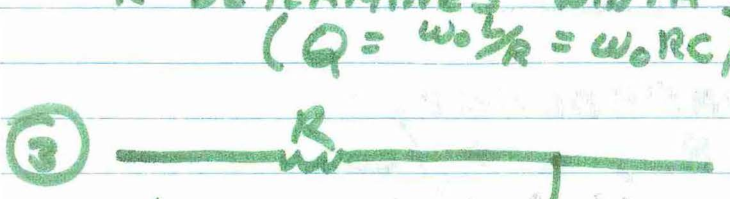
A HI PASS FILTER

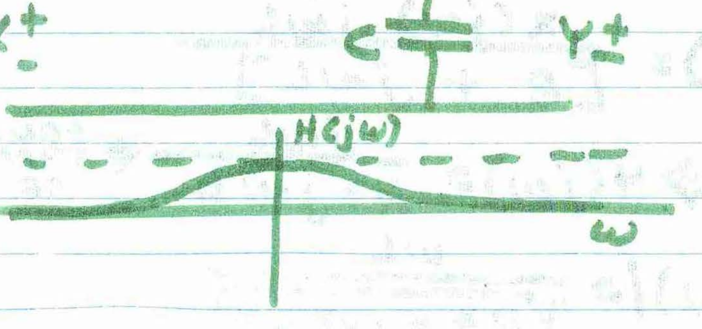






R DETERMINES WIDTH OF CURVE
$(Q = \omega_0 L/R = \omega_0 RC)$





10-29-30

! BODE PLOTS !

ENERGY SPECTRUM: $E = \int_{-\infty}^{\infty} p(t)dt$

$$= \int_{-\infty}^{\infty} f^2(t)dt$$

$$E = \int_{-\infty}^{\infty} f(t)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{j\omega t}d\omega\right]dt$$

CHANGING ORDER OF LIMITS

$$E = \int_{-\infty}^{\infty} \frac{1}{2\pi} F(j\omega)\Big[\underbrace{\int_{-\infty}^{\infty} f(t)e^{j\omega t}dt}\Big]d\omega$$

COMPLEX CONJUGATE OF $F(j\omega) = F^*(j\omega)$

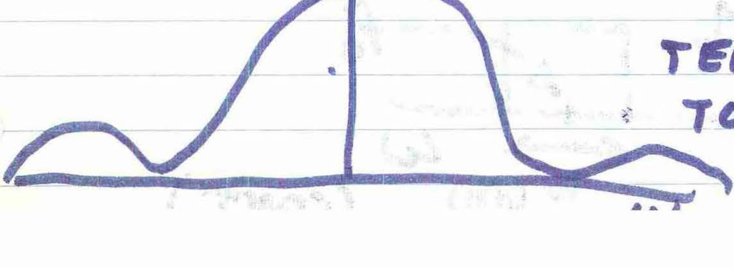
AND $f(t)$ MUST BE REAL

$$\therefore E = \int_{-\infty}^{\infty} \frac{1}{2\pi} F(j\omega)\, F^*(j\omega)d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$$

$$\Rightarrow E = \int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$$

PARSEVAL'S THEOREM



TELLS WHERE TO PUT THE FILTER

$|F(j\omega)|^2 \equiv$ ENERGY SPECTRAL DENSITY

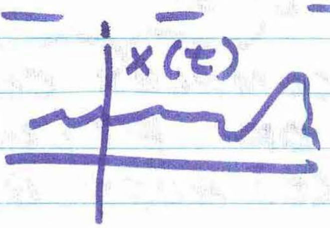
$$Y(t) = x(t) * h(t)$$

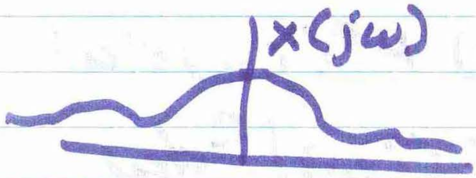
$$Y(j\omega) = X(j\omega)\, H(j\omega)$$

$$Y(j\omega)\, Y^*(j\omega) = [X(j\omega)\, H(j\omega)][X^*(j\omega)\, H^*(j\omega)]$$

$$\therefore [Y(j\omega)]^2 = [X(j\omega)]^2 \cdot [H(j\omega)]^2$$

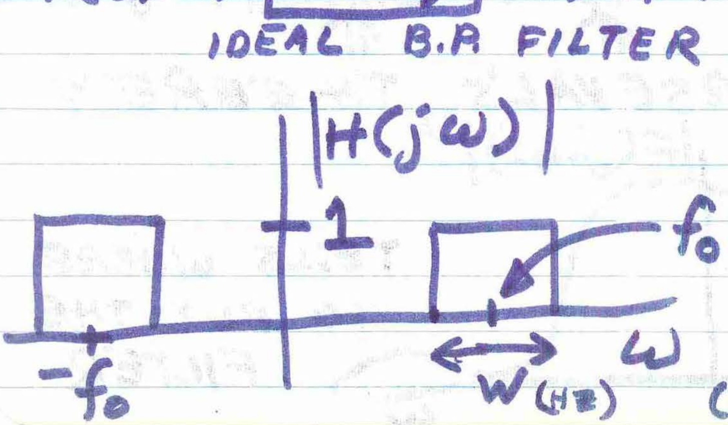
ENERGY TRANSFER FUNC.





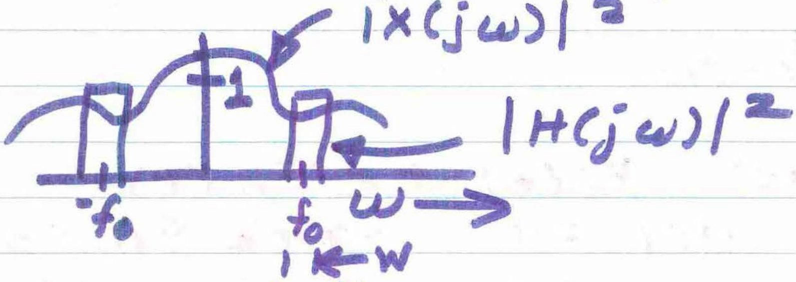
$x(t) \rightarrow \boxed{h(t)} \rightarrow Y(t)$

IDEAL B.P. FILTER



(CONT.)

$$E_{OUT} \propto \int |H(j\omega)|^2 \, |X(j\omega)|^2 \, d\omega$$



$$E_{OUT} = 2\int_{f_0-\frac{W}{2}}^{f_0+\frac{W}{2}} (1)\, |X(j\omega)|^2 \, d\omega$$

$$\cong 2 \, |X(j2\pi f_0)|^2 \cdot W$$

FOR SHORT B.W.

$$\Rightarrow |X(j2\pi f_0)|^2 = \frac{E_{OUT}}{2W}$$

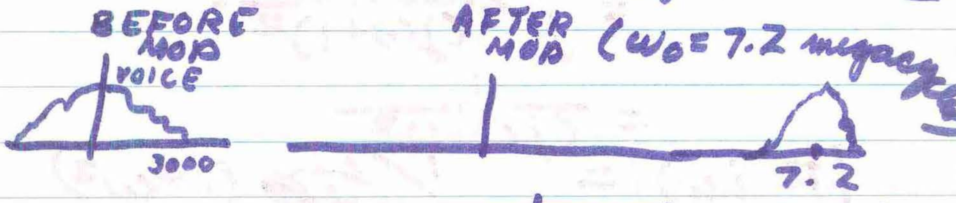
MATHEMATICAL THINGIES

FOR ~~[illegible]~~

TIME SCALE: $f(t) \Leftrightarrow F(j\omega)$; $f(at) \Leftrightarrow \frac{1}{|a|} F(j\omega$

DELAY: $f(t-t_0) \Leftrightarrow e^{-j\omega t_0} F(j\omega)$

MODULATION: $e^{-j\omega_0 t} f(t) \Leftrightarrow F[j(\omega-\omega_0)]$



$$\cos(\omega_0 t) f(t) \Leftrightarrow \frac{1}{2} F(j\omega - \omega_0) + \frac{1}{2} F(j\omega + \omega_0)$$

$$\sin(\omega_0 t) f(t) \Leftrightarrow \frac{j}{2} F(\omega-\omega_0) + \frac{-j}{2} F(\omega+\omega_0)$$

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$\mathcal{F}\{x\} = Y \Leftrightarrow \mathcal{F}\{Y\} = X$

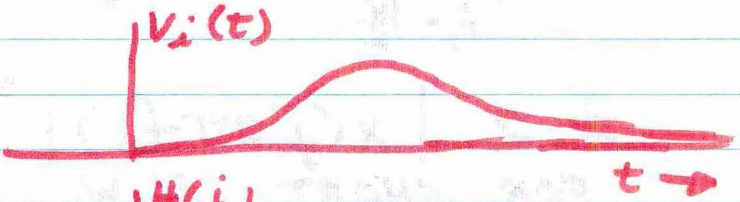
10-30-70 (29)

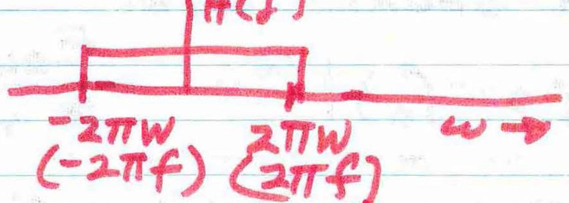
5-14) $v_i(t) = t e^{-t} \mu(t)$

$H(j\ ) = 1 \qquad -2\pi W \leq \omega \leq 2\pi W$

$= 0 \qquad$ ELSEWHERE

$(R = 1)$





$\mathcal{F}\{v_i(t)\} = j \dfrac{d}{d\omega} \dfrac{1}{(j\omega + 1)}$

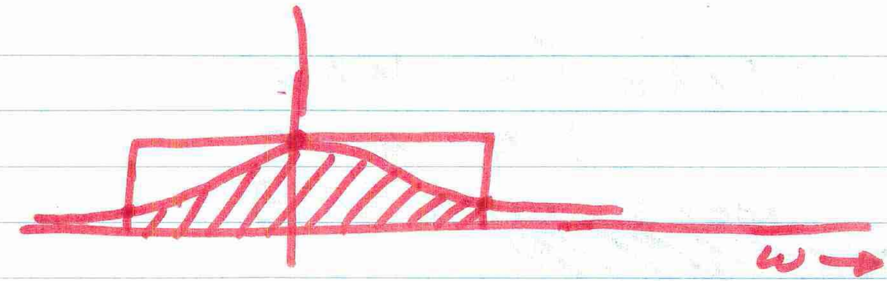
$= F(j\omega) = \int_0^\infty t e^{-t} e^{-j\omega t} dt$

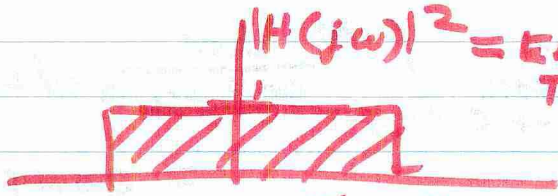
$= -j^2 \dfrac{1}{(j\omega+1)^2}$

$= \dfrac{1}{(j\omega+1)^2}$

$|F(j\omega)| = 1/(1+\omega^2)$

$|F(j\omega)| = \dfrac{1}{(1+\omega^2)2}$





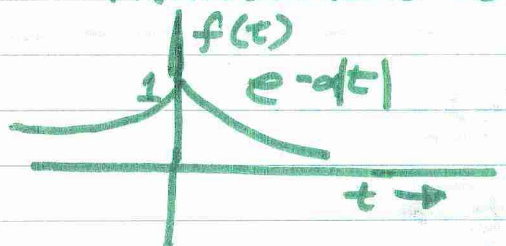
$|H(j\omega)|^2$ = ENERGY TRANSFER FUNCTION

$$\int_0^\infty \left(\frac{1}{1+\omega^2}\right)^2 d\omega = \int_{-2\pi f}^{2\pi f} \left(\frac{1}{1+\omega^2}\right)^2 d\omega$$

"TOTAL ENERGY"

10-30-70

TRANSFORMS OF POWER SIGNALS



AREA IS FINITE

AS $t \to 0$, $f(t) = 1$

WANT NOW TO TAKE LIMIT OF TRANSFORM

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} d\tau$$

$$= \int_{-\infty}^{\infty} e^{\alpha t} e^{-j\omega t} dt + \int_0^\infty e^{-\alpha t} e^{-j\omega t} dt$$

$$= \left.\frac{e^{(\alpha - j\omega)t}}{\alpha - j\omega}\right|_{-\infty}^{0} + \left.\frac{e^{(-\alpha - j\omega)t}}{-\alpha - j\omega}\right|_0^\infty$$

$$= \frac{1}{\alpha - j\omega} + \frac{1}{\alpha + j\omega} = \frac{2\alpha}{\alpha^2 + \omega}$$

$$Fr(1) = \lim_{\alpha \to 0} \frac{2\alpha}{\alpha^2 + \omega} = \begin{cases} 0 & \text{IF } \omega \neq 0 \\ \infty & \text{IF } \omega = 0 \end{cases}$$

(CONT)

$$\therefore \mathcal{F}\{1\} = \begin{cases} 0 & \omega \neq 0 \\ \infty & \omega = 0 \end{cases}$$

LOOKS LIKE $\delta(t)$

~~DOES $\int_{-\infty}^{\infty}$?~~

DOES $\lim_{\alpha \to 0} \int_{-\infty}^{\infty} \frac{2\alpha}{\alpha^2 + \omega^2} d\omega$

$= \text{CONST}$?

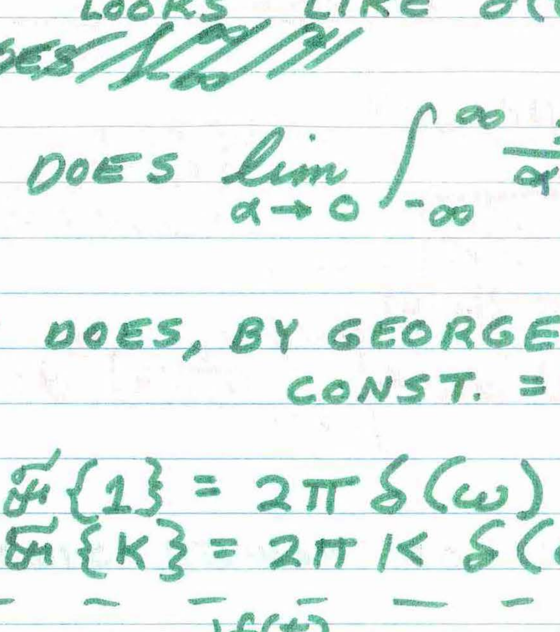
IT DOES, BY GEORGE $\Rightarrow$

CONST. $= 2\pi$

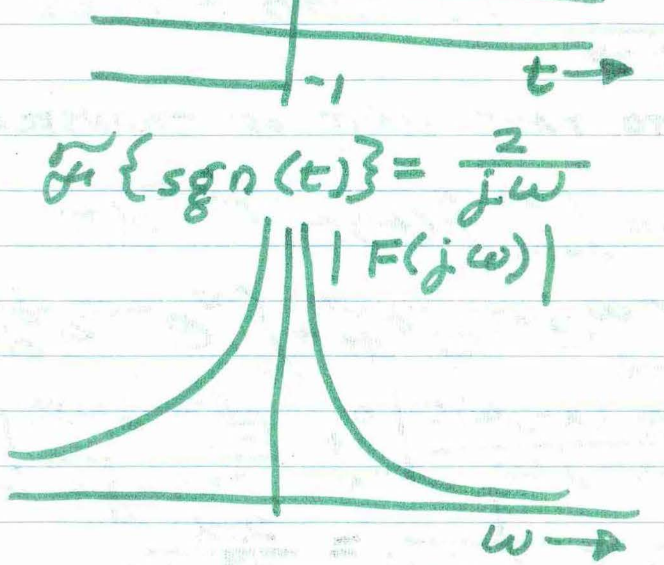
$$\therefore \mathcal{F}\{1\} = 2\pi\,\delta(\omega)$$

$$\mathcal{F}\{K\} = 2\pi K\,\delta(\omega)$$

- - - - - - - - - - - - - -

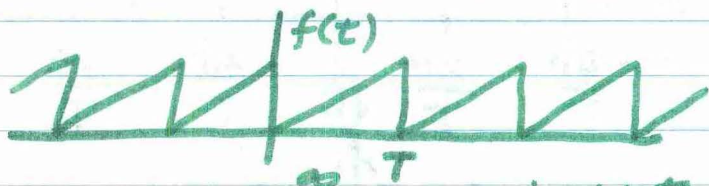


$$\mathcal{F}\{\text{sgn}(t)\} = \frac{2}{j\omega}$$



$$\mathcal{F}\{\mu(t)\} = \mathcal{F}\{\tfrac{1}{2}(sgn(t)+1)\}$$
$$= \frac{1}{j\omega} + \pi\delta(\omega)$$

$$\mathcal{F}\{e^{j\omega_0 t}\} = \mathcal{F}\{e^{j\omega_0 t}(1)\}$$
$$= \cancel{\mathcal{F}\{} 2\pi\,\delta(\omega-\omega_0)$$



$$f(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t}$$

$$\mathcal{F}\{f(t)\} = \mathcal{F}\{\sum_{n=\infty}^{\infty} a_n e^{jn\omega_0 t}\}$$
$$= \sum_{n=-\infty}^{\infty} a_n \mathcal{F}\{e^{jn\omega_0 t}\}$$
$$= \sum_{n=\infty}^{\infty} a_n \; 2\pi\delta(\omega-n\omega_0)$$

$$a_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\,e^{-jn\omega_0 t}\,dt$$
$$= \frac{1}{T}\int_{-\infty}^{\infty} f_T(t)\,e^{-jn\omega_0 t}\,dt$$

$$= \frac{1}{T} F_T(jn\omega_0)$$

(CONT)

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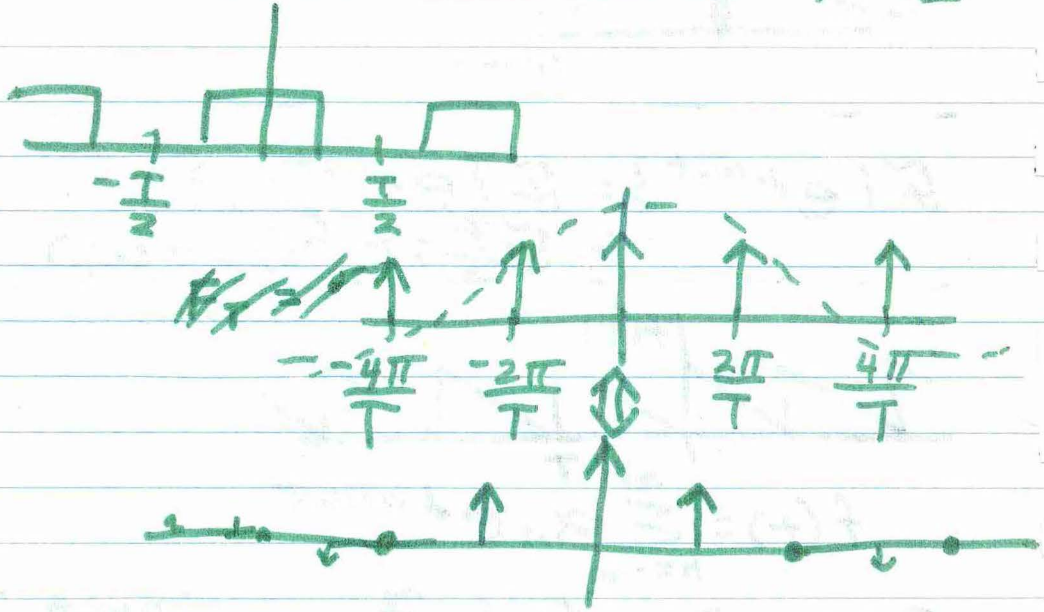
$\omega_0 = 2\pi T$

$\therefore \mathcal{F}[f(t)] = \sum_{n=-\infty}^{\infty} \left[\frac{2\pi}{T} F_T\left(i\frac{2\pi n}{T}\right) \cdot \delta\left(\omega - \frac{2\pi n}{T}\right) \right]$



11-3-70

$$\mathcal{F}\{f_1 * f_2\} = F_1(j\omega)\,F(j\omega)$$

$$\mathcal{F}\{f_1 \cdot f_2\} = F_1(j\omega) \circledast F(j\omega)$$

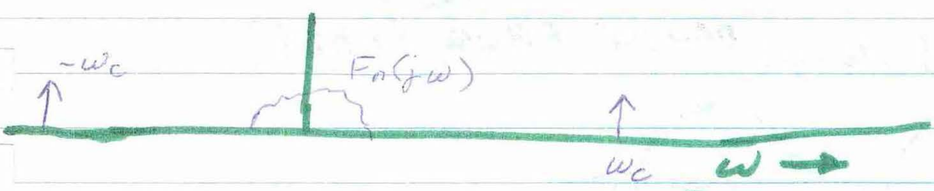
CONVOLUTION OF COMPLEX NUMBERS

LATTER CONCERNED WITH MODULATION (MULTIPLYING 2 FUNCTIONS TOGETHER

a.m. (AMPLITUTE MODULATION)

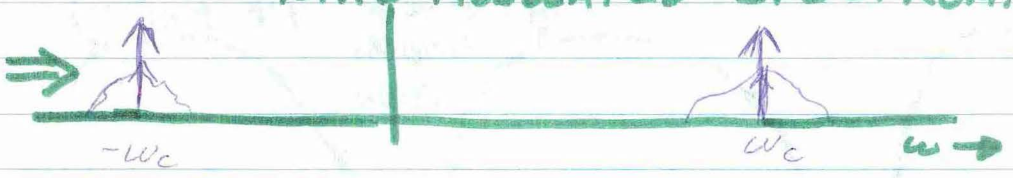
HIT RECORD / CARRIER

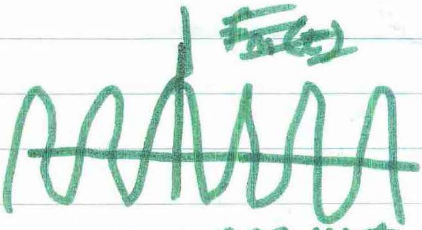
$$f(t) = \underbrace{\left(1 + f_m(t)\right)}_{f_1(t)} \underbrace{\cos(\omega_c t)}_{f_2(t)}$$

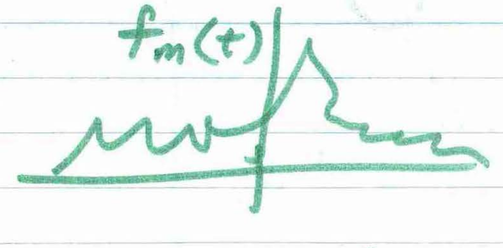


$$f(t) = \cos\omega_c t + f_n(t)\cos\omega_c t$$

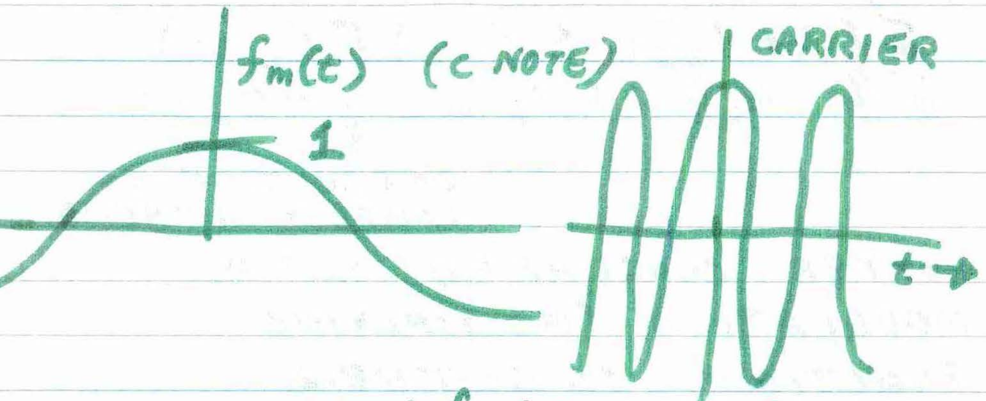
TOTAL MODULATED SPECTRUM.

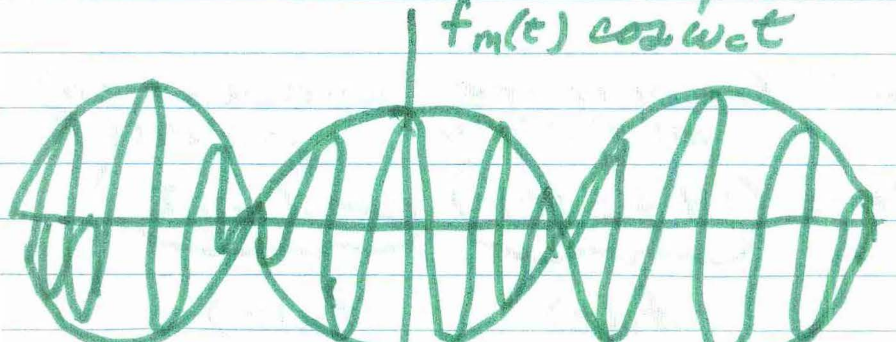




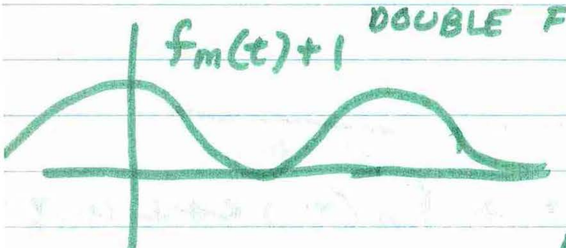


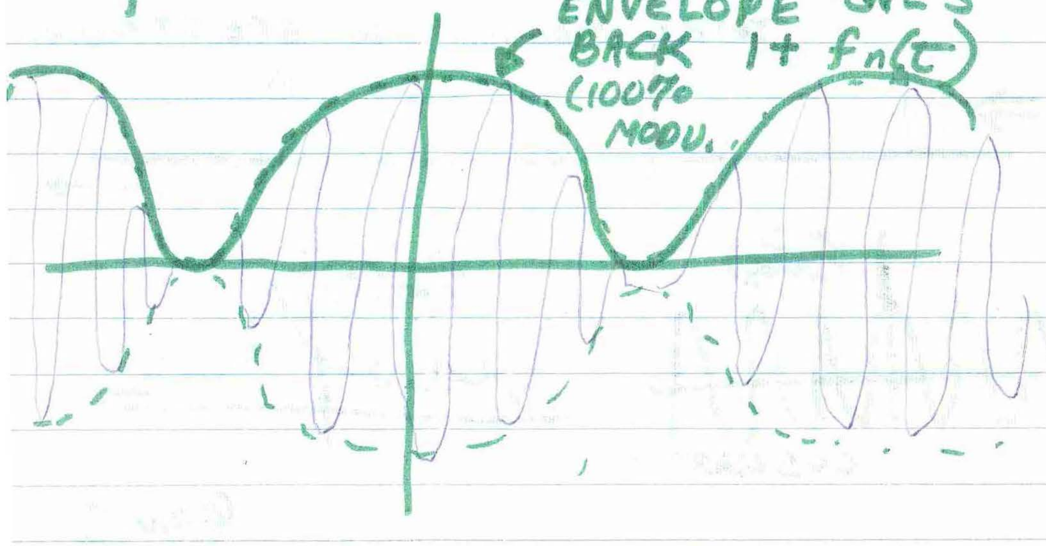
(CONT)



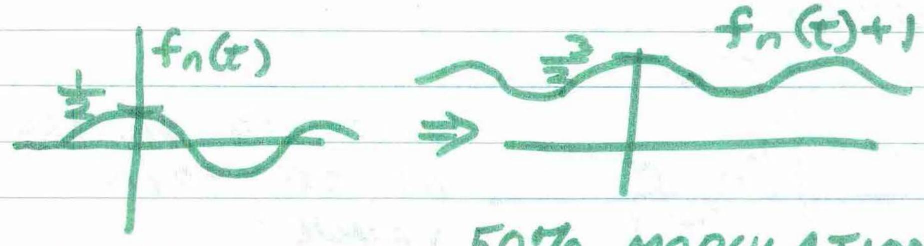


DOUBLE FREQ (BAD!)

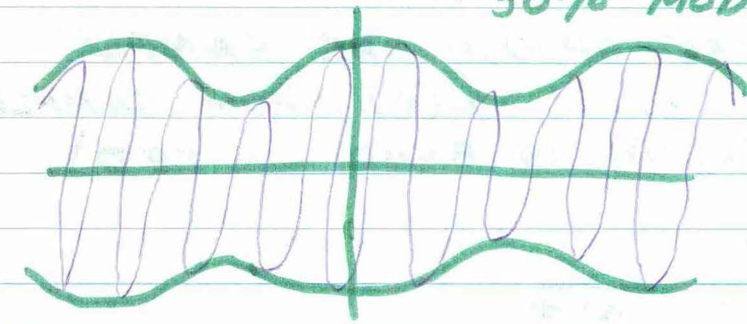


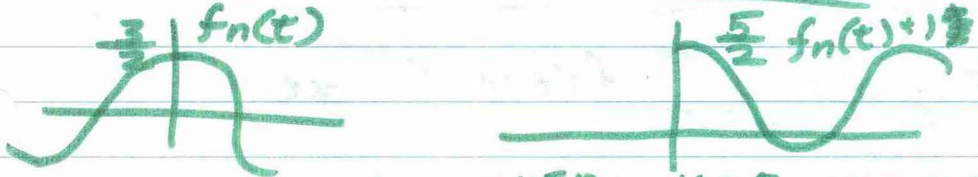


Ex)

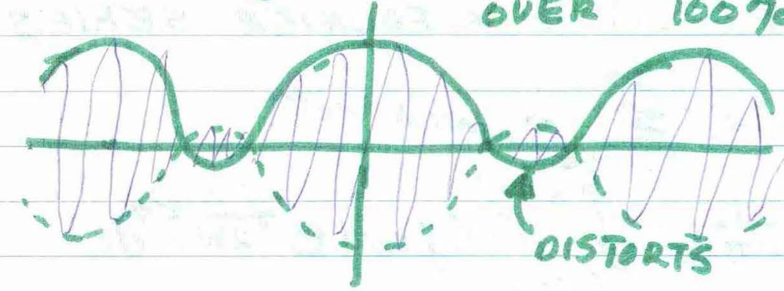


50% MODULATION

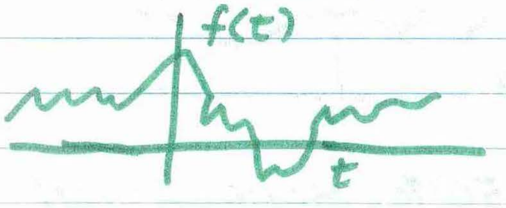




OVER 100% MODULATION

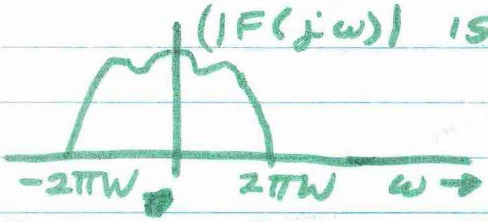


SAMPLING



WANT TO SAMPLE AT CERTAIN VALUES.

HOW CLOSE SHOULD ONE SAMPLE TO GET BACK $f(t)$ FROM SAMPLES?



($|F(j\omega)|$ IS BAND LIMITED)

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt$$

EXPAND $F(j\omega)$ IN A FOURIER SERIES

$$F(j\omega) = \sum_{a_n=-\infty}^{\infty} a_n e^{j\omega_0 t}$$

$$a_n = \frac{1}{4\pi W} \int_{-2\pi W}^{2\pi W} F(j\omega) e^{-\frac{jn\omega}{2W}}\, d\omega$$

COMPARE

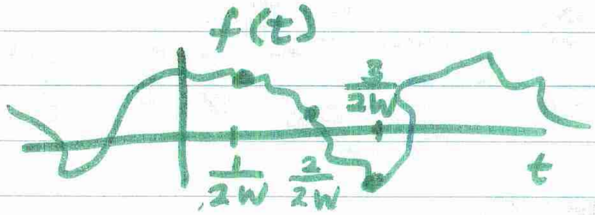
$$\omega_0 = \frac{n}{W}$$

BUT $f(t) = \frac{1}{2\pi} \int_{-2\pi W}^{2\pi W} F(j\omega) e^{j\omega t}\, d\omega$

LET $t = \frac{n}{2W}$

$$f(t) = f\left(\frac{n}{2W}\right) = \frac{1}{2\pi} \int_{-2\pi W}^{2\pi W} F(j\omega) e^{\frac{+j\omega n}{2W}}\, d\omega$$

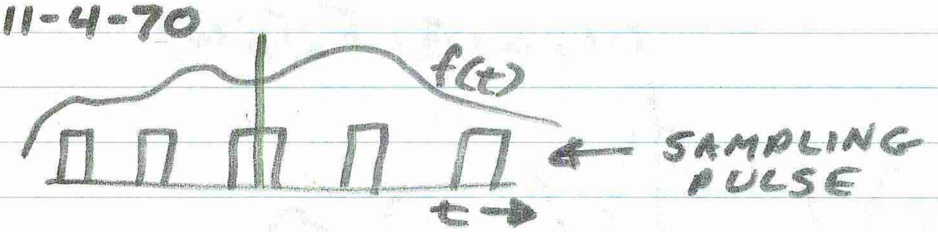
$$\Rightarrow a_n = \frac{1}{2W} f\left(\frac{-n}{2W}\right)$$

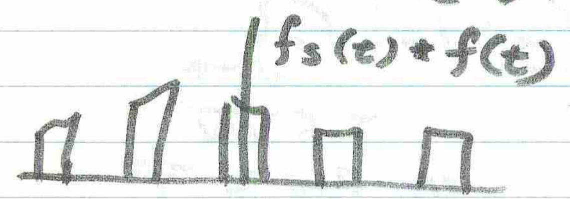


THUS: KNOWING $f(t)$ AT DISCRETE SAMPLING POINTS $\left(t = \frac{n}{2W}\right)$ DETERMANS a_n EXACTLY. KNOWING a_n EXACTLLY, WE CAN FIND $F(j\omega)$ EXACTLY KNOWING $F(j\omega)$ EXACTLY, WE CAN FIND $f(t)$ EXACTLY FOR <u>ALL</u> TIMES t.

SAMPLING RATE MUST BE $> 2W$ $\frac{\text{SAMPLES}}{\text{SEC}}$
" INTERVAL " " $< \frac{1}{2W}$ SECS

11-4-70

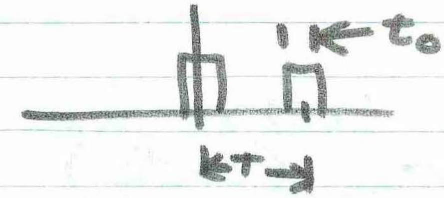


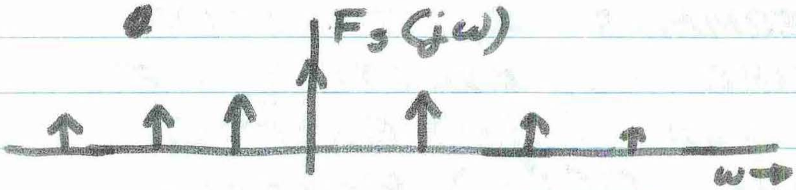


$$\mathcal{F}\{ f_s(t) f(t) \}$$

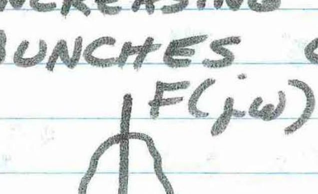
$$= F_s(j\omega) \circledast F(j\omega)$$

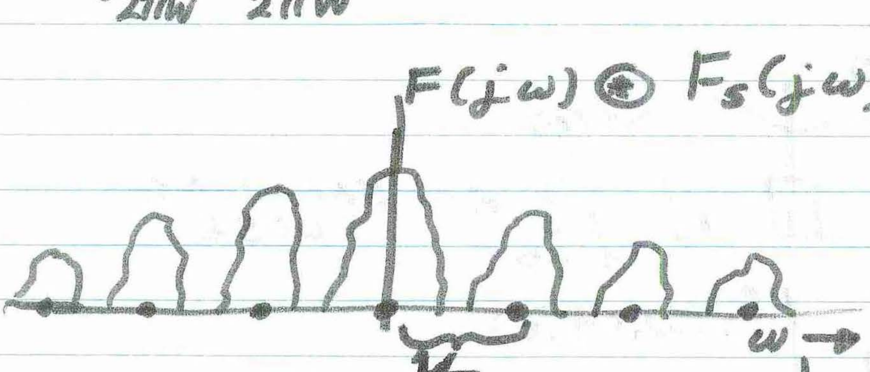
$$F_s(j\omega) = \frac{t_0}{T}\sum_{n=0}^{\infty} \frac{\sin\frac{n\pi t_0}{T}}{\frac{n\pi t_0}{T}}\,\delta\left(t - \frac{n}{t}\right)$$





INCREASING T FURTHER BUNCHES UP δ FUNCTIONS





$T < \frac{1}{2W}$

OR $2W < \frac{1}{T}$

FOR NO INTERFERENCE,

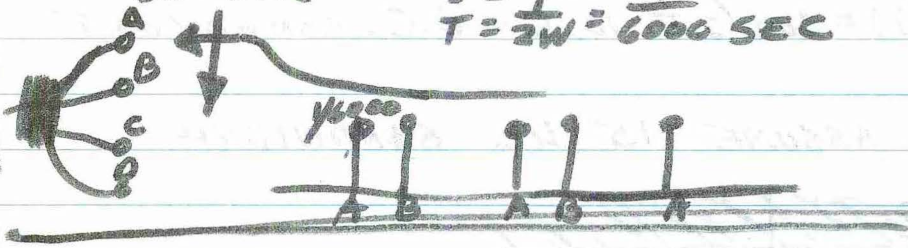
$\frac{1}{T} > 2W$

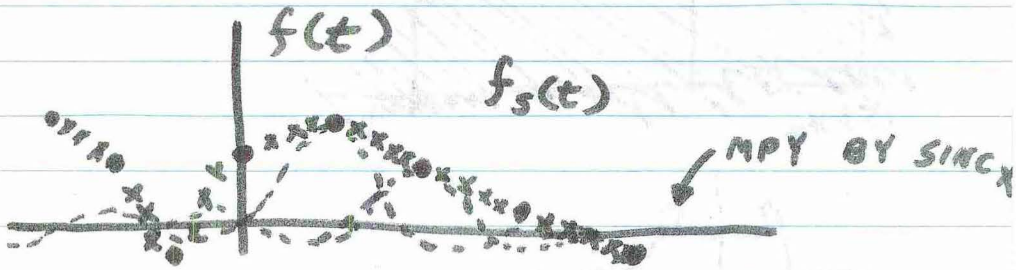
SPREADING OUT DELS (SAMPLES)
WILL SMASH SPECTRA THINGS TOGETHER

BW = 3KC

TRIVIAL

$T = \frac{1}{2W} = \frac{1}{6000}$ SEC





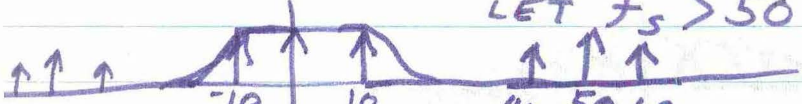
SUM UP ALL SINCX FOR
EVERY POINT, WILL GET
BACK f(t)

11-5-70

#31) $f_s > 20\ Hz = 2f(t)$

BUT USING A REAL FILTER

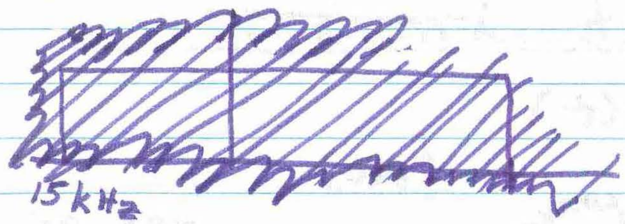
LET $f_s > 50$



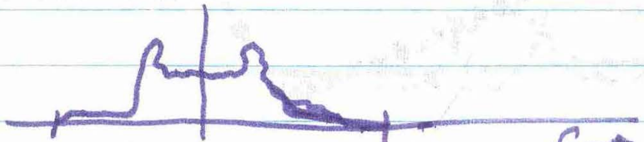
-10 10 40 50 60

$f(t) = \cos(2\pi \cdot 10)t$ + D.C. COMPONENT

#34) ASSUME 15 kc BANDWIDTH



15 kHz

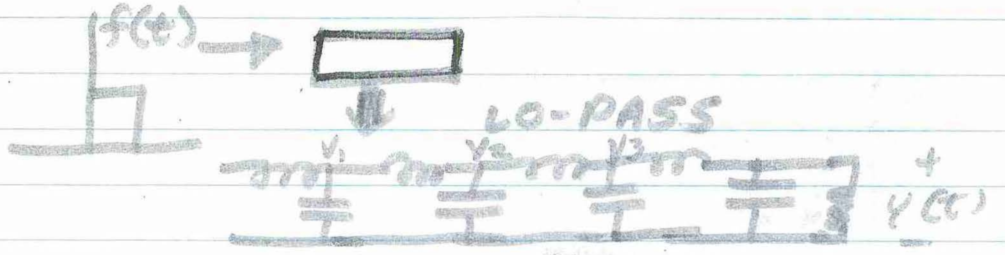


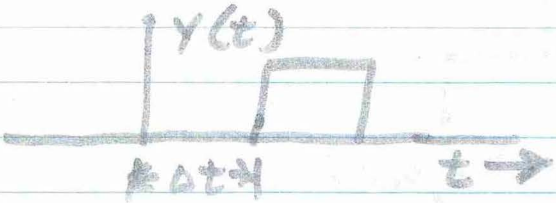
−15 kHz +15 kHz $f \rightarrow$

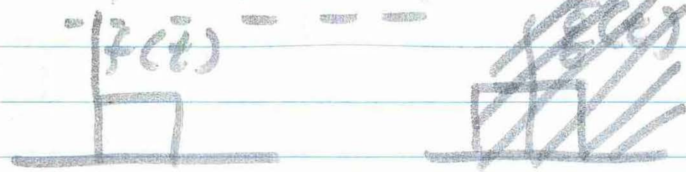
SAMPLING RT > 30 kHz

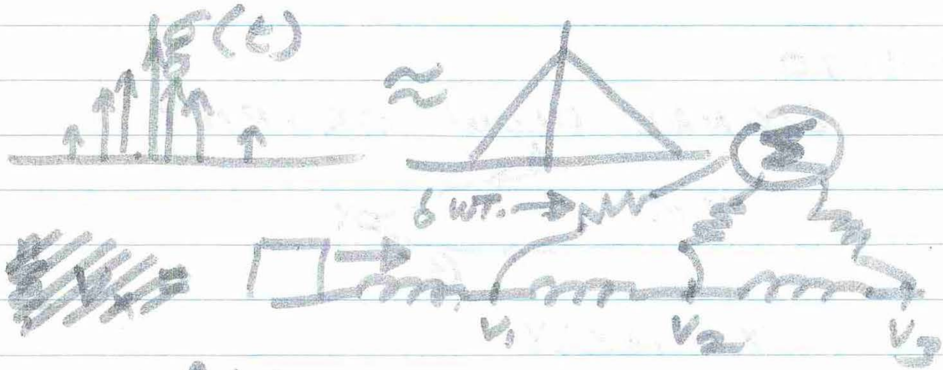
HIGHER FREQ. WOULD BE DISTORTED

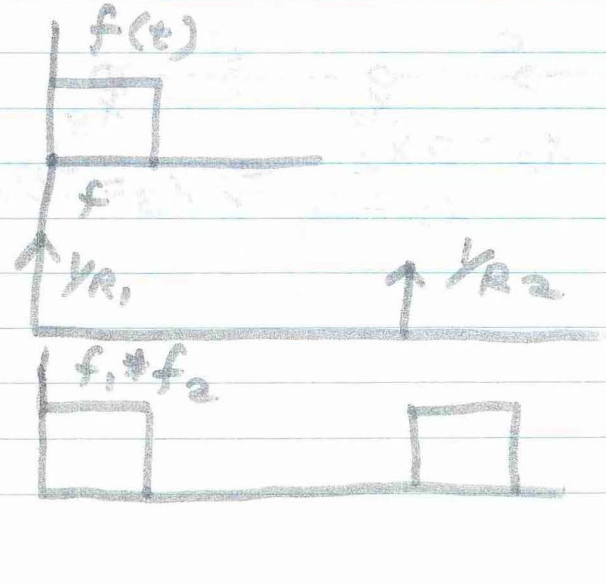
11-6-70

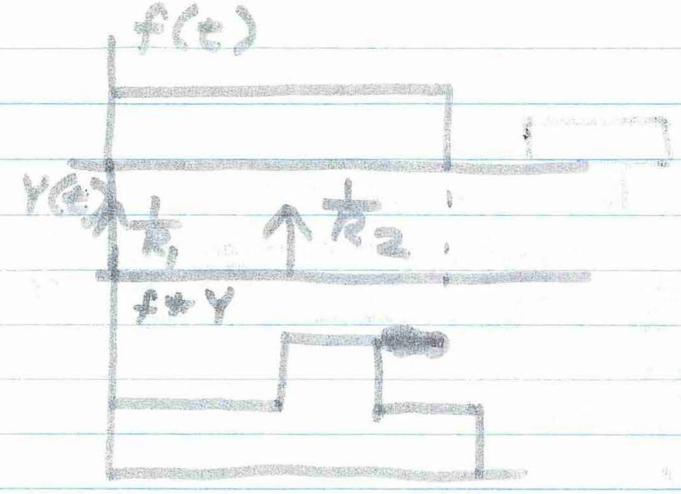








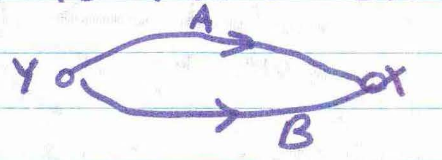




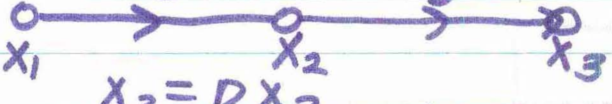
VARYING DEL WTS, MAY GET

11-9-70

SIGNAL FLOW GRAPHY

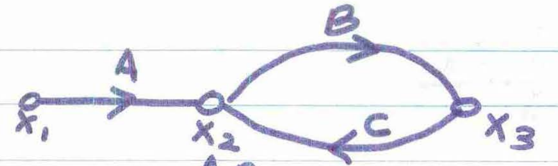


$X = (A+B)Y$



$X_3 = D X_2$

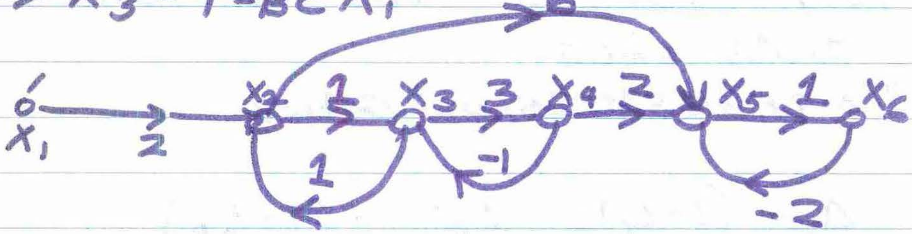
$X_2 = C X_1 \Rightarrow X_3 = C D X_1$



$X_3 = BX_2$

$X_2 = CX_3 + AX_1$

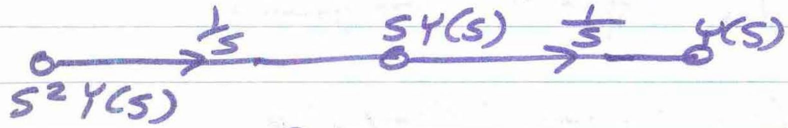
$\Rightarrow X_3 = \frac{AB}{1-BC} X_1$



$As^2X_1(s) + BsX_1(s) + CX_1(s) +$
$DY(s) = U(s)$

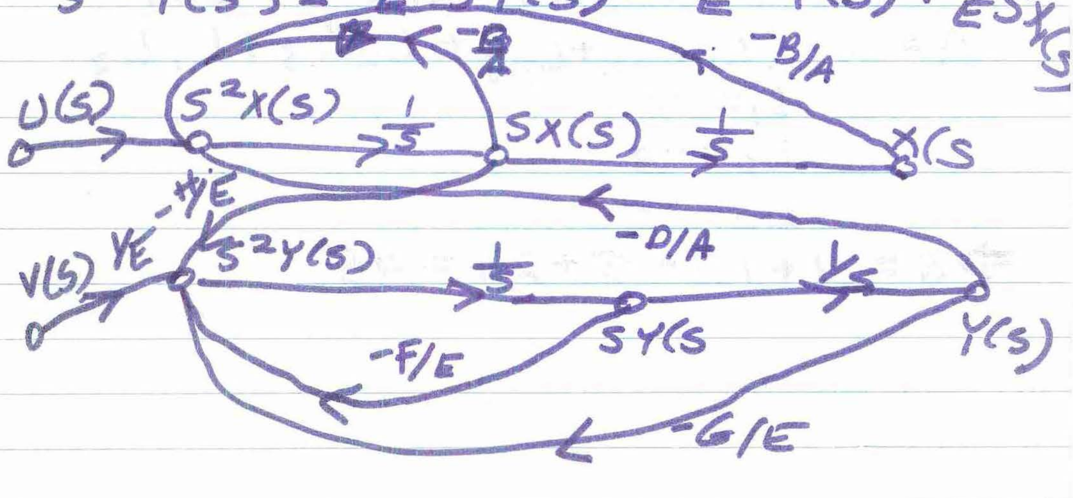
FACTOR OUT HIGHEST TERM

$s^2Y(s) + sY(s) + Y(s) + sX_1(s) = 0$



$s^2X(s) = -\frac{B}{A} sX_1(s) - \frac{C}{A}X_1(s)$
$- \frac{D}{A}Y(s) + U(s)/A$

$s^2Y(s) = -\frac{F}{E} sY(s) - \frac{G}{E}Y(s) + \frac{H}{E}sX(s)$

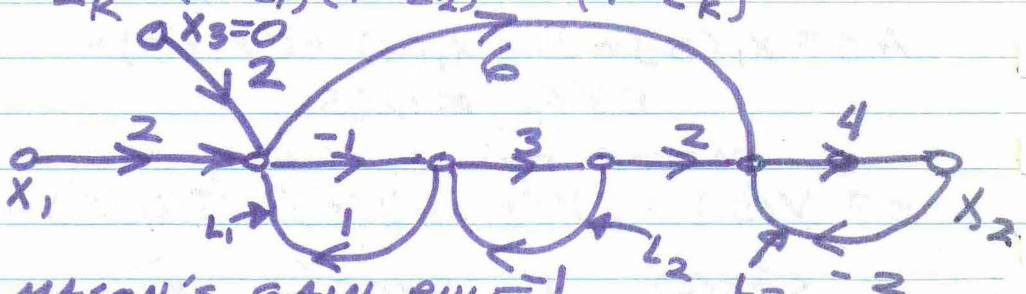


$$T = \frac{\sum P_x \Delta_K}{\Delta}$$

LOOP TOUCHES NODE NO MORE THAN ONCE

T = TRANSMISSION GAIN

$\Delta_K = (1-L_1)(1-L_2)\cdots(1-L_K)$



MASON'S GAIN RULE

$$T_A = \frac{X_2}{X_1} = \frac{\sum P_K \Delta_K}{\Delta}$$

$\Delta = (1-L_1)(1-L_2)(1-L_3)$

$= 1-(L_1+L_2+L_3) + L_1L_2 + L_2L_3 - L_1L_2L_3 + L_1L_3$

IF TWO LOOPS SHARE A NODE OR A BRANCH, STRIKE OUT

$\therefore \Delta = 1-(L_1+L_2+L_3) + L_2L_3 + L_1L_3$

$L_1 = -1$

$L_2 = -3$

$L_3 = -8$

$\Rightarrow \Delta = 1+12+8+24 = 45$

- STRIKE OUT ALL LOOPS WITH COMMON NODES

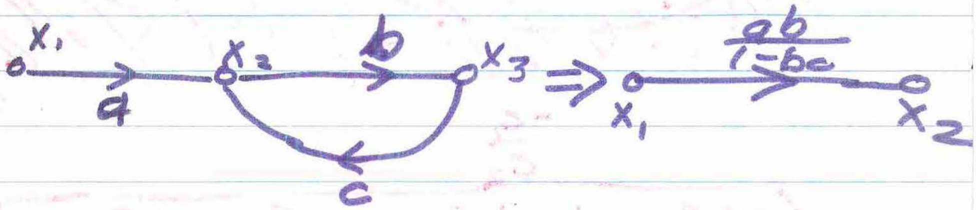
$\Rightarrow \Delta_1 = 4 \; ; \; \Delta_2 = 1$

$$\therefore T = \frac{X_2}{X_1} = \frac{\Sigma P_k \Delta_k}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

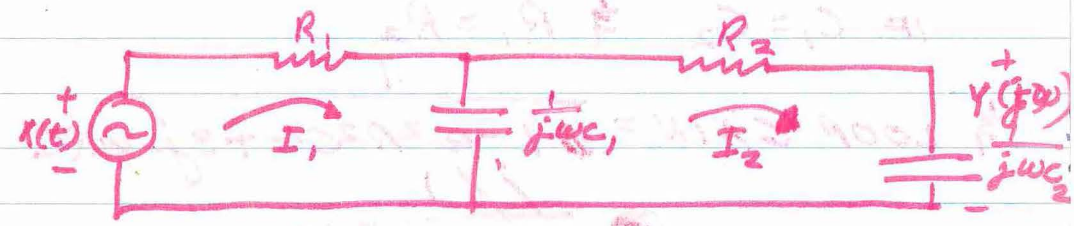
GAIN OF PATH 1 = $P_1 = 48$

" " " 2 = $P_2 = -48$

$$\Rightarrow T = \frac{(48 \cdot 4) + (-48 \cdot 1)}{45} = \frac{3 \cdot 48}{45} = 3\tfrac{1}{5}$$



11-10-70



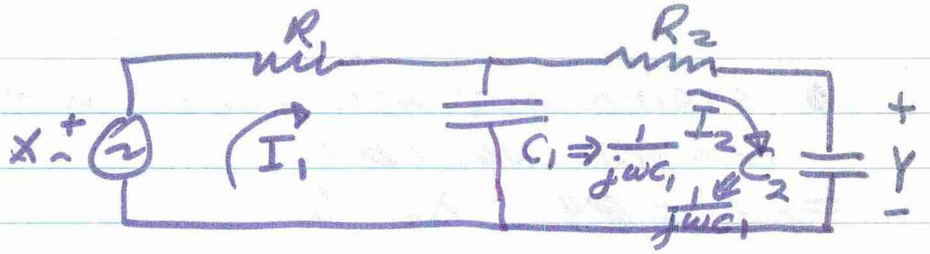
$$X(s) = I_1(s)\left[R_1 + \frac{1}{jwC_1}\right] + I_2\left(\frac{-1}{jwC}\right)$$

$$Y(t) = \frac{-1}{jwC_1}(I_2 - I_1) - R_2 I_3$$

$$Y(t) = \frac{1}{jwC_2} I_2$$

73

EX)

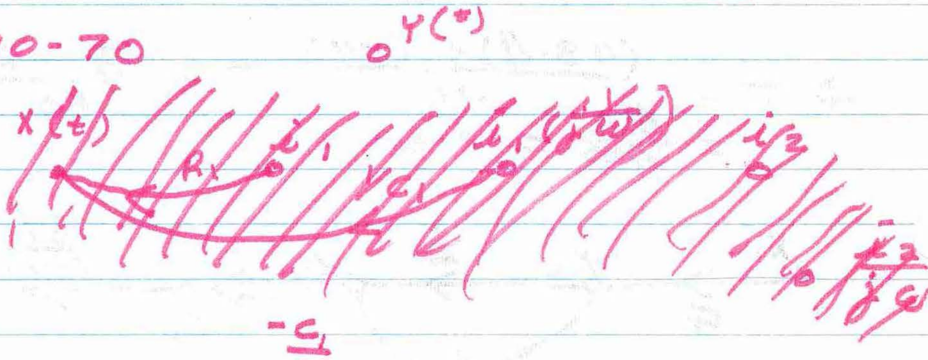


WILL GET 2 SIMUL. D.E.S

$$Y(j\omega) = \frac{1}{j\omega C_2} I_2(j\omega)$$

(PLAY WIT. IT)

11-10-70



$$\frac{-\frac{C_1}{C_2}}{(1+R_1C_1j\omega)(j\omega C_1R_1+1+C_1/C_2)}$$

IF $C_1 = C_2$ & $R_1 = R_2$

& LOOP GAIN $= \dfrac{1}{1-\omega^2R^2C^2+3j\omega RC}$

$$\frac{1}{(1+j\omega RC)^2}$$

5-11

10-12-70

$$Y(s) = H(s)X(s) = \frac{p(s)}{q(s)}$$

$$= \frac{K[s^m + a_1 s^{m-1} + \ldots + a_{m-1}s + a_m]}{s^n + b_1 s^{n-1} + \ldots + b_{n-1}s + b_n}$$

SHOULD MAKE $n \geq m$

EX) $Y(s) = \frac{s^2+2s+3}{s+1}$

$$s+1 \overline{)s^2+2s+3} = s + 1 + \frac{2}{s+1}$$

$\mathcal{L}\{\delta(t)\} = 1$; $\mathcal{L}\{\delta'(t)\} = s$

ASSUME $m < n$
THEN $Y(s) = \frac{K(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ldots(s+p_n)}$

Z = ZERO DETERMINES MAGNET. OF RES
POLE = DETERMINE "FORM" OF RESP.
POLES RTS. OF $Q(s) = -p_1, -p_2, \ldots, -p_n$

PARTIAL FRACTION EXPANSION:
$Y(s) = \frac{k_1}{s+p_1} + \frac{k_2}{s+p_2} + \ldots + \frac{k_i}{s+p_i} + \ldots \frac{k_n}{s+p_n}$
FOR DISTINCT, NON REPEATED RTS
k_i'S ARE <u>RESIDUES</u> AT THE POLES p_i

$$k_i = (s+p_i)Y(s)\Big|_{s=-p_i}$$

EX) $$Y(s) = \frac{24(s+1)(s+2)}{s(s+3)(s+4)(s^2+2s+4)}$$

$$(s+1+\sqrt{3}j)(s+1-j\sqrt{3})$$

$$= \frac{k_1}{s} + \frac{k_2}{(s+3)} + \frac{k_3}{s+4} + \frac{k_4}{s+1+j\sqrt{3}} + \frac{k_5}{s+1-j\sqrt{3}}$$

$$K_1 = 1\ ;\ K_2 = \frac{-16}{7}\ ;\ K_3 = 3\ ;\ K_4 = -.86 - j.99$$

$$K_4^* = K_5 = -.86 + j.99$$

REPEATED LINEAR FACTORS

$$Y(s) = K(s+z_1)\cdots(s+z_n) / (s+p_1)^r (s+p_2) \cdots (s+p_n)$$

$$= \frac{C_1}{s+p_1} + \frac{C_2}{(s+p_1)^2} + \cdots + \frac{C_{r-1}}{(s+p_1)^{r-1}} + \frac{C_r}{(s+p_1)^r}$$

$$+ \frac{K_2}{s+p_2} + \frac{K_3}{s+p_3} + \cdots + \frac{K_n}{s+p_n}$$

SOLVE FOR ALL K_i'S

TO FIND C_r MPY BY $(s+p_1)^r$

$$\Rightarrow \left. Y(s)(s+p_1)^r \right|_{s=-p_1} = C_r$$

$$\frac{d}{ds}(s+p_1)^r Y(s) = (r-1)(s+p_1)^{r-2} + \ldots 1 C_{r-1} + 0$$

$$\frac{d}{ds}\left[(s+p_1)^r Y(s)\right]_{s=-p_1} = C_{r-1}$$

$$C_{r-n} = \frac{1}{n!}\frac{d^n}{ds^n}\left[(s+p_1)^r Y(s)\right]_{s=-p}$$

10-13-70

$$F(s) = \frac{s^2}{(s+1)^2(s+2)}$$

ALTERNATE METHOD FOR REPEATED RTS.

$$\frac{s^2}{(s+1)^2(s+2)} \approx \frac{s^2}{(s+.99)(s+1.01)(s+2)}$$

$$= \frac{k_1}{s+.99} + \frac{k_2}{s+1.01} + \frac{k_3}{s+2} \quad A$$

REALLY $\frac{s^2}{(s+1)^2(s+2)} = \frac{c_1}{s+1} + \frac{c_2}{(s+1)^2} + \frac{K}{s+2}$ B

ARE ANTI. LAPLACE ABOUT THE SAME

FOR $\mathcal{L}^{-1}$ A $= K_1 e^{-\alpha T}$

$\mathcal{L}^{-1}$ B $K_2 e^{-\alpha T} + K_3 t e^{-\alpha t}$

GIVES ABOUT SAME THING

FROM A

$$k_1 = \frac{(-.99)^2}{(-.99+1.01)(-.99+2)} = 48.5$$

$K_2 = -51.5 \qquad K_3 = 4$

$f(t) = \mathcal{L}^{-1}\{f(s)\} = 48.5e^{-.99t} - 51.5e^{-1.01t} + 4e^{-2t}$

FOR A

$$f(s)_B = \frac{-3}{s+1} + \frac{1}{(s+1)^2} + \frac{4}{s+1}$$

$$\Rightarrow f_B(t) = -3e^{-t} + te^{-t} + 4e^{-2t}$$

ALTHOUGH THEY DON'T LOOK VERY EQUAL, BUT THEY JUST ABOUT ARE

(HOW ABOUT THAT)

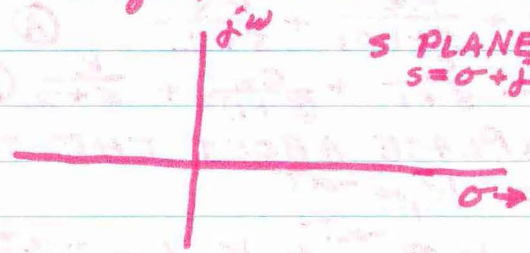
<10% ERROR

POLE - ZERO PLOTS

PHYSICAL REALIZABILITY & STABILITY IN TERMS OF $H(s)$

$h(t) \rightarrow$ CAUSAL (ALL LAPLACE $u(t)$)

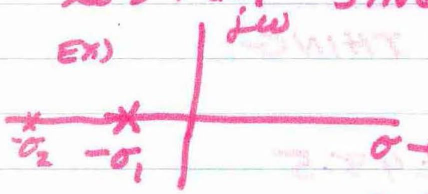
$H(j\omega) \rightarrow$ PALEY-WEINER SAYS OKAY



S PLANE
$s = \sigma + j\omega$

POLE-ZERO PLOT OF $H(S)$ IN S PLANE, ie) PLOT SINGULARITIES OF $H(S)$

EX)



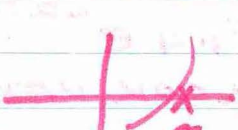
$\Rightarrow H_1(s) = \ldots + \frac{K}{s+\sigma_1}$

$\Rightarrow h_1(t) = \ldots + Ke^{-\sigma_1 t}$

$\sigma_2 > \sigma_1 \Rightarrow h_2(t) = \ldots + e^{-\sigma_2 t}$

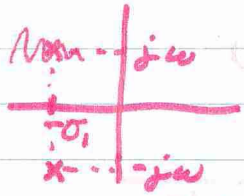
(DECAYS QUICKER)

$\Rightarrow h_0(t) = K[u(t)]$



$\Rightarrow h_4(t) = Ke^{\sigma_4 t}$

(BLOWS UP)



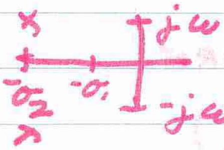
WILL ALWAYS HAVE CONJUGATE PAIRS

$$\Rightarrow H(s) = \dots + \frac{K}{(s+\sigma_1)^2+\omega_0}$$

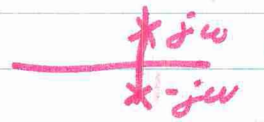
$$= \dots + \frac{K}{(s+\sigma_1+j\omega)(s+\sigma-j\omega)}$$

EX) $s^2+2s+3 \Leftrightarrow (s+1)^2+2 \Rightarrow \sigma = 1, \omega_0 = \sqrt{2}$

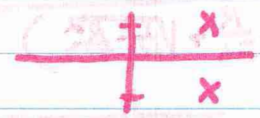
$h(t) = \dots +$ DECAYING SINUSOID



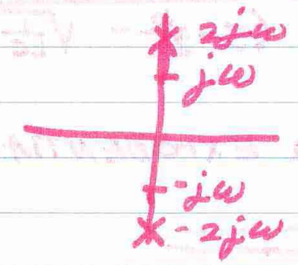
$\Rightarrow$ FASTER DECAY



$\Rightarrow$ PURE SINUSOIDAL



$\Rightarrow$ EXPOD. INCR. SINUSOID

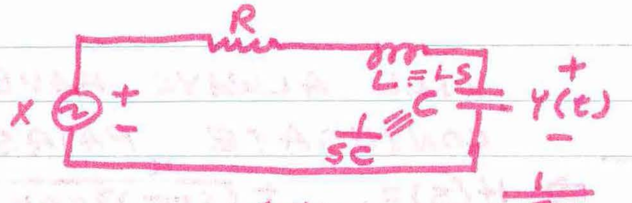


$\Rightarrow$ GREATER ω_0

EVERTHING IN LFT. HALF OF PLANE IS STABLE. ON $j\omega$ AXIS, MARGINALLY STABLE, ON RT HALF, UNSTABLE (THEY BLOW UP)

79)

EX)



$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{1}{sC}}{R + sL + 1/sC}$$

$$H(s) = \frac{1}{s^2LC + sCR + 1}$$

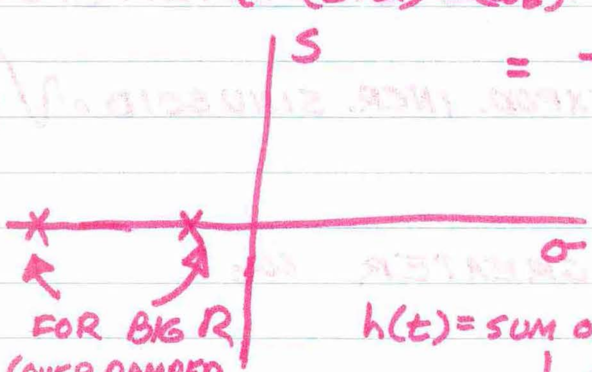
$$= \frac{1}{LC}\left[\frac{1}{s^2 + s\frac{R}{L} + \frac{1}{LC}}\right]$$

COMPLETEING SQUARE:

$$H(s) = \frac{1}{LC}\ \frac{1}{(s+\frac{R}{2L})^2 + (\frac{1}{LC} - \frac{R^2}{4L^2})}$$

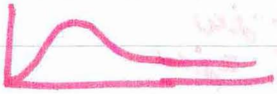
$$= \frac{1}{LC}\left[\frac{1}{(s+\alpha)^2 + (\omega_0)^2}\right]$$

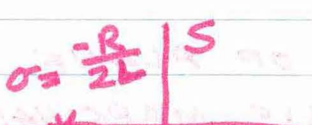
$$= \frac{1}{\left(s + \frac{R}{2L} + \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\right)} \cdot \frac{1}{\left(s + \frac{R}{2L} - \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\right)}$$



FOR BIG R
(OVER DAMPED

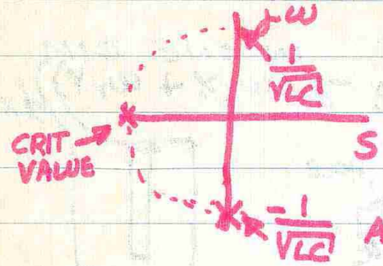
h(t) = SUM OF 2 EXPODENTIALS





IF 2 POLES EQUAL

CRITICAL DAMPING



POLES WILL SWEEP CIRCLE AS R IS DECREASED

AT R=0

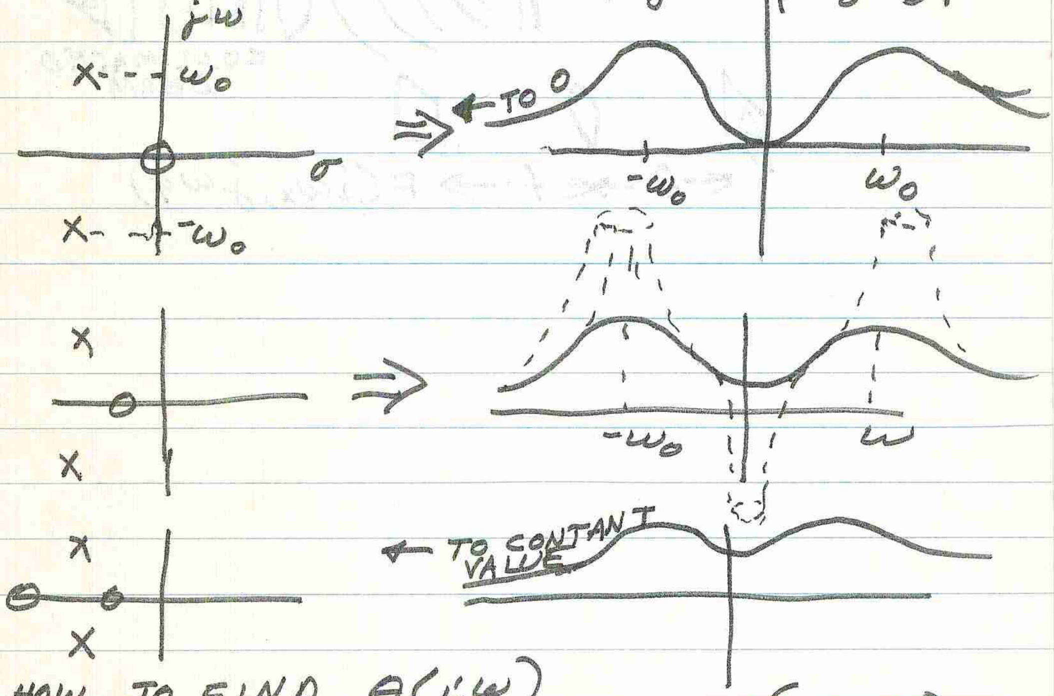
HAVE ZEROS AT $\pm\infty$

H(s) HAS EQUAL # OF POLES AND ZEROS

READ SECTION 7-8

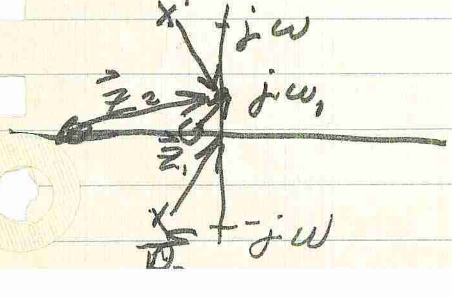
11-17-70

$s=\sigma+j\omega$ $|H(j\omega)|$



HOW TO FIND $\theta(j\omega)$

$$H(s)=\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)\cdots\cdots(s+p_n)}$$



$$H(j\omega)=\frac{\vec{Z}_1\,\vec{Z}_2}{\vec{P}_1\,\vec{P}_2}$$

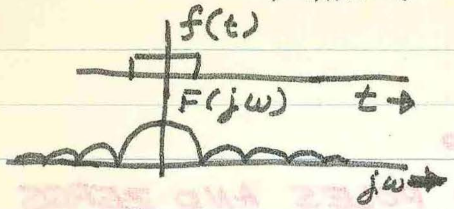
$$H(j\omega)=\frac{|Z_1||Z_2|\angle Z_1+Z_2}{|P_1||P_2|\angle P_1+P_2}$$

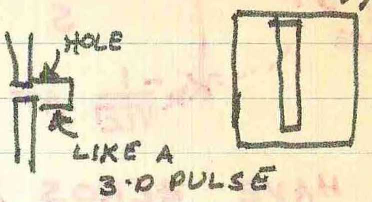
$$=\left|\frac{Z_1\,Z_2}{P_1\,P_2}\right|\angle Z_1+Z_2-P_1-P_2$$

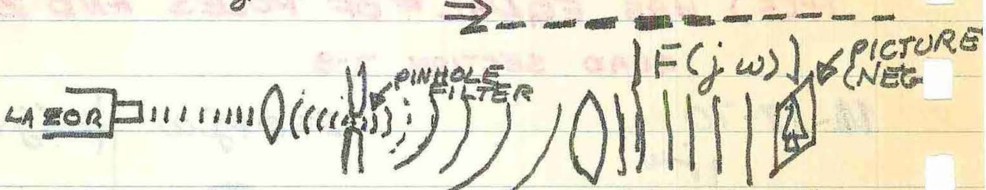
$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt \quad \text{IN ONE DIM}$$

$$F(\omega_x, \omega_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-j(\omega_x x + \omega_y y)}\, dx\, dy$$

(HAIRY!)



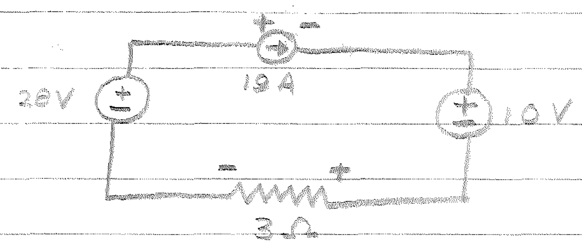




	1 7:50	2 8:45	3 9:40	4 10:35	5 11:30	6 12:25	7 1:20	8 2:15	9 3:15	10 4:05
MON	1	2 INTRO TO PROB D01 AVS	ADV. 3 CALC. I F208 DFO	4	5	PROG PRINC A241 TFK	7	8	9	10
TUES	1	2 INTRO TO PROB D01 AVS	3 ADV. CALC I A212 DFO	4 CONVO	5 CONVO	6	7	E SCI III 8 TFK	9	GL. CLUB
WED	1	2	3	4	5	6 PROG PRINC A241 TFK	7	8 E SCI III TFK	9	10
THURS	1	2 INTRO TO PROB D01 AVS	ADV 3 CALC. I A212 DFO	4	5	PROG 6 PRINC. A241 TFK	7	8 E SCI. III TFK	9	GLEE CLUB 10
FRI	1	2 INTRO TO PROB D01 AVS	ADV 3 CALC. I F208 DFO	4	5	PROG 6 PRINC A241 TFK	7	E SCI III 8 TFK	9	10

CHAPT. 2, pp. 57-9

12) a)



$P_{28VVS} = -Vi = (28V)(19A) = -532$ WATTS

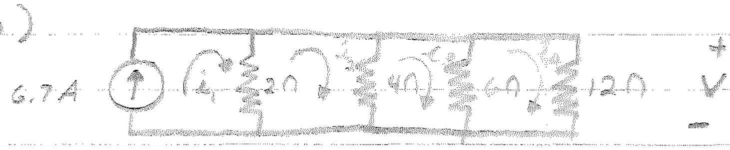
$P_{10VVS} = Vi = (10V)(19A) = 190$ WATTS

$P_{CS} = Vi = (10V - 28V)(19A) = -242$ WATTS

$P_{R} = Vi = i^2R = (19A)^2(3\Omega) = 1083$ WATTS

$P_S = 39(19) = 741V$

b.)



$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 \\ 2 & 6 & -4 & 0 \\ 0 & -4 & 10 & -6 \\ 0 & 0 & -6 & 18 \end{bmatrix}$$

$$2(i_2 - i_1) + 4(i_2 - i_3) = 0 \Rightarrow -2i_1 + 6i_2 - 4i_3 + 0 = 0$$
$$4(i_3 - i_2) + 6(i_3 - i_4) = 0 \Rightarrow 0 - 4i_2 + 10i_3 - 6i_4 = 0$$
$$6(i_4 - i_3) + 12i_4 = 0 \Rightarrow 0 + 0 - 6i_3 + 18i_4 = 0$$
$$i_1 + i_2 + i_3 + i_4 = 6.7$$

$$2i_1 = 4i_2 = 6i_3 = 12i_4 \Rightarrow i_1 = 2i_2 = 3i_3 = 6i_4$$
$$6.7 = i_4 + 2i_4 + 3i_4 + 6i_4 = 12i_4 \Rightarrow i_4 = .556\ A$$
$$i_3 = 2i_4 = 2(.556) = 1.01\ A$$
$$i_2 = 3i_4 = 3(.556) = 1.67\ A$$
$$i_1 = 6i_4 = 6(.556) = 3.33\ A$$

$P_{2\Omega} = R_1 i_1^2 = (2)(3.33)^2 = 22.2 = Vi_1 \Rightarrow V = 6.67\ V$

$P_{CS} = Vi = (6.67)(6.7) = 44.6$ WATTS

$P_{4\Omega} = R_{4\Omega} i_2^2 = (4)(1.67)^2 = 11.16$ WATTS

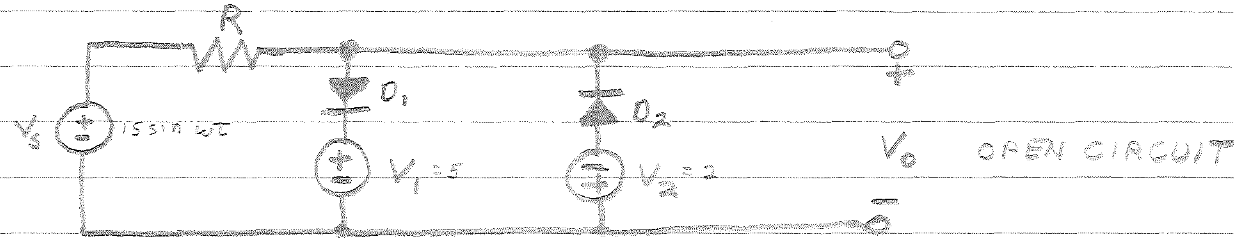
$R_{6\Omega} = R_{6\Omega} i_3^2 = (6)(1.01)^2 = 6.12$ WATTS

$R_{12} = R_{12\Omega} i_4^2 = (12)(.556)^2 = 3.70$ WATTS

18) BOTH WOULD BE STORED OPEN-
CIRCUITED (A DRY CELL)

CHAPT. 2; Pg. 59 1-19-70

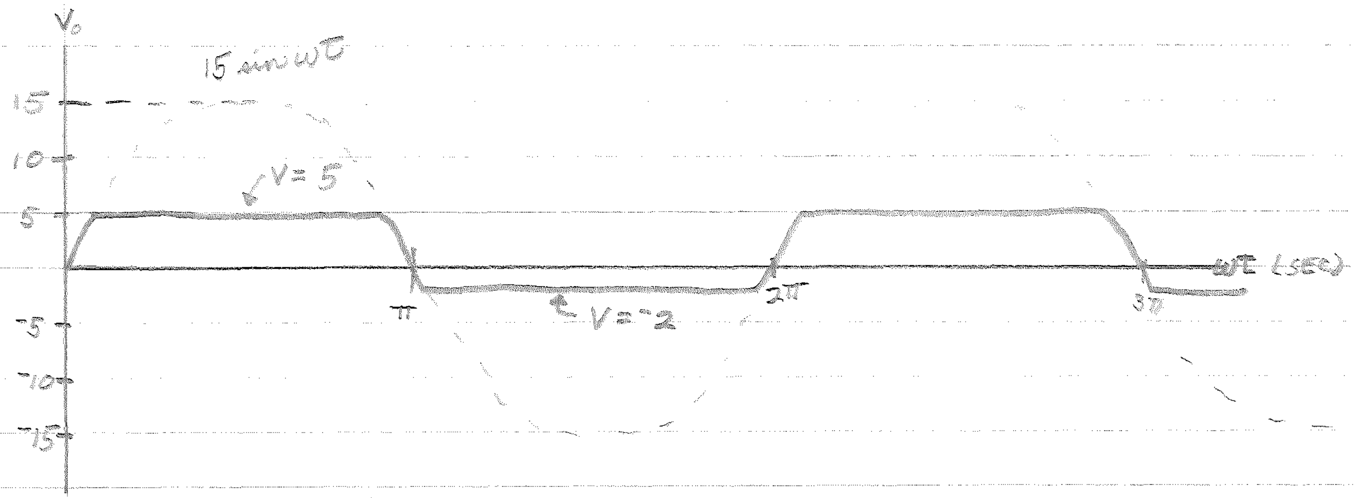
19)



$V_s = 15 \sin \omega t$ VOLTS

$V_1 = 5$ VOLTS

$V_2 = 2$ VOLTS



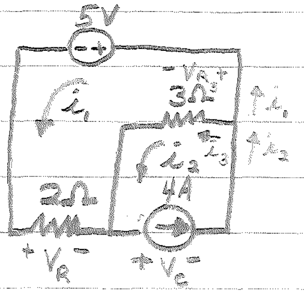
$-2 \leq 15 \sin \omega t \leq 5$

D_1 OPERATES WHEN $0 < V_s \leq 5$

D_2 OPERATES WHEN $-2 \leq V_s < 0$

CHAPT. 3 pp. 91-2

3)



$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \begin{bmatrix} 5 & -3 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} -5 \\ -V_c \end{bmatrix}$$

$$2(i_1) + 3(i_1 - i_2) + 5 = 0 \Rightarrow 5i_1 - 3i_2 = -5$$

$$3(i_2 - i_1) + V_c = 0 \Rightarrow -3i_1 + 3i_2 = -V_c$$

$$i_2 = 4$$

$$5i_1 - 12 = -5 \rightarrow 5i_1 = 7 \rightarrow i_1 = 7/5$$

$$V_c = 3\left(\tfrac{7}{5}\right) - 12 = \tfrac{21}{5} - \tfrac{60}{5} = \tfrac{-39}{5}$$

$$V_R = i_1 R$$

$$V_R = \left(\tfrac{7}{5}\right)(2) = \tfrac{14}{5}$$

$$(i_1 + i_3) = i_2 \Rightarrow i_3 = i_2 - i_1 = 4 - \tfrac{7}{5} = \tfrac{13}{5}$$

$$V_{R_3} = R_3 i_3 = (3)\left(\tfrac{13}{5}\right) = 39/5$$

	V_{source}	i_{source}	R_3	R_2
i	$(7/5)$ Amp	4 Amp	$(13/5)$ Amp	$(7/5)$ Amp
V	4 V	$(-39/5)$ V	$(39/5)$ V	$(14/5)$ V

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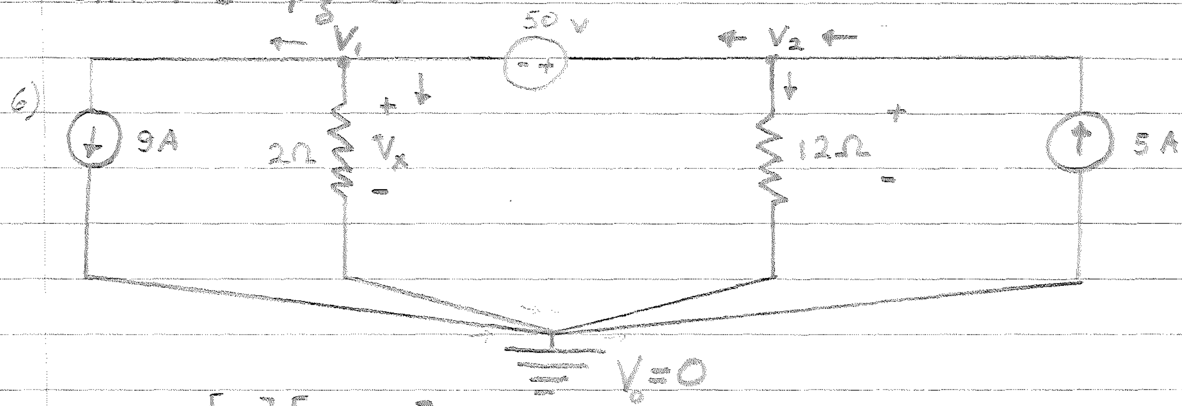
6)



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 12 \end{bmatrix}$$

$$9 + \frac{V_1}{2} - \left(5 + \frac{V_2}{12}\right) = 0 \qquad V_1 - V_2 = 50$$

$$9 + \frac{V_1}{2} - 5 + \frac{V_1 - 50}{12} = 0 \qquad V_2 = V_1 - 50$$

$$108 + 6V_1 - 60 + V_1 - 50 = 0$$

$$7V_1 = 12$$

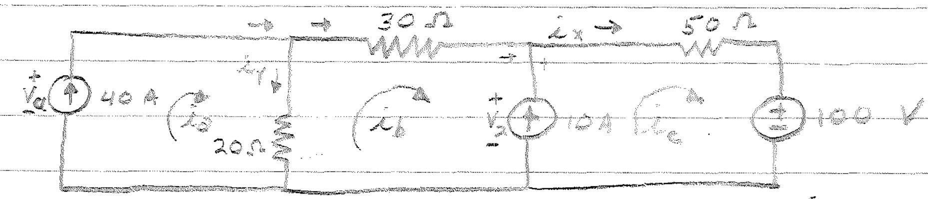
$$V_1 = 12/7$$

$$V_1 = V_x = 12/7 \text{ VOLTS}$$

-14 ✓

Pg 95

13)



$i_a = 40$

$20(i_a - i_b) - V_a = 0 \Rightarrow 20 i_a - 20 i_b = V_a$

$20 i_b = 800 - V_a$

$20(i_b - i_a) + 30 i_b + V_2 = 0 \Rightarrow -20 i_a + 50 i_b = -V_2$

$50 i_b = 800 - V_2$

$50 i_c + 100 - V_2 = 0 \Rightarrow 50 i_c = V_2 - 100$

$i_y = 40 - i_b$; $i_c = i_x$

$$\begin{bmatrix} i_b \\ i_c \end{bmatrix} \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix} = \begin{bmatrix} 800 - V_a \\ V_2 - 100 \end{bmatrix} \qquad i_b + 10 = i_c$$

$800 - 50 i_b = 50 i_c + 100$

$800 - 50 i_b = 50 i_b + 600$

$100 i_b = -200 \Rightarrow i_b = -2 \Rightarrow i_y = 42$ AMPS

$50 i_c = 800 \Rightarrow i_c = 40 \Rightarrow i_x = 40$ AMPS

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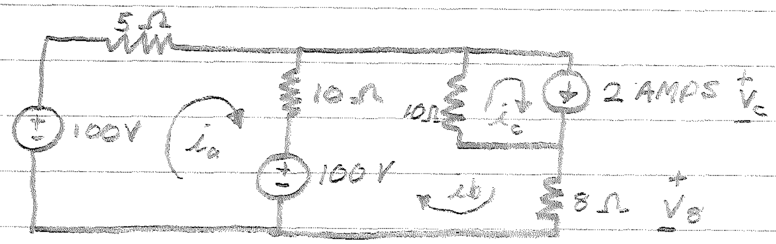
14)



$i_c = 2$ AMPS

$V_8 = 8\, i_b$

$P_{8\Omega} = (i_b)^2 R = 8(i_b)^2$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \begin{bmatrix} 15 & -5 & 0 \\ -10 & 28 & -10 \\ 0 & -10 & 10 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ -V_c \end{bmatrix}$$

$5 i_a + 10(i_a - i_b) = 0 \Rightarrow 15 i_a - 10 i_b = 0$

$10(i_b - i_a) - 100 + 10(i_b - i_c) + 8 i_b = 0$

$\Rightarrow -10 i_a + 28 i_b - 10 i_c = 100$

$10(i_c - i_b) + V_c = 0$

$15 i_a - 10 i_b = 0 \Rightarrow 3 i_a = i_b$

$-10 i_a + 28 i_b = 120 \Rightarrow -5 i_a + 14 i_b = 60$

$-5 i_a + 14(3 i_a) = 60$

$-5 i_a + 42 i_a = 37 i_a = 60 \Rightarrow i_a = 1.62$ AMPS

$i_b = 48.6$ AMPS

$V_8 = 8 i_b = 8(48.6) = 389$ VOLTS

$P_{8\Omega} = i_b^2\, 8 = 18{,}900$ WATTS

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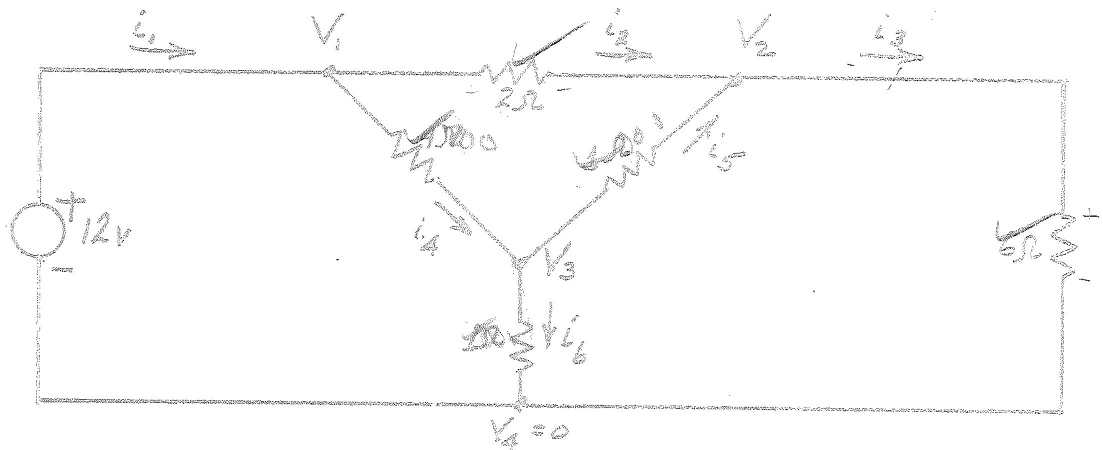
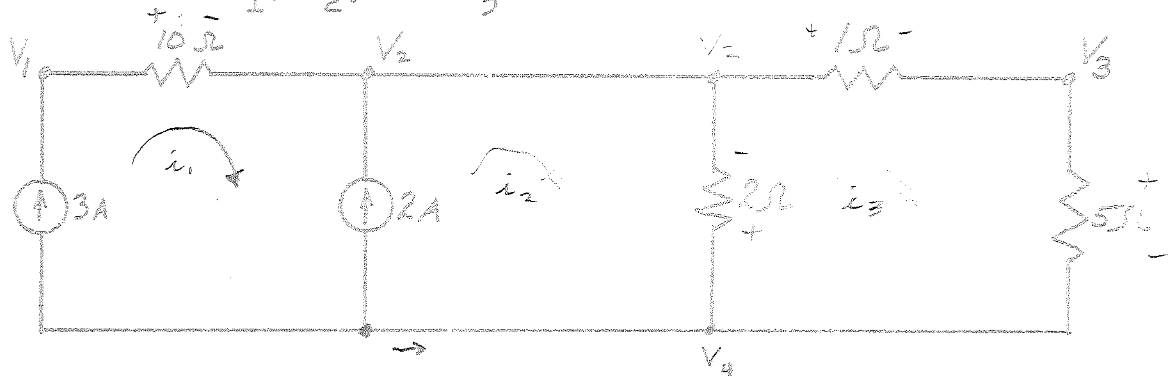
Special Problems

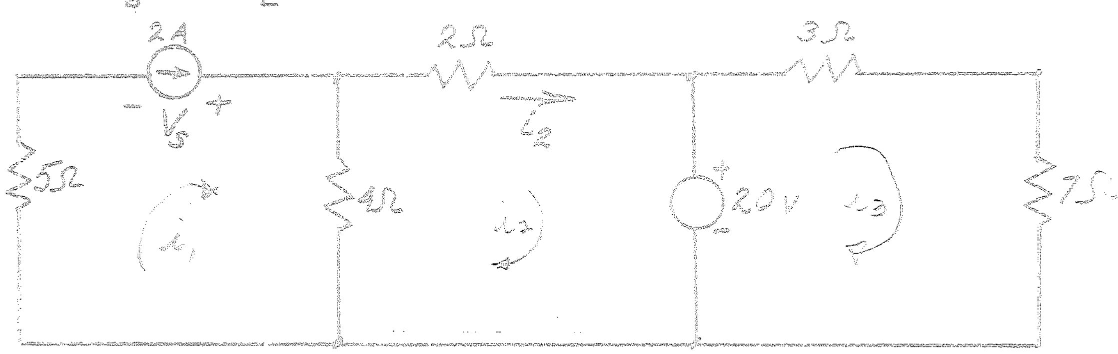
(1) Find all currents and voltages in the following circuit.



(2) Solve for v_1, v_2, and v_3.



(3) Find V_s and i_2.



SPECIAL PROBLEM

1) $\frac{V_1 - V_3}{4} + \frac{V_1 - V_2}{2} = i \Rightarrow V_1 - V_3 + 2V_1 - 2V_2 = 4i$

$3V_1 - 2V_2 - V_3 = 4i$

$\frac{V_2 - V_1}{2} + \frac{V_2}{6} + \frac{V_2 - V_3}{1} = 0 \Rightarrow 3V_2 - 3V_1 + V_2 + 6V_2 - 6V_3 = 0$

$-3V_1 + 10V_2 - 6V_3 = 0$

$\frac{V_3}{1} + \frac{V_3 - V_2}{1} + \frac{V_3 - V_1}{4} = 0 \Rightarrow 4V_3 + 4V_3 - 4V_2 + V_3 - V_1 = 0$

$-V_1 - 4V_2 + 9V_3 = 0$

$V_1 = 12 \Rightarrow 10V_2 - 6V_3 = 36 \Rightarrow 60V_2 - 36V_3 = 216$

$\Rightarrow -4V_2 + 9V_3 = 12 \Rightarrow \underline{-16V_2 + 36V_3 = 48}$

$44V_2 = 264 \Rightarrow V_2 = 6$

$60 - 6(V_3) = 36 \Rightarrow V_3 = 4$

$V_1 = 12\text{ V}$; $V_2 = 6\text{ V}$; $V_3 = 4\text{ V}$; $V_4 = 0.0\text{ V}$

$i_2 = \frac{V_1 - V_2}{2} = \frac{12-6}{2} = 3$ AMPS

$i_3 = \frac{V_2 - V_4}{6} = \frac{6}{6} = 1$ AMP

$i_4 = \frac{V_1 - V_3}{4} = \frac{12-4}{4} = 2$ AMPS

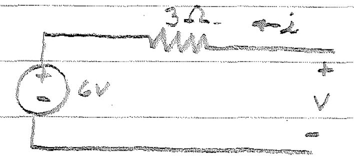
$i_5 = \frac{V_3 - V_2}{1} = \frac{4-6}{1} = -2$ AMPS

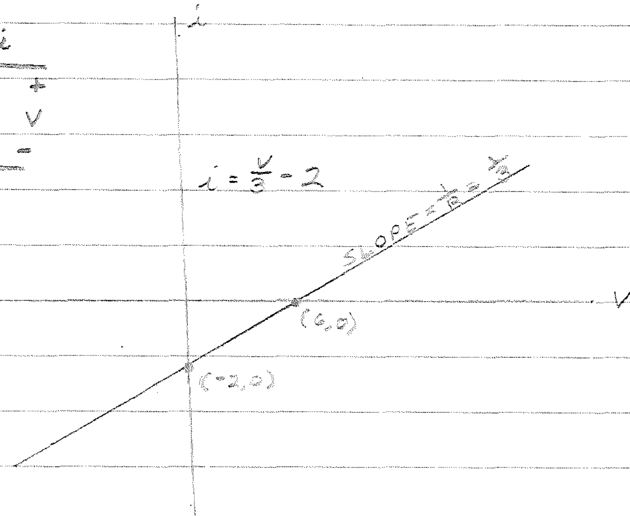
$i_6 = \frac{V_3 - V_4}{1} = \frac{4}{1} = 4$ AMPS

$i_1 = i_2 + i_4 = 3 + 2 = 5$ AMPS

Pp. 100-1

28) a)

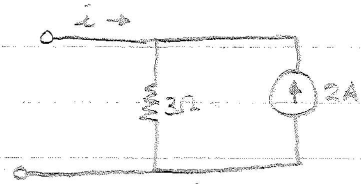


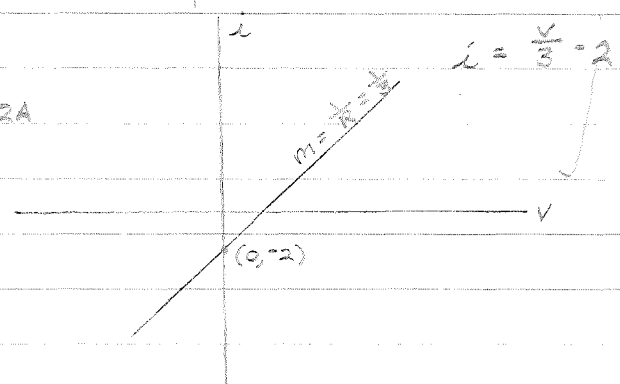


$V = 3i + 6$

$i = \frac{V}{3} - 2$

b)

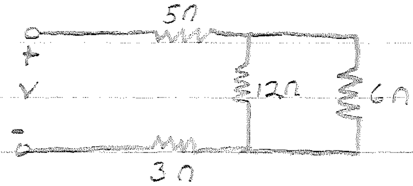


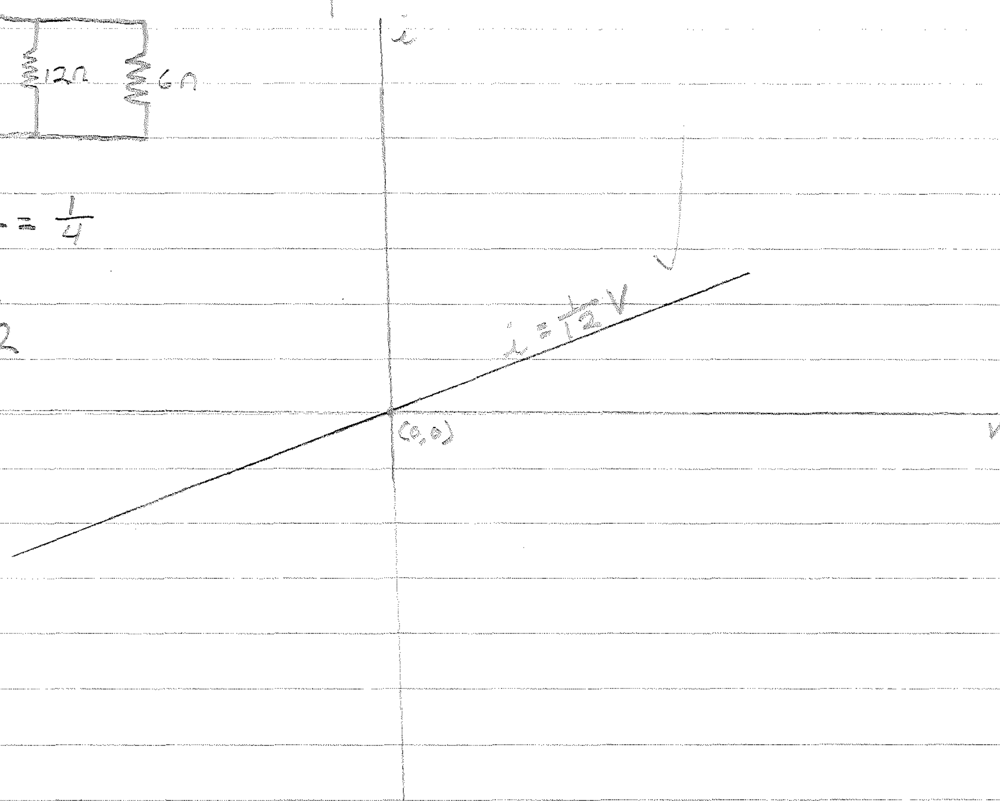


$V = 3(i + 2)$

$i = \frac{V}{3} - 2$

c)





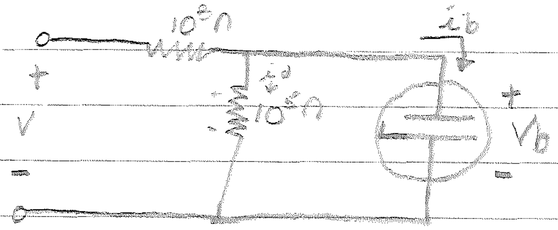
$\frac{1}{12} + \frac{1}{6} = \frac{3}{12} = \frac{1}{4}$

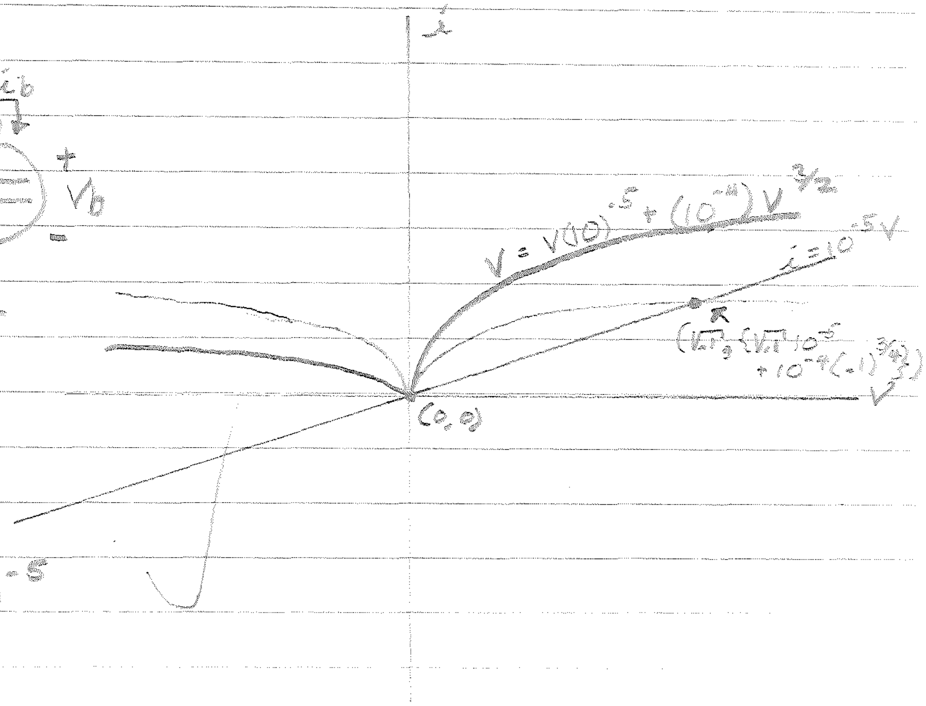
$\therefore R_{Eq} = 12$

$V = i\,12$

$i = \frac{1}{12}V$

d)





$i_b = 10^{-4} V_b^{3/2}$

$V = V_b$

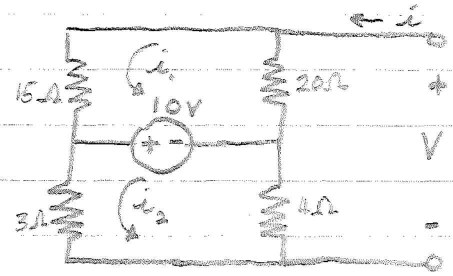
$i_b = 10^{-4} V^{3/2}$

$i = i_b + i_d$

$i_d = V10^{-5}$

$i = 10^{-4} V^{3/2} + V10^{-5}$

e)



by MESH CURRENT:

$35i_1 - 20i = -10 \Rightarrow 35i_1 - 20i = -10$

$7i_2 - 4i = +10 \Rightarrow -35i_2 + 20i = -50$

$\therefore 35i_1 - 35i_2 = -60 \Rightarrow i_1 = i_2 - \frac{12}{7}$

$24i - 20i_1 - 4i_2 = V$

$24i - 24i_2 + \frac{240}{7} = V$

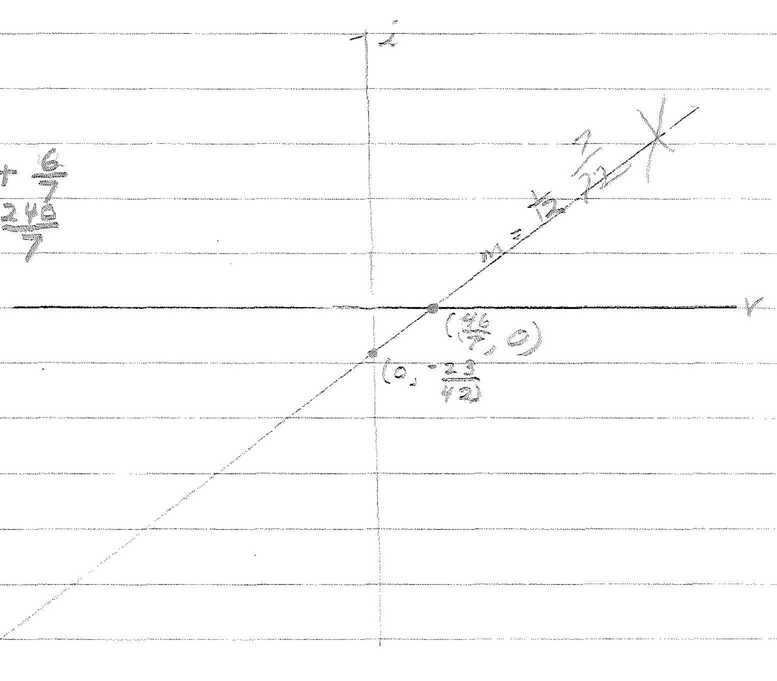
ALSO $i = i_1 + i_2$

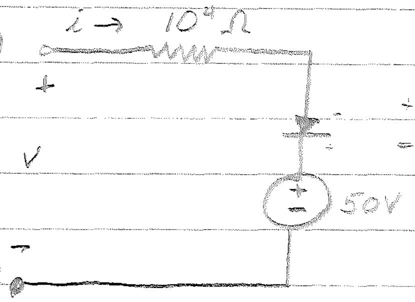
$i = 2i_2 - \frac{12}{7} \Rightarrow i_2 = \frac{i}{2} + \frac{6}{7}$

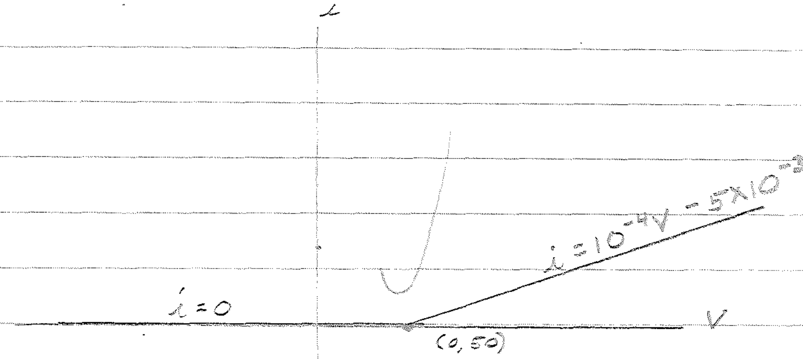
$\therefore V = 24i - 24\left(\frac{i}{2} + \frac{6}{7}\right) + \frac{240}{7}$

$V = 12i + \frac{46}{7}$

$\Rightarrow i = \frac{V}{12} - \frac{23}{42}$



f) 



FOR $i > 0$

$V = 10^4 i + 50$

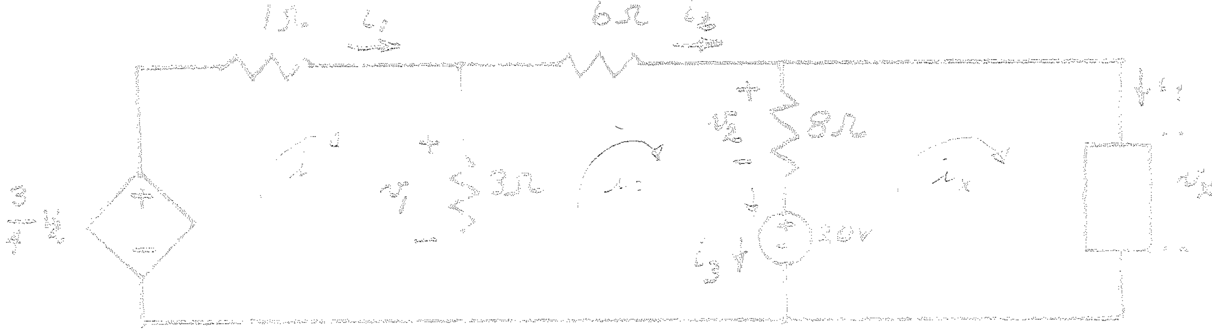
$i = 10^{-4} V - 5\times10^{-3}$

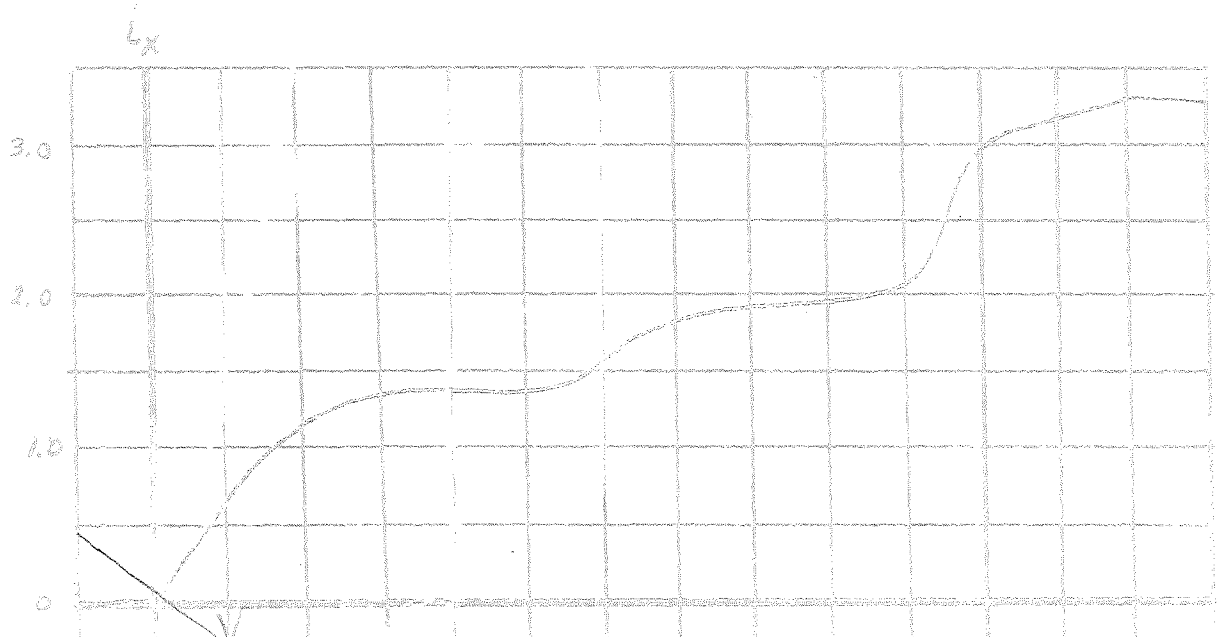
FOR $i = 0$, $V = 50$

FOR $i < 0$, $V = 0$

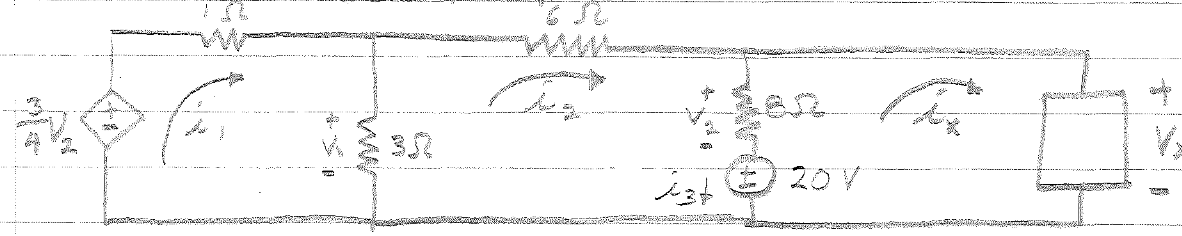
Special Problem 4

Element NL is a nonlinear device whose characteristics are sketched below. Find i_x, i_1, i_2 and i_3. Also find V_1, V_2, and V_x. Determine the power associated with each element in the circuit.





SPECIAL PROBLEM #4



1) $4i_1 - 3i_2 = \frac{3}{4}V_2$; $V_2 = 8i_3 = 8(i_x - i_2)$

$\therefore 4i_1 - 3i_2 = 6(i_x - i_2) = 6i_x - 6i_2$

$4i_1 + 3i_2 - 6i_x = 0$

2) $^-17i_2 + 3i_1 + 8i_x = {}^-20$

3) $^-8i_x + 8i_2 + V_x = {}^-20$

$$i_1 = \frac{3}{2}i_x - \frac{3}{4}i_2 = \frac{-20}{3} + \frac{17}{3}i_2 - \frac{8}{3}i_x$$
$$= \frac{18}{12}i_x - \frac{9}{12}i_2 = \frac{-20}{3} + \frac{68}{12}i_2 - \frac{32}{12}i_x$$
$$\Rightarrow \frac{18}{12}i_x + \frac{32}{12}i_x + \frac{20}{3} = \frac{68}{12}i_2 + \frac{9}{12}i_2$$
$$= \frac{77}{12}i_2 = \frac{50}{12}i_x + \frac{20}{3}$$
$$\Rightarrow i_2 = \frac{50}{77}i_x + \frac{80}{77}$$

FROM 3:

$$i_2 = \frac{-20}{8} + \frac{V_x}{8} + i_x \left(= \frac{50}{77}i_x + \frac{80}{77}\right)$$
$$\Rightarrow i_x - \frac{50}{77}i_x = \frac{-V_x}{8} + \frac{545}{154}$$
$$\frac{22}{77}i_x = \frac{-V_x}{8} + \frac{545}{2(77)}$$
$$\Rightarrow i_x = \frac{-77V_x}{8(22)} + \frac{545}{2(77)}\frac{(77)}{(22)}$$
$$= \frac{-77}{176}V_x + \frac{545}{44}$$
$$= {}^-.437\,V_x + .124$$

FROM GRAPH:

$i_x \cong .1$; $V_x \cong .1$

a) $V_x - 20 - V_2 = 0$

$V_2 = (.1 - 20) = 19.9\ V$

b) $V_2 = 8 i_3 \Rightarrow i_3 = 2.49$ AMPS

c) $i_3 + i_x = i_2$

$(2.49 + .1) = i_2 = 2.59$ AMPS

d) $4 i_1 - 3 i_2 - \frac{3}{4} V_2 = 0$

$4 i_1 = 2.21 + 7.78 = 9.99$

$i_1 = 2.49$

e) $V_1 = 3(i_1 - i_2)$

$= 3(2.49 - 2.59)$

$= -.30$ VOLTS

$V_1 = -.30$ VOLTS

$V_2 = 2.49$ VOLTS

$V_x = .1$ VOLTS

$i_1 = 2.49$ AMPS

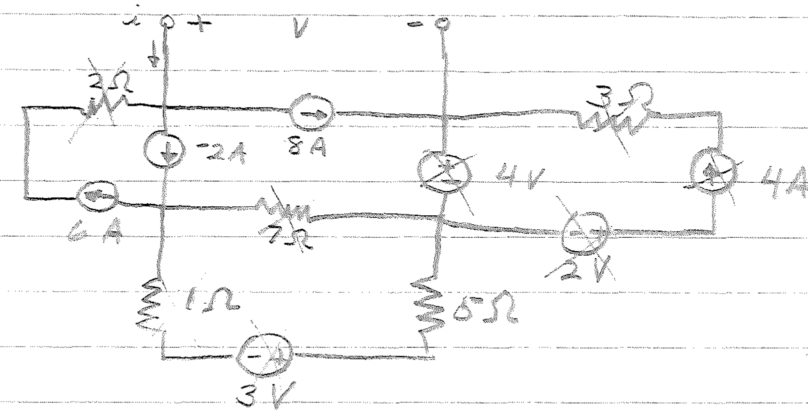
$i_2 = 2.59$ AMPS

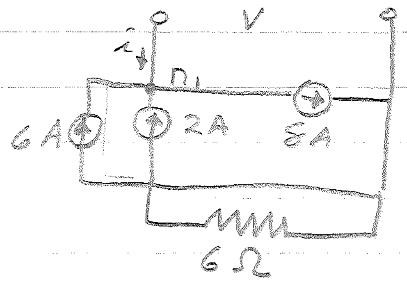
$i_x = .1$ AMP

Pounds?

Pg. 143

4)

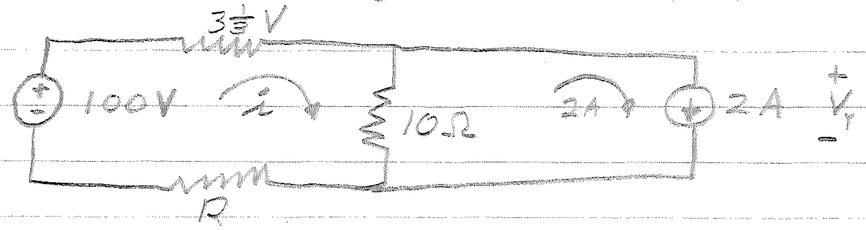




$\Sigma i_{n_1} = 6 + 2 + i - 8 = 0 \Rightarrow i = 0$ FOR ALL V

Pg 146

9) CIRCUIT REDUCES TO:



$$i\left(R+\frac{10}{3}+10\right)-20=100 \Rightarrow i\left(R+\frac{40}{3}\right)=120$$

$$V_Y+10(2-i)=0 \Rightarrow V_Y-10i=-20$$

SOLVE FOR i IN BOTH EQUATIONS:

$$i = 120\Big/\left(R+\frac{40}{3}\right)=\frac{360}{3R+40}$$

$$i=\frac{V_Y+20}{10}$$

$$\therefore \frac{V_Y+20}{10}=\frac{360}{3R+40} \Rightarrow (V_Y+20)(3R+40)=3600$$

$$\Rightarrow 3V_YR+40V_Y+60R+800=3600$$

$$\Rightarrow R(3V_Y+60)=-40V_Y+2800$$

$$\Rightarrow R=\frac{2800-40V_Y}{3V_Y+60}$$

$$\Rightarrow \frac{dR}{dV_Y}=\frac{-40}{3V_Y+60}-\frac{3(2800-40V_Y)}{(3V_Y+60)^2}$$

$$\Rightarrow \frac{1600-8400+120V_Y}{(3V_Y+60)^2}=0$$

$$\Rightarrow 120V_Y=6800 \Rightarrow V_Y=\frac{170}{3}$$

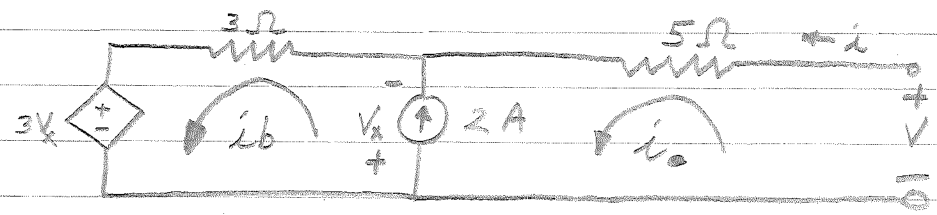
$$\therefore R=\frac{2800-40\left(\frac{170}{3}\right)}{170+60}=\frac{533}{230}=2.32\,\Omega$$

$$\left(\frac{d^2R}{dV_Y^2}\right)_{V_Y=\frac{170}{3}}<0 \Rightarrow \text{MAXIMUM}$$

$$i=\frac{120}{15.6}=7.67$$

$P=i^2R=$ ~~395~~ 136 WATTS

4-12)



FOR V_{oc}

$i_b = 2$

$3V_x + V_x + 3i_b = 0 \Rightarrow 4V_x = -6$

$-V_{oc} + 6 + 3V_x = 0 \Rightarrow V_{oc} = -(6 + 3V_x) = -(6 + 3(-\frac{3}{2}))$
$= -\frac{3}{2}$

FOR i_{sc}

$5i_{sc} = V_x$

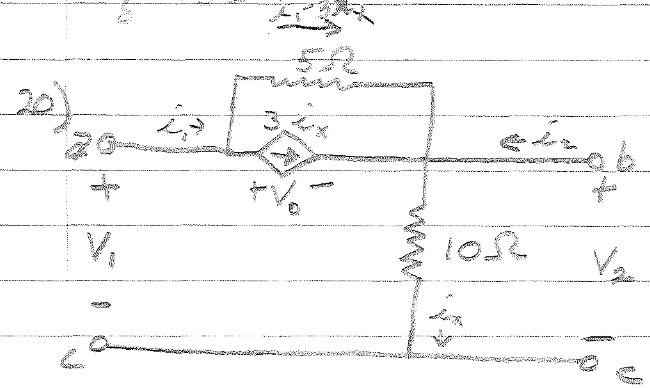
$V_x + 3i_b + 3V_x = 0 \Rightarrow 4V_x = -3i_b$

$i_b - i_{sc} = 2 \Rightarrow 4V_x = -3(2 + i_{sc})$
$= -(6 + i_{sc}) = 20i_{sc}$
$\Rightarrow -6 = 21 i_{sc} \Rightarrow i_{sc} = \frac{-6}{21} = -\frac{2}{7}$

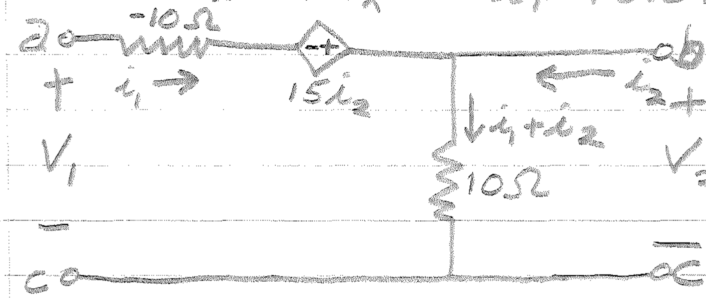
$\Rightarrow I_{eq} = \frac{2}{7}$ AMPS
$V_{Eq} = -\frac{3}{2}$ VOLTS
$R_{EQ} = \frac{21}{4}\ \Omega$

Pg 150

20)



$$10 i_x = V_2$$
$$V_0 + 10 i_x = V_1$$
$$5(i_1 - 3i_x) + 10 i_x = V_1$$
$$5 i_1 - 5 i_x = V_1$$
$$i_1 - \cancel{3i_x} + i_2 + \cancel{3i_x} - i_x = 0$$
$$i_1 + i_2 = i_x$$
$$V_1 = 5i_1 - 5(i_1 + i_2) = -5 i_2$$
$$V_2 = 10 i_x = 10 i_1 + 10 i_2$$



$$V_1 = -10 i_1 - 15 i_2 + 10 i_1 + 10 i_2 = -5 i_2$$
$$V_2 = 10 i_1 + 10 i_2$$

∴ THE TWO CIRCUITS ARE EQUIVALENT

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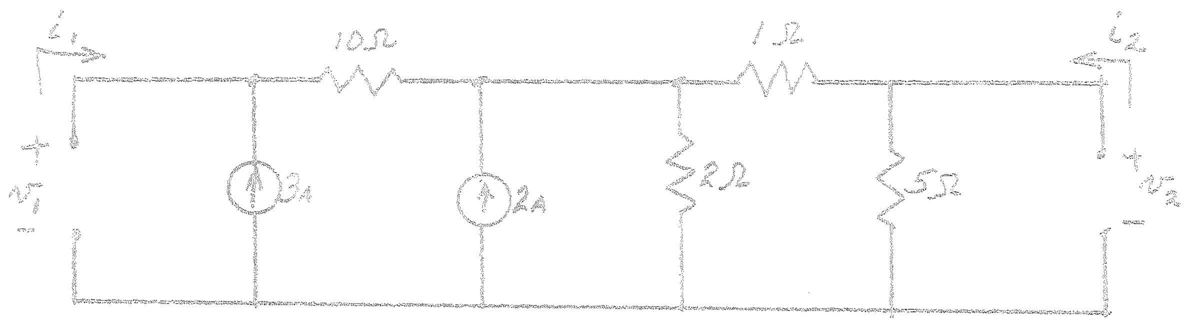
Special Problem 6

Determine the equivalent three terminal network (in both T and π forms) for the following circuit.



Pg 230

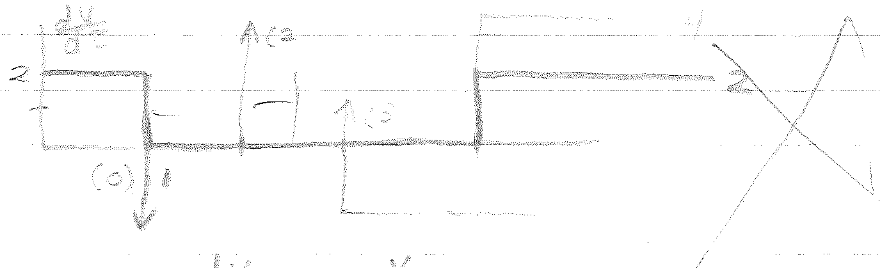
12) a) $C = \frac{q}{V}$

$i = C\frac{dV}{dt}$

$V = 2t - 2t\mu(t-1) + 2\mu(t-1) - 2\mu(t-2) + 4\mu(t-2) - 4\mu(t-3)$
$+ (-2t+10)\mu(t-3) - (-2t+10)\mu(t-4)$

$V = 2t + (2-2t)\mu(t-1) + 2\mu(t-2) + (-2t+6)\mu(t-3) + (2t-10)\mu(t-4)$

$\frac{dV}{dt} = 2 - 2\mu(t-1) + (2-2t)\delta(t-1) + 2\delta(t-2) - 2\mu(t-3) + (-2t+6)\delta(t-3)$
$+ 2\mu(t-4) + (2t-10)\delta(t-4)$



sketches?

AT $t > 4$; $\frac{dV}{dt} = 2\frac{V}{SEC}$

$i = C\frac{dV}{dt} = (2F)(2\frac{V}{SEC}) = 4$ AMPS

b) $V = \frac{1}{C}\int_0^{4^+} i\,dt$
$= \frac{1}{2}[2+2-2+1-3] = 0$

c) $V = L\frac{di}{dt}$

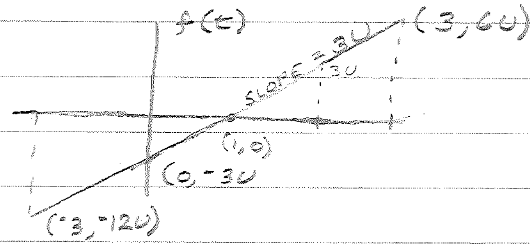
$\frac{di}{dt} = 2\frac{AMP}{SEC}$ AT $t > 4$ (FROM PART a)

$\therefore V = 2$ VOLTS

d) $i = 0$ (FROM MATH IN PART b)

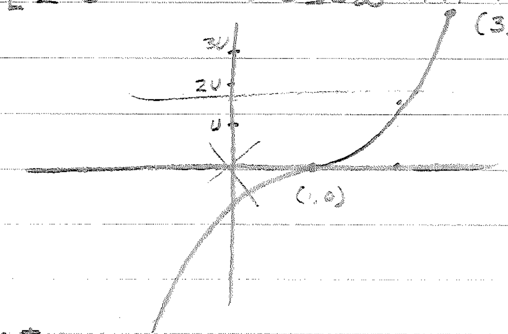
5) a) $f(t) = 3U(t-1)$

$= 3Ut - 3U$



~~b)~~ $\#(t) = \int_{-\infty}^{t} (3Ut - 3U)\,dt$

$= \left[\frac{3}{2}Ut^2 - 3Ut\right]_{-\infty}^{t}$ - MAY NOT BE EVALUATED IN THIS MANNER



b) $\#(t) = \int_{-\infty}^{t} (3Ut - 3U)\,dt$

$= \int_{-\infty}^{-3} (3Ut - 3U)\,dt + \int_{-3}^{t} (3Ut - 3U)\,dt$

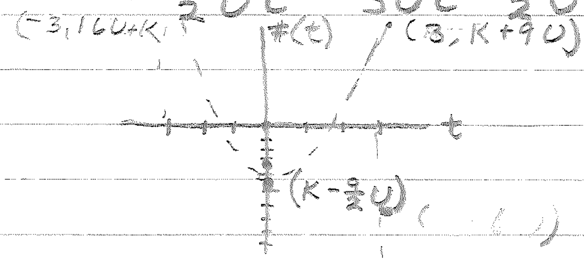
IF $\int_{-\infty}^{-3} (3Ut - 3U)\,dt$ IS FINITE

$\#(t) = \int_{-3}^{t} (3Ut - 3U)\,dt + K$

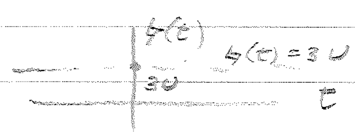
$= \left[\frac{3}{2}Ut^2 - 3Ut\right]_{-3}^{t} + K$

$= \frac{3}{2}Ut^2 - 3Ut - \frac{27}{2}U + 9U + K$

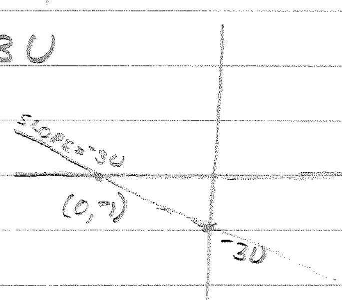
$= \frac{3}{2}Ut^2 - 3Ut - \frac{9}{2}U + K$



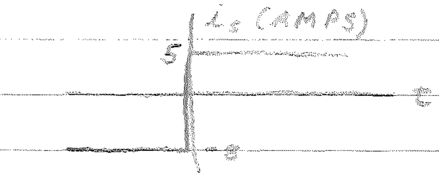
c) $g(t) = 3U$



d) $f(-t) = -3Ut - 3U$

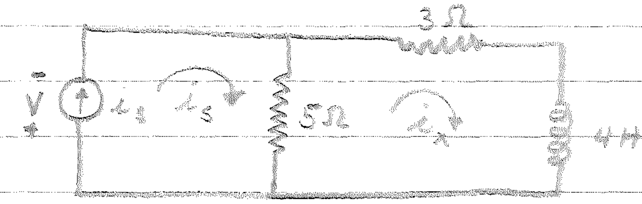


3) $i_s = -8 + 13u(t)$



$V + 5i_s - 5i_x = 0$

$8i_x - 5i_s + 4\frac{di_x}{dt} = 0$



FOR $t > 0$; $i_s = 5$

$$8i_x + 4\frac{di_x}{dt} = 25$$

$$8\bar{i}_x + 4s\bar{i} - i_0 = \frac{25}{s} \qquad (i_0 = i \text{ at } t=0)$$

$$\bar{I}_x\, 4(s+1) = \frac{25}{s} + i_0$$

$$\bar{i}_x = \frac{25}{4(s+1)s} + \frac{i_0}{4(s+1)}$$

$$= \frac{25}{4s} - \frac{25}{4(s+1)} + \frac{i_0}{4(s+1)}$$

$$i_x = \frac{25}{4} + e^{\left(\frac{i_0-25}{4}\right)t}$$

$$P_L = V_L i_x = -4\left(\frac{i_0-25}{4}\right)e^{-\left(\frac{i_0-25}{4}\right)t}$$

$$P_{3\Omega} = 3i_x^2 = 3\left(\frac{25}{4} + e^{\frac{25-i_0}{4}t}\right)^2$$

$$P_{5\Omega} = 5(i_x - i_s)^2 = 5\left(\frac{-75}{4} + e^{\frac{25-i_0}{4}t}\right)^2$$

$$\underset{t>0}{P(t)} = (25-i_0)e^{\frac{25-i_0}{4}t} + 3\left(\frac{25}{4} + e^{\frac{25-i_0}{4}t}\right)^2 + 5\left(\frac{-75}{4} + e^{\frac{25-i_0}{4}t}\right)^2$$

IX-1) a) $F = \frac{q_1 q_2}{4\pi \varepsilon_0 r^2}$

$= \frac{(10^{-13})(1.06\times10^{-9})(9\times10^{9})}{10^{-12}}$

$=$

b) $E = 3\times10^{6}\ \frac{V}{M}$

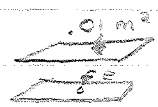
$F = Eq = (3\times10^{6})(1.06\times10^{-19})$

$=$

c) $E = \frac{Q}{A\varepsilon_0}$; $F = eQ$

$\therefore F = \frac{(10^{-6})(1.06\times10^{-19})}{(8.85\times10^{-12})(.1)}$

$=$

X-2) 

$Q = 10^{-6}\,c$ a) $E = \frac{Q}{\varepsilon_0 A} \Rightarrow F = \frac{q_e Q}{\varepsilon_0 A} = ma$

$a_e = \frac{q_e Q}{\varepsilon_0 A m_e}$

$x = \frac{1}{2}at^2$; $v = at \rightarrow x = \frac{v^2}{2a}$

$\therefore x = \frac{v^2 \varepsilon_0 A m_e}{2 q_e Q}$

$= \frac{(5\times10^{7})(8.85\times10^{-12})(10^{-2})(9.11\times10^{-31})}{2(1.06\times10^{-19})(10^{-6})}$

$=$

b) $K = 10^{-15}\ J = \frac{1}{2} m_e v^2$

$v = \sqrt{\frac{2K}{m_e}}$

$x = \frac{v^2 \varepsilon_0 A m_e}{2 q_e Q}$

$= \frac{2K \varepsilon_0 A m_e}{2 m_e q_e Q}$

$= \frac{K \varepsilon_0 A}{q_e Q} = \frac{(10^{-15})(8.85\times10^{-12})(10^{-2})}{(1.06\times10^{-19})(10^{-6})}$

$=$

c) $t = 10^{-12}$

$a_e = \frac{q_e Q}{\varepsilon_0 A m_e}$

$x = \frac{1}{2}at^2$; $v = at \Rightarrow a = \frac{2x}{t^2}$

$\frac{2x}{t^2} = \frac{q_e Q}{\varepsilon_0 A m_e}$

$x = \frac{q_e Q t^2}{2 \varepsilon_0 A m_e}$

$= \frac{(1.06\times10^{-19})(10^{-6})(10^{-24})}{2(8.85\times10^{-12})(10^{-2})(9.11\times10^{-31})}$

$=$

XI-6) a) $q = .02 \cos(100t - 1.5\pi)$

$\frac{dq}{dt} = i = -2.\sin(100t\ 1.5\pi)$

at $t=0$

$i = -2\sin(-1.5\pi) = -2.0$ AMPS

b) $\left(4 \times 10^{20} e^{-.2t}\ \text{elec}\right)\left(\frac{1.6 \times 10^{-19}\ \text{coul}}{\text{elec}}\right)$

$q = 64\ e^{-.2t}$

$\frac{dq}{dt} = i = 12.8$ AMPS at $t=0$

c) $V = 10t - 3$; $W = Vi$

$i = \frac{4}{10t-3}$

at $t=0$, $i = -1.33$ AMPS

XI-7) $P(=W) = Vi$

a) -7.79

b) -8.19

c) 7.59

XI-8) a) $U_e = 5.93 \times 10^5 \sqrt{V_o}$

$V = \frac{U_e^2}{(5.93 \times 10^5)^2}$

$= \frac{(1.5 \times 10^6)^2}{(5.93 \times 10^5)^2}$

$= 640$ VOLTS

Pg. 67

X3-2) $40 - 15 + V_3 - V_2 = 0 \Rightarrow V_3 - V_2 = {}^-25$

$60 - V_2 - V_7 = 0 \Rightarrow V_2 + V_7 = 60$

$-15 - 12 + V_6 = 0 \Rightarrow V_6 = 27$ ✓

$27 - 40 - V_7 = 0 \Rightarrow V_7 = {}^-13$ ✓

$60 + 13 + V_2 = 0 \Rightarrow V_2 = 73$ ✓

X3-3) $i_3 = 1$ A

$i_1 - 2 + 8 + 1 = 0 \Rightarrow i_1 = 7$ A

$i_8 = {}^-7$

Pg. 79

X3-7) ${}^-11 + .2i + V_s = 0$

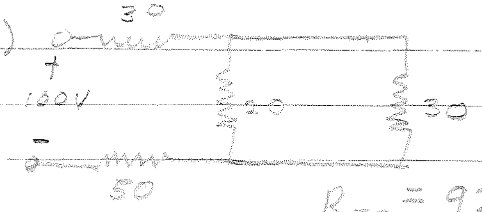
${}^-V_s + .1(5 - i) + 12 = 0$

$V_s = 11 - .2i = {}^-12 + .1(i - 5)$

$11 - .2i = {}^-12 + .1i - .5$

$23.5 = .3i \Rightarrow i = 6$

X4.3) a)



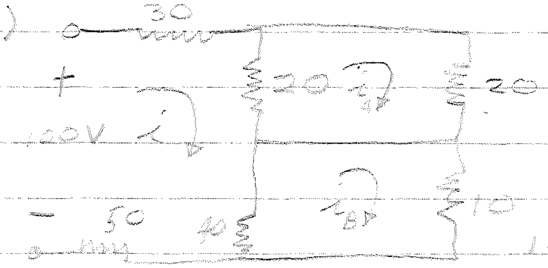
$$\frac{1}{30} + \frac{1}{20} = \frac{1}{R}$$

$$\frac{5}{60} = \frac{1}{R} \Rightarrow R = 12$$

$$R_{EQ} = 92$$

$$i = \frac{V}{R} = \frac{100}{92} = 1.09 \text{ AMPS}$$

b)



$$100 = 140i - 20i_A - 40i_B$$

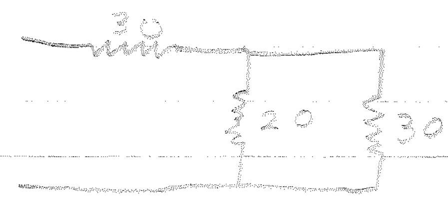
$$40i_A = 20i$$

$$50i_B = 40i$$

$$100 = 140i - 10i - 80i$$

$$50i = 100 \Rightarrow i = 2 \text{ AMPS}$$

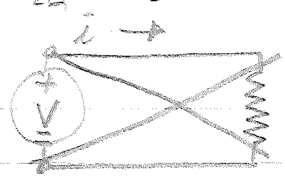
c)

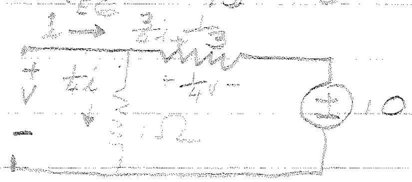


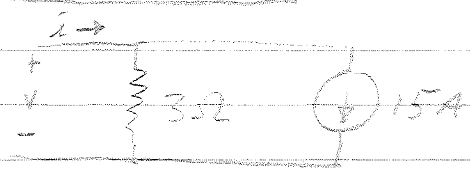
$$i = \frac{V}{R} = \frac{100}{42} = 2.38 \text{ A}$$

X4.4) $I_{EQ} = -15$ $V_{EQ} = 10$ $R_{EQ} = -\frac{10}{15} = \frac{1}{3}$

a)

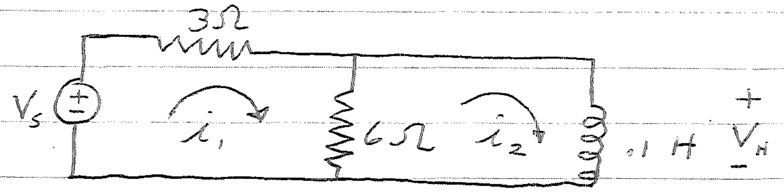




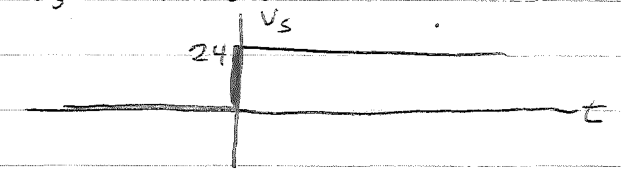


Pg 259

7)



$V_s = 24\,u(t)$



$$-V_s + 9i_1 - 6i_2 = 0 \Rightarrow i_1 = \frac{6i_2 + V_s}{9}$$

$$6i_2 + L\frac{di_2}{dt} = 6i_1 \Rightarrow 6i_2 + .01\frac{di_2}{dt} = \frac{12i_2 + 2V_s}{3}$$

AT $t > 0$

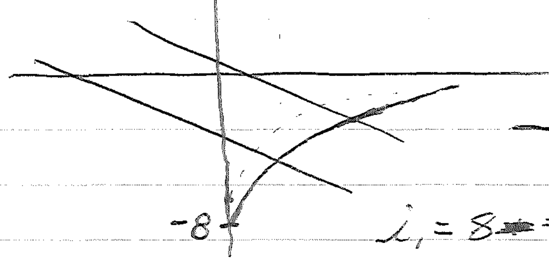
$$2i_2 + .01\frac{di_2}{dt} - 16 = 0$$

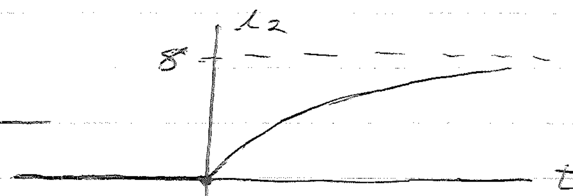
$$2\bar{I} + .01 s\bar{I} = \frac{16}{s}$$

$$\bar{I} = \frac{16}{s(.01s+2)} = \frac{16}{.01\,s(s+200)} \doteq \frac{1600}{s(s+200)}$$

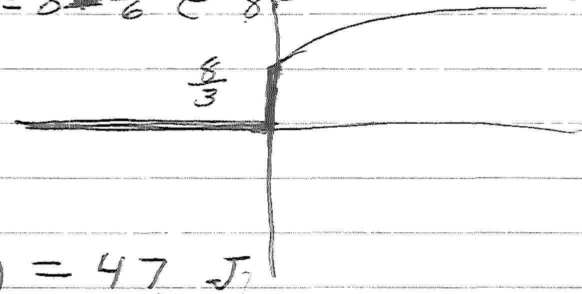
$$= \frac{8}{s} - \frac{8}{s+200}$$

$$i_2 = 8 - 8e^{-200t}$$





$$i_1 = 8 - \frac{32}{6}e^{-200t}$$



$E = Pt$

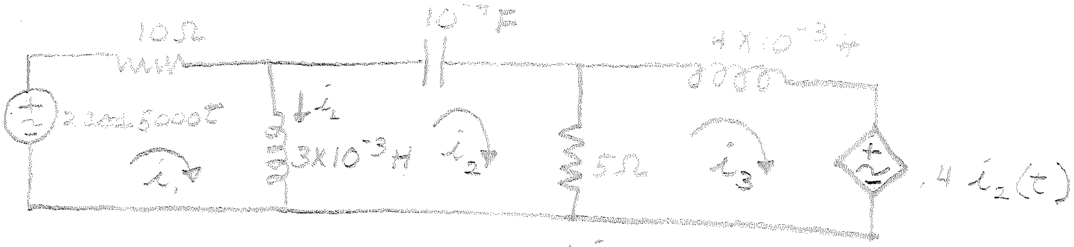
$E_{3\Omega} \cong \left(\frac{8}{3}\right)^2 3(2.2) = 47\ J$

$E_{6\Omega} \cong \left(\frac{8}{3}\right)^2 6\ (2.2) = 94\ J$

$E_{V_s} \cong V_s i_1 \cong (24)\left(\frac{8}{3}\right) \cong 64\ J$

$E_{IND} = V_H i_2 \cong 0$

5) a)



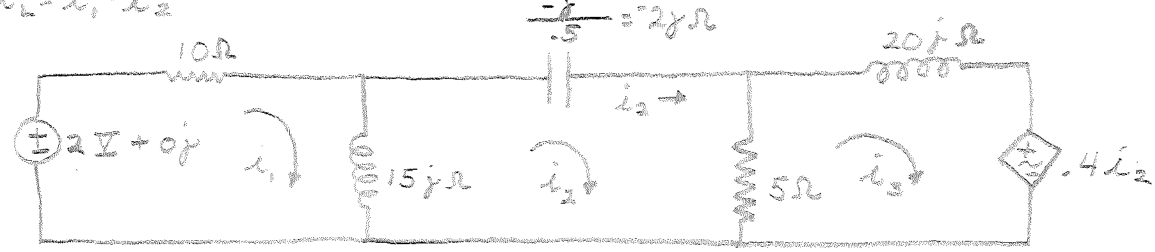
$$-2\cos 5000t + 10i_1 + 3\times10^{-3}\frac{di_L}{dt} = 0$$

$$-3\times10^{-3}\frac{di_L}{dt} + 10^4\int i_2(t)\,dt + 5i_2 - 5i_3 = 0$$

$$5i_3 - 5i_2 + 4\times10^{-3}\frac{di_3}{dt} + 4i_2 = 0$$

$$i_L = i_1 - i_2$$

b)



$$2 = 10i_1 + 15ji_1 - 15ji_2$$

$$15ji_2 - 15ji_1 - 2ji_2 + 5i_2 - 5i_3 = 0$$

$$5i_3 - 5i_2 + 20ji_3 + .4i_2 = 0$$

TABLE OF LAPLACE TRANSFORMS

f (t)	F (s)
$f(t)$	$F(s) = \int_0^{\infty} e^{-st} f(t)\, dt$
$A f_1(t) + B f_2(t)$	$A F_1(s) + B F_2(s)$
unit impulse $\delta(t)$	1
unit step $u(t)$	$1/s$
A	A/s
t	$1/s^2$
t^n	$n!/s^{n+1}$
$d/dt\,[f(t)]$	$sF(s) - f(0+)$
$d^2/dt^2\,[f(t)]$	$s^2F(s) - sf(0+) - f'(0+)$
$d^n/dt^n\,[f(t)]$	$s^nF(s) - s^{n-1} f(0+) - s^{n-2} f'(0+) - \dots - f^{(n-1)}(0+)$
$\int_a^t f(t)\, dt$	$[F(s) - \int_0^a f(t)dt\,] / s$
$f(t-a)$ $[f(t-a)=0,\ t<a]$	$e^{-as} F(s)$
$e^{-at} f(t)$	$F(s+a)$
$f(t/a)$	$aF(as)$
$\text{Lim}\,[f(t)]\ ;\ t \to 0$	$\text{Lim}\,[sF(s)]\ ;\ s \to \infty$
$\text{Lim}\,[f(t)]\ ;\ t \to \infty$	$\text{Lim}\,[sF(s)]\ ;\ s \to 0$
e^{-at}	$1/(s+a)$
$t\, e^{-at}$	$1/(s+a)^2$
$t^{n-1} e^{-at} / (n-1)!$	$1/(s+a)^n$

$\sin wt$	$w/(s^2+w^2)$
$\cos wt$	$s/(s^2+w^2)$
$e^{-at}\sin wt$	$w/[(s+a)^2+w^2]$
$e^{-at}\cos wt$	$(s+a)/[(s+a)^2+w^2]$
$\dfrac{1}{T\sqrt{1-\zeta^2}}\, e^{-\zeta t/T} \sin[\sqrt{1-\zeta^2}\,(\frac{t}{T})]$; $0<\zeta<1$	$\dfrac{1}{T^2s^2+2\zeta Ts+1}$
$1+\dfrac{e^{-\zeta t/T}\sin[\sqrt{1-\zeta^2}(\frac{t}{T})-\phi]}{\sqrt{1-\zeta^2}}$; $\phi=\tan^{-1}[\sqrt{1-\zeta^2}/-\zeta]$; $0<\zeta<1$	$\dfrac{1}{s(T^2s^2+2\zeta Ts+1)}$
$A\cos(wt+\phi)$; $A=\sqrt{T^2+(1/w)^2}$; $\phi=\tan^{-1}[-1/wT]$	$\dfrac{Ts+1}{s^2+w^2}$

$\frac{82}{100} = 82\%$

ROSE POLYTECHNIC INSTITUTE

EE 201
Winter 1970
NEM

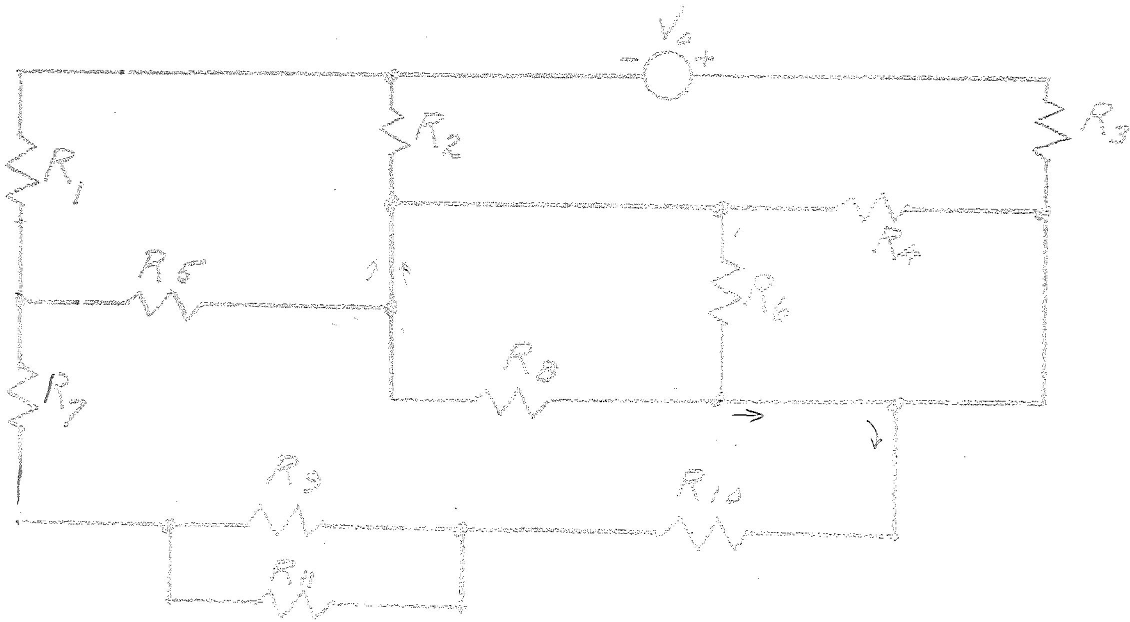
EXAM I

BOB MARKS
Name
I.D. #8069

(20 pts) 1. Fill in the following table by placing an "X" in the appropriate column for each combination of elements.

-2

ELEMENTS		SERIES	PARALLEL	NEITHER
R_1	R_2			X
V_0	R_3	X		
R_5	R_8			X
R_4	R_8		X	~~X~~
R_6	R_8		X	~~X~~
R_7	R_9			X
R_9	R_{10}			X
R_9	R_{11}		X	
R_{10}	R_7	X		



~~(46 pts)~~ 2. Suppose you are the manager of a baseball team where all the players are electrons. The score is tied when your leadoff batter raps a single into short left (magnetic) field. The next batter sacrificed, moving the runner to second base. You are trying to decide whether to call for a steal or a hit-and-run. From previous observations you know that the catcher can throw to third base in 17 ns. (1 ns = 10^{-9} sec.) If the voltage at third base is +300 v with respect to second base how long will it take a runner to reach third?

DISTANCE BETWEEN BASES 90mm

Is it wise to call for a steal?

−6

+ V −

3RD 2nd

$\longleftarrow d \longrightarrow$

$d = 90 \times 10^{-3}\, m$

$V = 300\ V$

$V = EL$ (3)

$E = \frac{\cancel{300 \times 10^{2}}}{9.00 \times 10^{-3}} = \frac{1}{3} \times 10^{5} = 3.33 \times 10^{4}$

$F = Eq = (3.33 \times 10^{4})(\cancel{1.06}\,1.60 \times 10^{-19})$ −3

$= 3.53 \times 10^{-15}$ Nt.

$F = ma$

$a = \frac{3.53 \times 10^{-15}}{9.11 \times 10^{-31}} = .387 \times 10^{16} \frac{m}{SEC^2}$

$x = \frac{1}{2} a t^{2}$

$t = \sqrt{\frac{2x}{a}} = \left(\frac{2(9.0 \times 10^{-2})}{3.87 \times 10^{15}}\right)^{\frac{1}{2}} = \left(\frac{1.8}{3.87}\right)^{\frac{1}{2}} \times 10^{-8}$

$= \sqrt{.464} \times 10^{-8}$

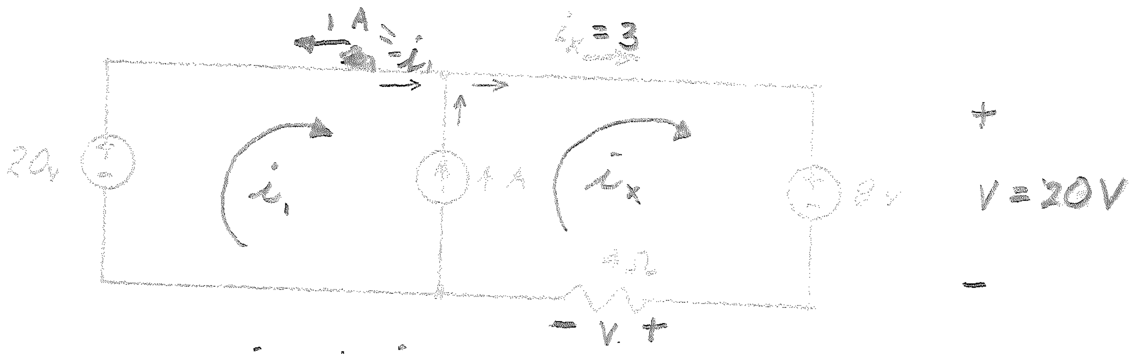
$t = 6.82 \times 10^{-8}$ SEC X

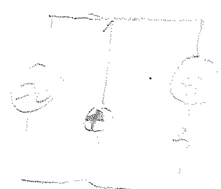
NO

EE 20[illegible] [illegible] M 1 [illegible]

(40 pts) 3. Find i_x <u>and</u> the power delivered or absorbed (state which) by the 20 volt source.

−10





$$i_1 + 4 = i_x$$

$$\cancel{20 + 8 + 4 i_x = 0}$$

$$4 i_x = (20 - 8)$$

$$i_x = 3 \text{ AMPS} \rightarrow i_1 = {}^{-}1 \text{ AMP}$$

$P = iV$

$$\boxed{P_{20V} = (1)(20) = {}^{+}20 \text{ WATTS}} -$$

$$P_{4A} = -iV = {}^{-}(20)(4) = {}^{-}80 \text{ WATTS} -1$$

$$P_{8V} = (8)(3) = {}^{+}24 \text{ WATTS}$$

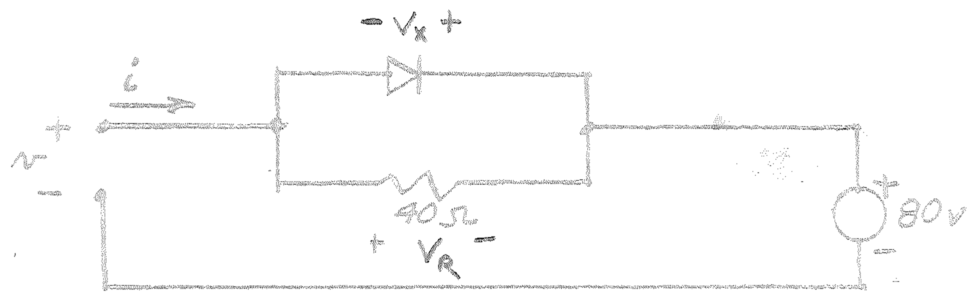
$$P_{4\Omega} = i^2 R = (3)^2(4) = {}^{+}36 \text{ WATTS}$$

(+ INDICATES DELIVERED)
(− INDICATES ABSORBED) } Wrong Conclusions

−10

(40 pts) 4. Sketch and label the i-v curve for the following circuit. The diode is ideal.

10



$i<0 \rightarrow$ DIODE IS OPEN CIRCUIT

$V = V_1 + 40i$

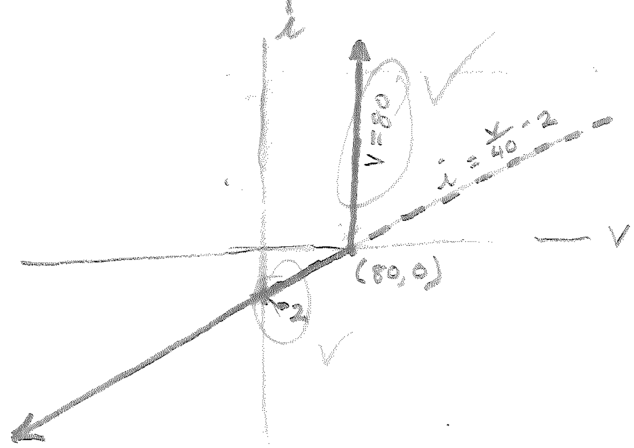
$= 80 + 40i$

$i = \frac{V}{40} - 2$

$i>0 \rightarrow V_x = 0 \rightarrow$ DIODE IS SHORT CIRCUIT

$V_R = 0$

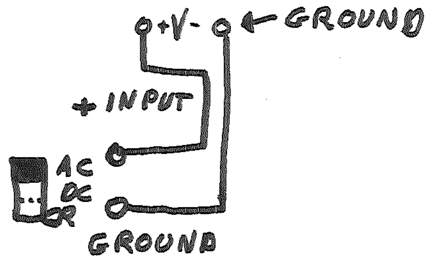
$\therefore\ V = 80\,V$



EE 201, EXAM 1 (cont'd)

(20 pts) 5. What settings on a type 503 oscilloscope should be used to display 3 cycles of normal line voltage (nominal value 115v, RMS) ?

115 V A.C.



SENSITIVITY: 20 $\frac{\text{VOLTS}}{\text{CM}}$

HORIZONAL DISPLAY: NORMAL

TIME SWEEP: 5 m SEC cm

SLOPE = + ((ARBITRARY)

COUPLING - AC

SOURCE - LINE

LEVEL - AUTO

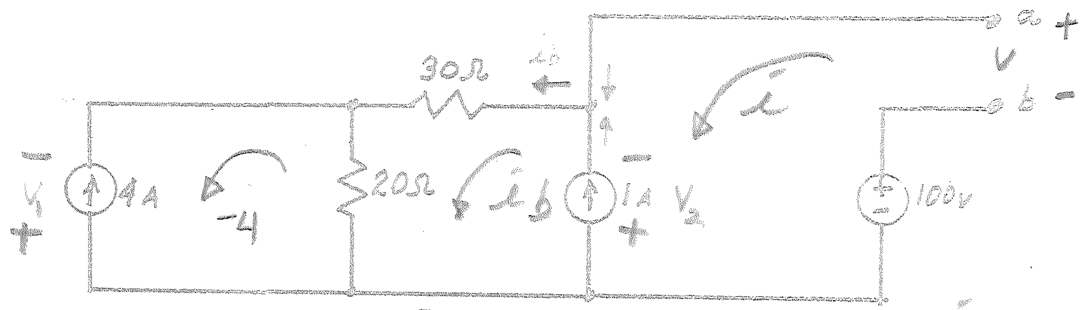
$\frac{184}{200} = 92\%$

ROSE POLYTECHNIC INSTITUTE

EE 201
Winter 1970
NEM

EXAM 2 BOB MARKS
Name

(60 pts) 1. Find either the Thevenin or the Norton equivalent circuit, viewed from terminals a & b.



$$-V_1 + 20(-4 - i_b) = 0 \Rightarrow V_1 = 20(4 + i_b)$$

$$V_1 = 80 + 20 i_b$$

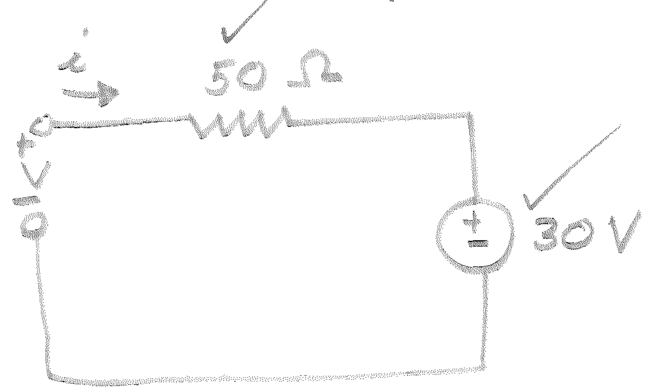
$$50 i_b + 4(20) + V_2 = 0 \Rightarrow V_2 = -50 i_b - 80$$

$$+V_2 + 100 + V = 0 \Rightarrow V = -V_2 - 100$$

$$V = 50 i_b - 20$$

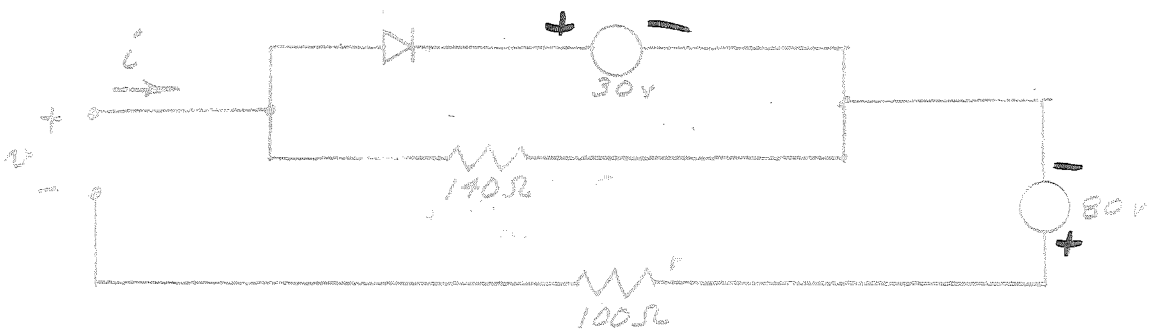
$$i + 1 = i_b \Rightarrow V = 50(i+1) - 20$$

$$V = 50i + 30$$



EE 201, EXAM 2 (cont'd)

-15 (40 pts) 3. Sketch and label the i-v curve for the following circuit. The diode is ideal.



−5

~~$i < 0$~~ $140i - 80 + 100i =$

$240i = V + 80$

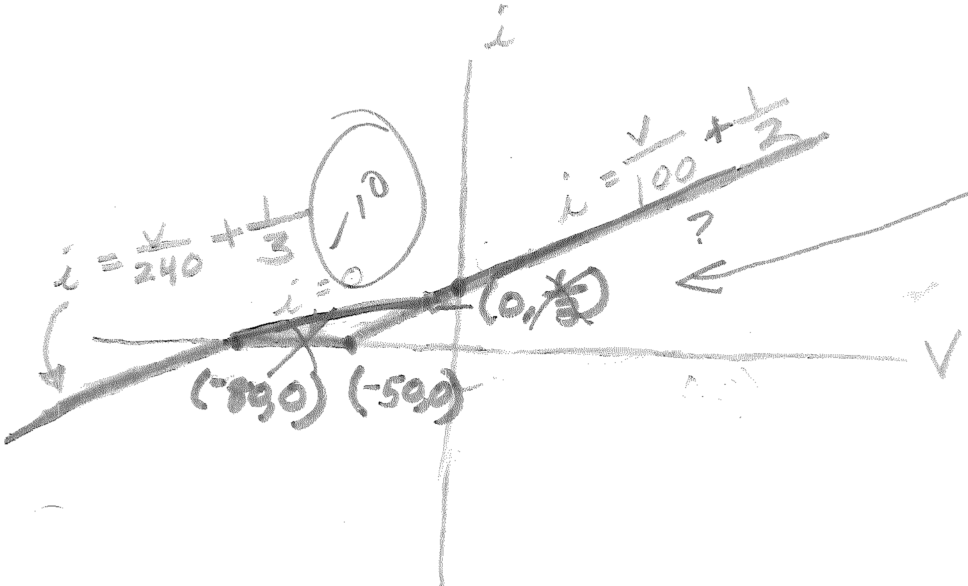
$i = \frac{V}{240} + \frac{1}{3}$

~~$i > 0$~~ ⇒ DIODE IS SHORT CIRCUIT

$(30 - 80 + 100i) = V$

$100i = V + 50$

$i = \frac{V}{100} + \frac{1}{2}$



$V_1 - 50 + 100 i_1 = 0 \Rightarrow i_1 = \frac{50 - V_1}{100}$

(BROWN) $30 i_3 = V_1 \Rightarrow i_3 = \frac{V_1}{30}$

(OARNCE) $-V_2 + 3V_1 - V_1 = 0 \Rightarrow V_2 = 2V_1$

$V_2 = (3 - i_2)20 \Rightarrow i_2 = \frac{60 - V_2}{20}$

-1

Σi AROUND $N = 0$

a) $\Rightarrow i_3 + i_2 - i_1 = 3$

$$\frac{V_1}{30} + \frac{60 - V_2}{20} + \frac{V_1 - 50}{100} = 3$$

$$\frac{V_1}{30} + \cancel{3} - \frac{V_1}{10} + \frac{V_1}{100} - \frac{1}{2} = \cancel{3}$$

$$\frac{10V_1 - 30V_1 + 3V_1}{300} = \frac{1}{2}$$

$$\frac{-17 V_1}{300} = \frac{1}{2} \Rightarrow V = \frac{750}{17} = -8.82\ V$$

b) $V_2 = 2V_1 = 2(-8.82) = -17.64\ V = V_2$

c) $30 i_3 = V_1 = -8.82 \Rightarrow i_3 = -.294$ AMPS $= i_3$

d) $i_1 = \frac{50 - V_1}{100} = \frac{58.8}{100} = .588$ AMPS $= i_1$

e) $i_2 = \frac{60 - V_2}{20} = \frac{77.6}{20} = 3.88$ AMPS $= i_2$

$V_x + 3V_1 = 0$

$V_x = -3(8.82) = 26.5\ V$

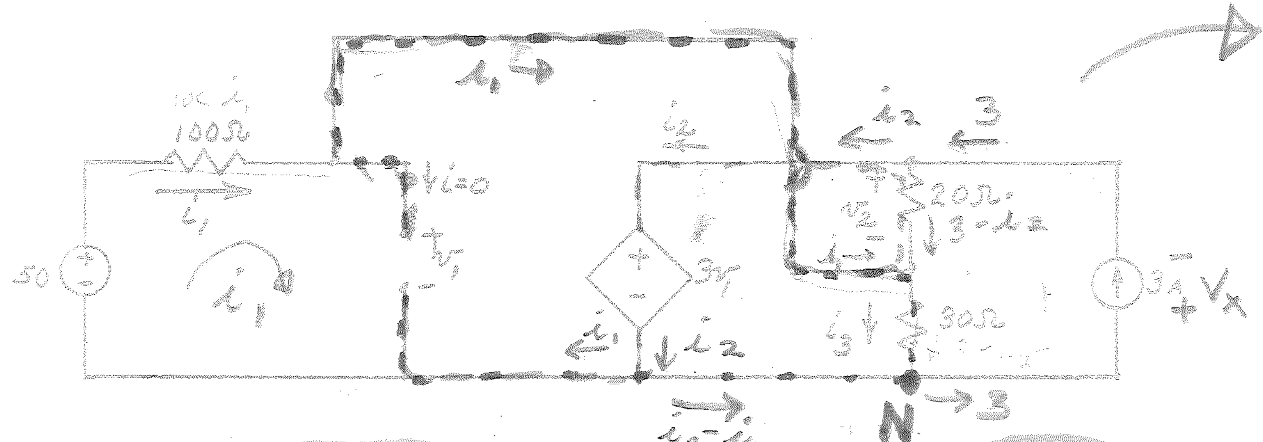
$P = Vi = (26.5)(3) = 79.5$ WATTS ABSORBED

Good

EE 201, EXAM 2 (cont'd)

-1

(120pts) 2. Find i_1, i_2, i_3, v_1, and v_2. Also find the power associated with the current source. State whether the source is absorbing or delivering power.



1) $-50 + 100 i_1 + V_1 = 0$

5) $3V_1 + V_x = 0$

$V_x = -3V_1$

2) $30 i_3 - V_1 = 0$

7) $i_3 = i_1 - i_2 + 3$

3) $3V_1 - 30 i_3 - V_2 = 0$

$3V_1 - 30 i_3 - V_2 = 0$

6) $V_2 + 30 i_3 + V_x = 0$

4) $V_2 = (3 - i_2)20 = 60 - 20 i_2$

2) $V_1 = 30 i_3 \Rightarrow 3V_1 = 90 i_3$

3) $90 i_3 - 30 i_3 - V_2 = 0$

$60 i_3 = V_2 = 60 - 20 i_2$

$6 i_3 = 6 - 2 i_2$

$6 i_3 = 6 i_1 - 6 i_2 + 18$

$-12 - 6 i_1 + 4 i_2 = 0$

$3V_1 - 30 i_3 - V_2$

$60 i_3 - V_2 = 0$

messy

114/140 81 70

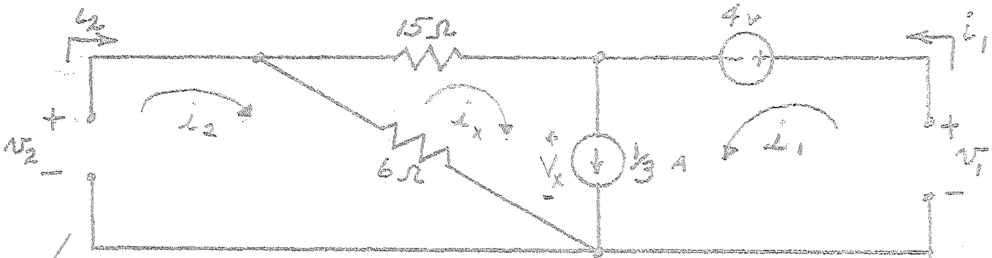
ROSE POLYTECHNIC INSTITUTE

EE 201
Winter 1970
NEM

EXAM 3 Bob Marks
Name

-21

(60 pts) 1. Find the three terminal π equivalent of the following network. Express your answer both as a set of equations and as a circuit. In the circuit, use resistances rather than conductances.



$V_2 = 6i_2 - 6i_x$ $i_x = \frac{1}{3} - i_1$

$21 i_x + V_x - 6 i_2 = 0$

$V_1 = 4 + V_x$

$V_2 = 6i_2 - 6i_x \Rightarrow \frac{V_2}{6} = i_2 - i_x$ (A)

$21 i_x + V_x = 6 i_2 \Rightarrow i_x = \frac{6i_2 - V_x}{21}$

$V_1 = 4 + V_x \Rightarrow \frac{6i_2 + 4 - V_1}{21}$

FROM (A): $\frac{V_2}{6} = i_2 - \frac{6i_2 + 4 - V_1}{21}$

$= i_2 - \frac{6i_2}{21} - \frac{4}{21} + \frac{V_1}{21}$

$\frac{V_2}{6} = \frac{15 i_2}{21} - \frac{4}{21} + \frac{V_1}{21}$

$\frac{15 i_2}{21} = \frac{-V_1}{21} + \frac{V_2}{6} + \frac{4}{21}$

$15 i_2 = -V + \frac{V_2 21}{6} + 4$

$i_2 = \frac{-V_1}{15} + \frac{7}{30} V_2 + \frac{4}{15}$

FROM (A)

$\frac{V_2}{6} = i_2 - i_x \Rightarrow i_x = i_2 - \frac{V_2}{6}$

ALSO: $i_x = \frac{1}{3} - i_1$

$\therefore i_2 - \frac{V_2}{6} = \frac{1}{3} - i_1$

OR $i_2 = \frac{1}{3} - i_1 + \frac{V_2}{6}$

$i_2 = \frac{-V_1}{15} + \frac{7}{30} V_2 + \frac{4}{15}$

$\therefore \frac{1}{3} - i_1 + \frac{V_2}{6} = \frac{-V_1}{15} + \frac{7}{30} V_2 + \frac{4}{15}$

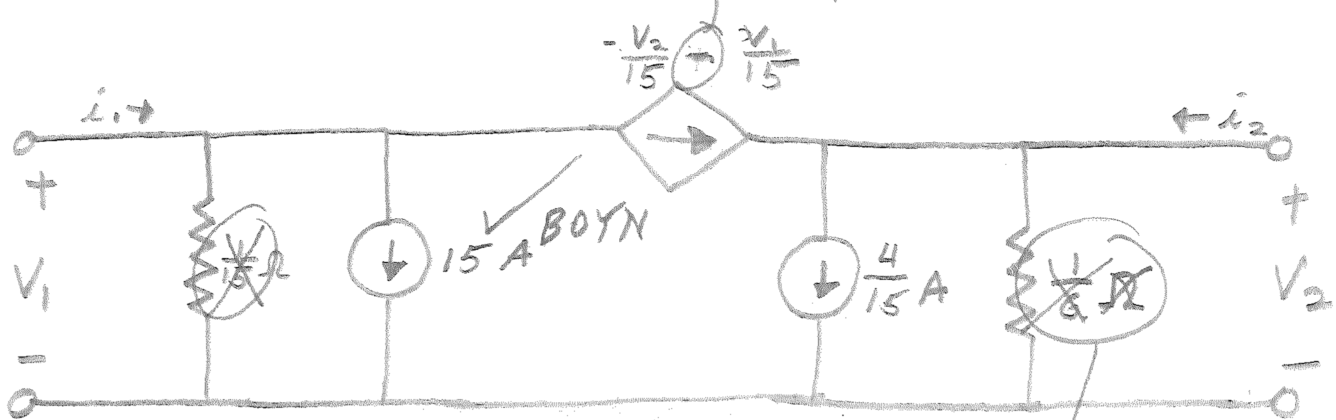
OR

$i_1 = \frac{V_1}{15} - \frac{V_2}{15} + \frac{1}{15}$

-3

-3

sign



THIS CIRCUIT HAS RECIPROCITY

not ~~resistance~~ -15

EE 201, EXAM 3 (cont'd)

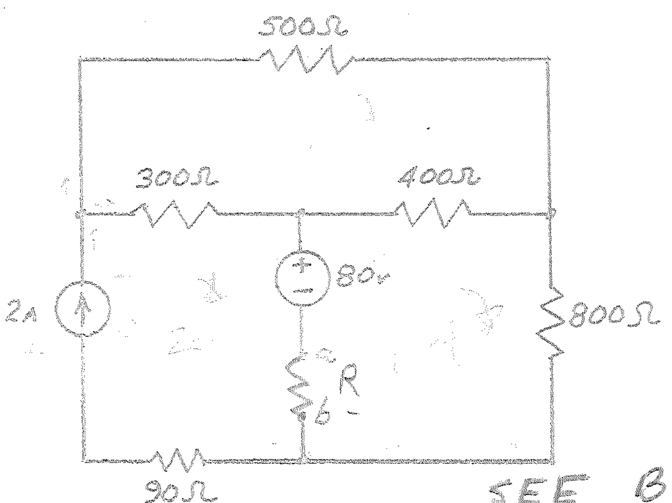
05

(80 pts) 2. A Mystery Problem! (Aren't they all!)

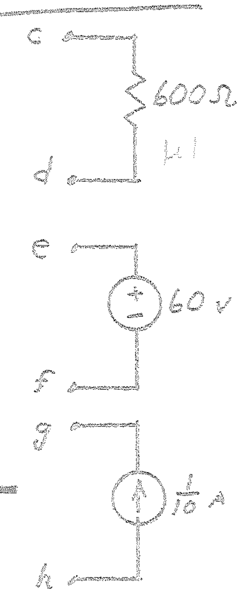
The following network has one of the listed subcircuits connected to terminals a and b.

If the power delivered to the subcircuit is 3.0 w which subcircuit is connected and how is it connected (what terminal is connected to terminal a) ?

Explain your method for solving the problem.



SUBCIRCUITS



SEE BACK OF TEST

1) PUT IN V.S.; V

$0 = -80(\pm V) + 1200 i_x - 400 i_y$

$0 = (1200 i_y - 400 i_x - 600)$

$0 = -240 \pm 3V + 3600 i_x - 1200 i_y$

$-(3200) i_x = -840 \pm 3V$

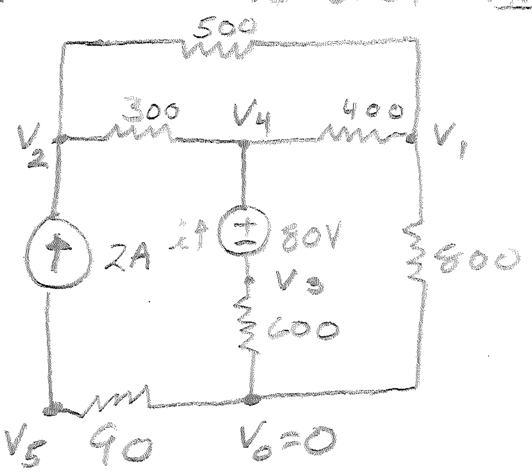
$i_x = \frac{84}{320} \pm 3V$

$|V| = 60.$

$V = +60 \Rightarrow i_x \approx 180A \Rightarrow P \neq 3$

$V = -60 \Rightarrow i_x \approx -180A \Rightarrow P \neq 3$

2) PUT IN ~~R~~ESISTORS



$$-\frac{V_3}{600} - \frac{V_1}{800} - \frac{V_5}{90} = 0$$

$$V_4 - V_3 = 80$$

$$\frac{V_2 - V_4}{300} - 2 + \frac{V_2 - V_1}{500} = 0$$

$$\cancel{Fi}\ -i + \frac{V_4 - V_1}{400} + \frac{V_4 - V_2}{300} = 0$$

$$i = \frac{V_4}{300} + \frac{V_4}{400} - \frac{V_1}{400} - \frac{V_2}{300}$$

$$\frac{V_1 - V_2}{500} + \frac{V_1 - V_4}{400} + \frac{V_1}{800} = 0$$

$$\cancel{\frac{V_4}{400}=}\quad V_1 - V_4 = -80 \qquad V_1 = V_4 - 80$$

$$\frac{V_1 - V_2}{500} + \frac{V_1}{800} = 80$$

$$\frac{3}{3}\frac{V_2}{500} = \frac{V_1}{500} + \frac{V_1}{800} - 80$$

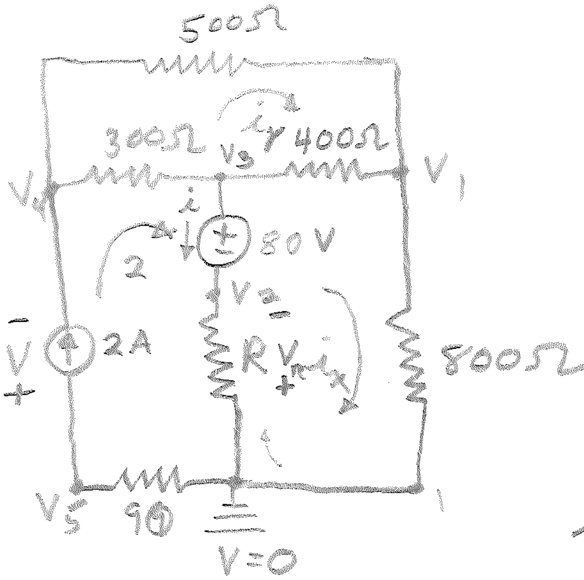
$$\frac{V_2}{300} = \frac{V_1}{300} + \frac{5V_1}{2400} - \frac{400}{3}$$

$$\frac{400}{3} + i = \frac{V_4}{300} + \frac{V_4}{400} - \left(\frac{V_1}{400} + \frac{V_1}{300} + \frac{5V_1}{2400}\right)$$

$$i = \cancel{\frac{V_4}{300}} + \cancel{\frac{V_4}{400}} - \cancel{\frac{V_4}{400}} + \frac{80}{400} - \cancel{\frac{V_4}{300}} + \frac{80}{300} - \frac{5V_4}{2400} + \frac{400}{24}$$

$$i = \frac{1}{5} + \frac{8}{30} - \frac{5V_4}{2400}$$

$$= \cancel{\frac{1}{5} + \frac{8}{30} - \frac{V_4}{480}}$$



$R(i_x-2)-80+1200i_x-400i_Y=0$ ✓

$1200i_Y-400i_x-600=0$ ✓

$\cancel{3R(i_x-2)}-\cancel{240}+\cancel{3600}i_x-\cancel{1200i_Y}=0$

$3200i_x+3R(i_x-2)-840=0$

$3200i_x+3Ri_x-6R=840$

$i_x(3200+3R)=840+6R$

$$i_x=\frac{840+6R}{3200+3R}$$

$V_R=(i_x-2)R$

$$P=\left(\frac{840+6R}{3200+3R}-2\right)^2R=3$$

~~$R(i_x-2)-(80)+1200i_x-400i$~~

$i_x(3200+3R)=840+6R$

BY N.T.D.

$\cancel{\frac{V_2}{R}=i}$ $\qquad \frac{V_3-V_1}{400}+i+\frac{V_3-V_4}{300}=0$

$\cancel{R=\frac{3}{i}}$ $\qquad \frac{V_5}{90}+2=0$

$\cancel{3V_2=i^3}$ $\qquad \frac{V_4-V_3}{300}-2+\frac{V_4-V_1}{500}=0$

−5

Incomplete

so wot!

$\frac{107}{130} = 82\%$

ROSE POLYTECHNIC INSTITUTE

EE 201
Winter 1970
NEM

EXAM 4   BOB MARKS
Name

(10 pts) 1. Find A and B.

$$\frac{10\,s}{(s+3)(s+4)} = \frac{A}{s+3} + \frac{B}{s+4}$$

$$10s \equiv A(s+4) + B(s+3)$$

$$s = -4 \Rightarrow -40 = -B \Rightarrow B = 40$$

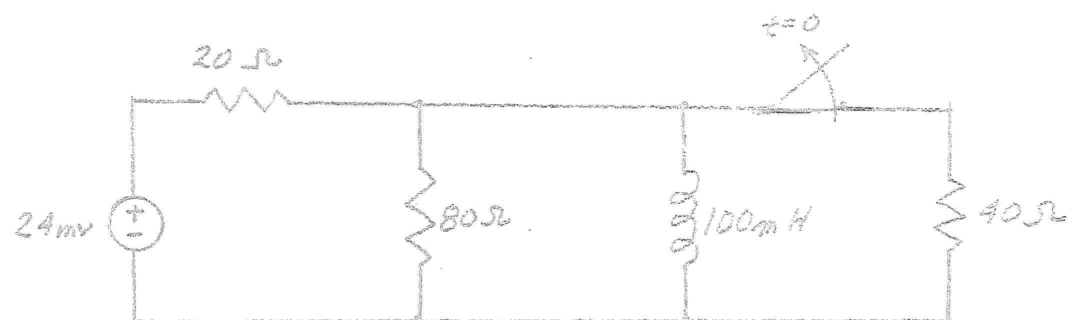
$$s = -3 \Rightarrow -30 = A$$

$$B = 40$$

$$A = -30$$

$$\left[\mathcal{L}^{-1}\left\{\frac{10s}{(s+3)(s+4)}\right\} = -30e^{-3t} + 40e^{-4t}\right]$$

-10 (20 pts) 2. What is the time constant of the following circuit?



$$\tau = \frac{L}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{80} + \cancel{\frac{1}{40}}$$

$$\frac{10}{R_{eq}} = \frac{1}{2} + \frac{1}{8} + \frac{1}{4}$$

$$= \frac{4+1+2}{8} = \frac{7}{8}$$

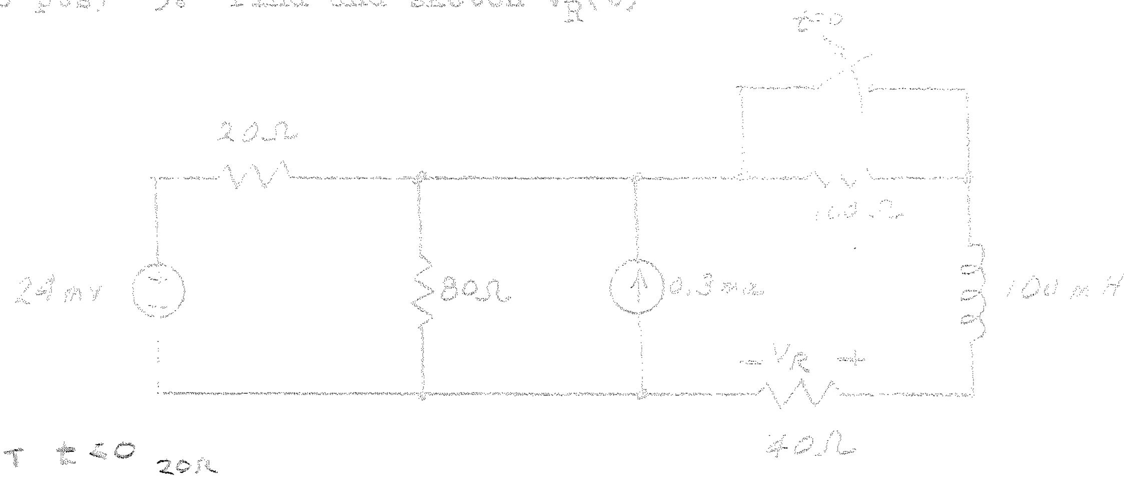
$$R_{eq} = \cancel{\frac{80}{7}}\,\Omega$$

$$\tau = \frac{(.1)\,7}{80} = \cancel{.0875}\ \text{SEC}$$

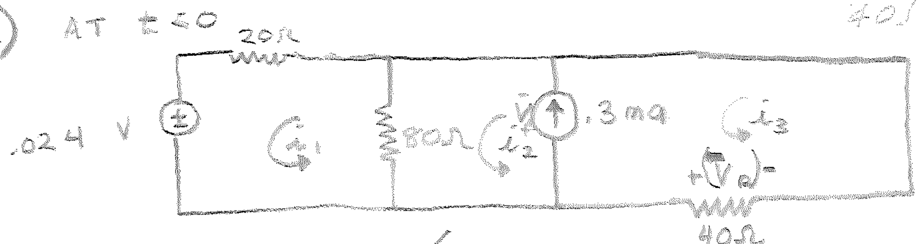
-10

40 Ω not in circuit during transient portion.

-08 (60 pts) 3. Find and sketch $v_R(t)$



I) AT $t<0$



?

1) $.024 + 100i_1 - 80i_2 = 0$
2) $80i_2 - 80i_1 + V_S = 0$
3) $-V_S + V_R = 0$
4) $40i_3 = -V_R$
5) $i_2 - i_3 = .3 \times 10^{-3}$

6) $V_S = -V_R$ (FROM 3)

7) $i_1 = \frac{-.024 + 80i_2}{100}$ (FROM 1)

8) $i_1 = \frac{80i_2 + -V_R}{80}$ (FROM 2 & 6)

9) $\frac{-.024 + 80i_2}{10} = \frac{80i_2 + -V_R}{8}$ (7 & 8)

10) $i_2 = 3 \times 10^{-4} + i_3$ (5)

11) $\frac{-.024 + 80(3 \times 10^{-4} + i_3)}{10} = \frac{80(3 \times 10^{-4} + i_3) + -V_R}{8}$ (10 & 9)

12) $i_3 = \frac{-V_R}{40}$ (4)

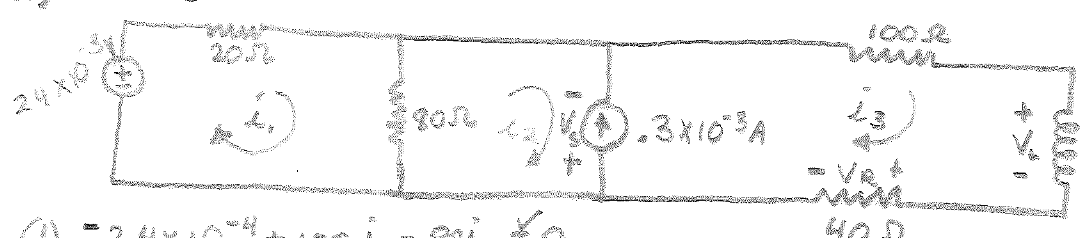
13) $\frac{-.024}{10} + \cancel{24 \times 10^{-4}} + \frac{8 V_R}{40} = 30 \times 10^{-4} + \frac{10 -V_R}{40}$ (10 & 9)

14) $\frac{2}{40} V_R = -3 \times 10^{-3}$

15) $-V_R = -60 \times 10^{-3} \Rightarrow V_R = \cancel{60} \times 10^{-3}$ VOLTS

SR -3

II) At $t>0$



Why change i's ?

1) $-2.4\times10^{-4}+100i_1-80i_2=0$

2) $-V_s+80i_2-80i_1=0$

3) $V_s+V_R+100i_3+(.10)\frac{di_3}{dt}=0$

4) $i_3-i_2=3\times10^{-4}$

5) $V_R=40i_3$

$\mathcal{L}$

6) $-\frac{2.4\times10^{-4}}{s}+100I_1-80I_2=0$

7) $-V_s+80I_2-80I_1=0$

8) $V_s+V_R+100I_3+(.10)sI_3-(.10)i_3(0+)=0$ $\left\{ i_3(0+)=\frac{V_R}{R}=\frac{60\times10^{-3}}{40}=\cancel{1.5\times10^{-3}\text{ A}}\right.$

8b) $V_s+V_R+100I_3+(.10)sI_3-1.5\times10^{-3}=0$

9) $I_3-I_2=\frac{3\times10^{-4}}{s}$

10) $V_R=40I_3$

11) $V_s=1.5\times10^{-3}-V_R-100I_3-(.10)sI_3$ (8b)

12) $V_s=80I_2-80I_1$ (7)

13) $1.5\times10^{-3}-V_R-100I_3-(.10)sI_3=80I_2-80I_1$ (11 & 12)

14) $I_1=\frac{80I_2}{100}+\frac{2.4\times10^{-4}}{s100}$ (6)

15) $1.5\times10^{-3}-V_R-100I_3-(.10)sI_3=80I_2-64I_2-\frac{1.92\times10^{-4}}{s}$ (13 & 14)

16) $I_2=I_3-\frac{3\times10^{-4}}{s}$ (9)

17) $1.5\times10^{-3}-V_R-100I_3-(.10)sI_3=16I_3-\frac{48\times10^{-4}+1.92\times10^{-4}}{s}$ (15, 16)

18) $I_3=\frac{V_R}{40}$ (10)

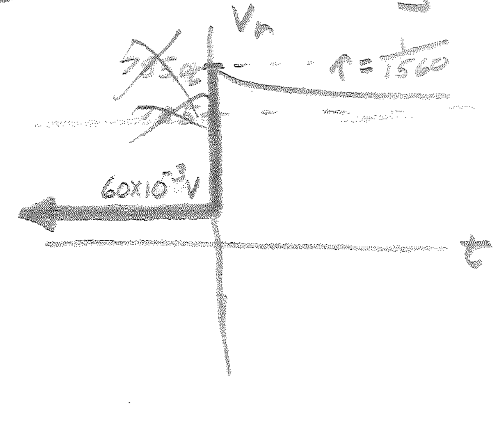
19) $1.5\times10^{-3}-V_R-\frac{100V_R}{40}-(.10)s\frac{V_R}{40}=16\frac{V_R}{40}-\frac{48\times10^{-4}+1.92\times10^{-4}}{s}$ (17, 18)

20) $V_R\left[+1+\frac{5}{2}+\frac{s}{400}+\frac{16}{40}\right]=+1.5\times10^{-3}+\frac{48\times10^{-4}+1.92\times10^{-4}}{s}$

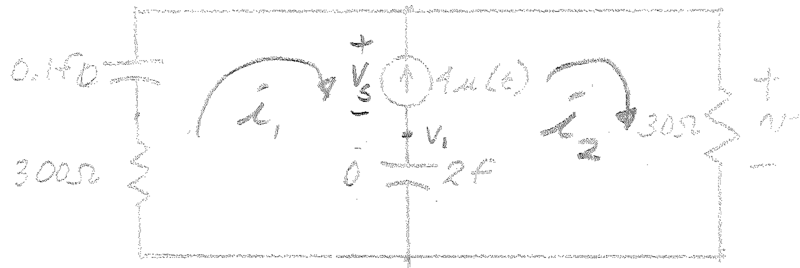
20) $V_R\left[\frac{39}{10}+\frac{s}{400}\right]=1.5\times10^{-3}+\frac{50\times10^{-4}}{s}$ -5

21) $V_R=\frac{600\times10^{-3}}{1560+s}+\frac{2}{s(1560+s)}=\frac{600\times10^{-3}+1.28\times10^{-3}}{1560+s}+\frac{\cancel{7.85}}{s}$

$V_r=.601e^{-1560t}+\cancel{7.85}$ For $t>0$



-3 (20 pts) 4. What is v(0+) ? (Initial capacitor voltages are zero.)



A CAPACITOR CANNOT HAVE AN <u>IMEDIATE</u> VOLTAGE CHANGE ACROSS IT.

SP -1

AT $t=(0+)$

$300 i_1 + V_s = 0$

$30 i_2 - V_s = 0$

$-300 i_1 = 30 i_2$

$-10 i_1 = i_2$

ALSO $i_2 - i_1 = 4$

$i_1 = i_2 - 4$

$\therefore -10(i_2 - 4) = i_2$

$-10 i_2 + 40 = i_2$

$40 = 11 i_2$

SR -2

$i_2(0+) = \frac{40}{11} = 3.53$ AMPS

$V(0+) = R\, i_2(0+)$

$= (30)(3.53)$

$= 106\,V$

REQ OF PULSE & SINOSOIDAL GENERATORS

I) PULSE GENERATOR PRF = 300 τ = 1 ms

A) AT AMP. ON FULL

V_{eq} = 50 V ; R_{eq} = 620 Low

B) AT 60%

V_{eq} = 30 V ; R_{eq} = 550

C) AT 30%

V_{eq} = 13 V R_{eq} = 240

II) FOR SINOSOIDAL, f = 30,000 HZ

A) AMPLITUDE ON FULL

V_{eq} = 6.0 V ; R_{eq} = 500 Ω

B) AT 50%

√ R_{eq} = ~~8.25~~ V ; R_{eq} = ~~180~~ Ω Nope!

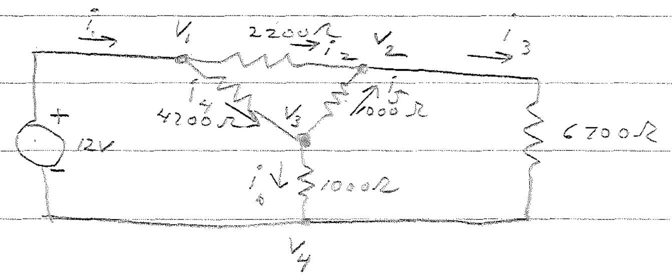
BOB MARKS

Dennis English

Mike Tatum

RUSS MAGERS

E-sci I



$i_1 = 4.1mA$	$V_1 = 12V$
$i_2 = 2.5mA$	$V_2 = 5.8V$
$i_3 = +.8mA$	$V_3 = 3.8V$
$i_6 = 3.4mA$	
$i_4 = 1.6mA$	
$i_5 = -1.6mA$	

Bob Marks
[illegible]
Dennis Engler
Russ Magers

Electrical Engineering Dept.
Rose-Hulman Institute of Tech.
Terre Haute, Indiana
February 11, 1971

EE208 Laboratory
Frequency response experiment
Uses software, not hardware.

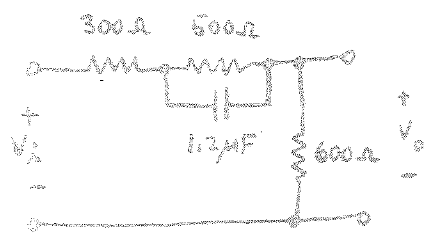
Use the computer program stored in EE204A to compute $|H(j\omega)|$ and $\angle H(j\omega)$ for the particular circuit shown below. In each case $H(s) = V_o(s)/V_i(s)$.

Plot $|H(j\omega)|$ on log-log paper as a function of f.

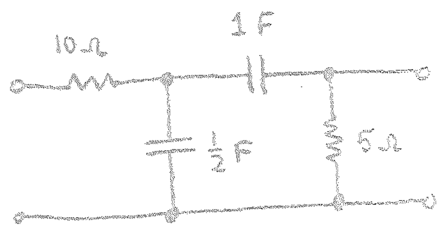
Plot $\angle H(j\omega)$ on semi logarithmic graph paper af a function of f.

Show your entire computer program to the lab instructor or his assistant by 4:00 pm Thursday, February 11, 1971, in order to receive credit for this assignment.

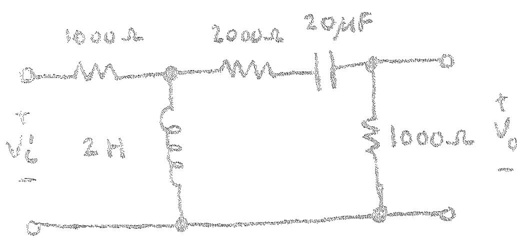
"A"



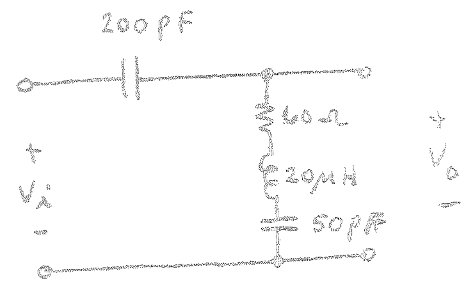
"B"



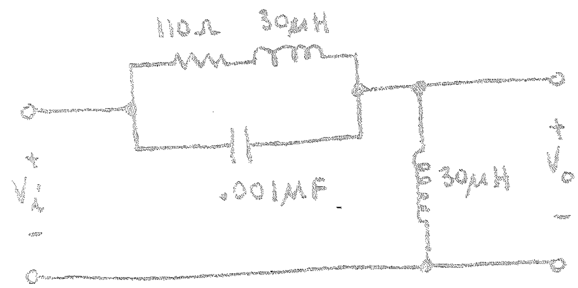
"C"



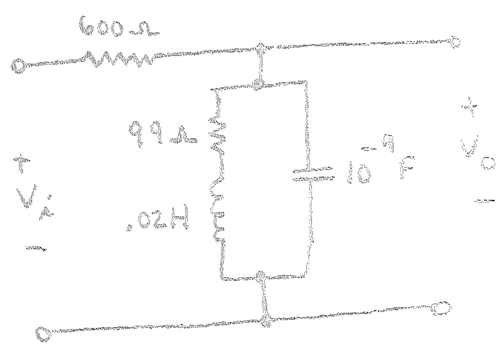
"D"



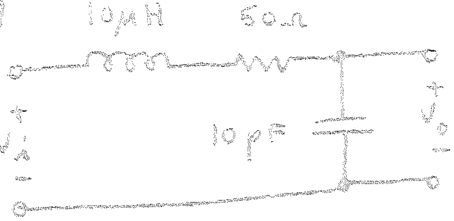
"E"



"F"

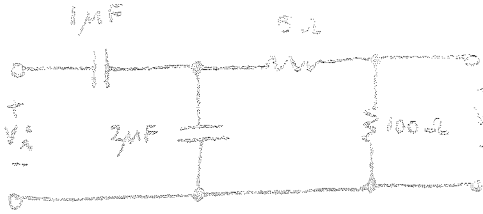


"G"



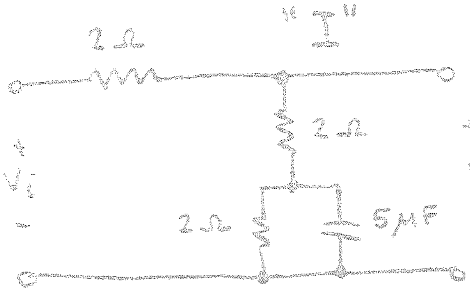
10μH
50Ω
10pF
V_i
V_o

"H"



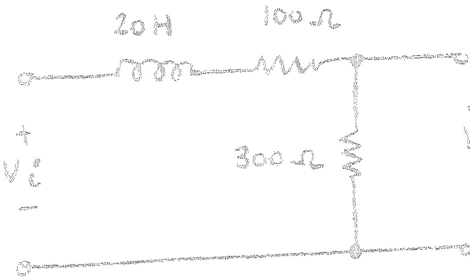
1μF
5Ω
2μF
100Ω
V_i
V_o

"I"



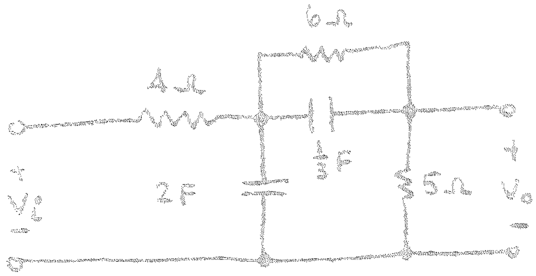
2Ω
2Ω
2Ω
5μF
V_i
V_o

"J"



20H
100Ω
300Ω
V_i
V_o

"K"



6Ω
4Ω
$\frac{1}{3}$F
2F
5Ω
V_i
V_o

[illegible]
Rose Polytechnic [illegible]
Terre Haute, [illegible]

EE204 and EE206 Laboratory

Helpful hints for happy lab
books.

1. Always keep the notebook in ink.

2. No loose pages will be allowed.

3. Always keep all lab notes in the notebook as you are doing the experiment.

4. Keep accurate records which would enable you to re[illegible] your experiment at a later date and get the same re[illegible]

5. Make sure a complete circuit diagram is included fo[illegible] measurements

6. Record instrument information and numbers. Same [illegible] used in calculations.

7. All tables should have column headings and appropr[illegible]

8. Tell what it is that you are doing preferably in [illegible] simple. When in doubt write it down.

9. All graphs should be on appropriate graph paper [illegible] attached in the notebook. Each graph should have [illegible] labelled axes, and units. The preparer of the gra[illegible] should put his name on the graph.

10. Always use straight edge or curves to draw the gra[illegible]

11. In general put all writing in the lab notebook. I[illegible] much easier to keep track of all your notes this [illegible]

12. The experiment instruction sheets are merely to a[illegible] guide in most cases. Do not "Parrot" these instru[illegible] in your notebook

Electrical Engineering Dept
Rose Polytechnic Institute
Terre Haute, Indiana

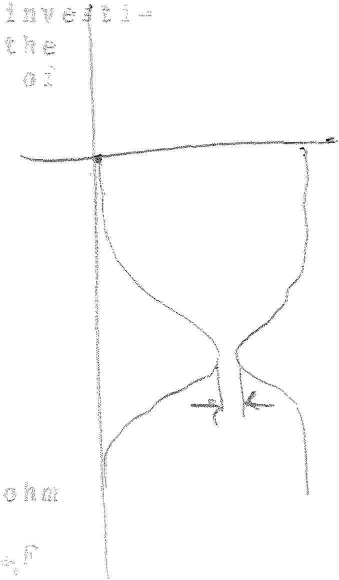
EE 208 ELECTRICAL SCIENCE LABORATORY

EXPERIMENT NO. ______

SERIES RESONANCE

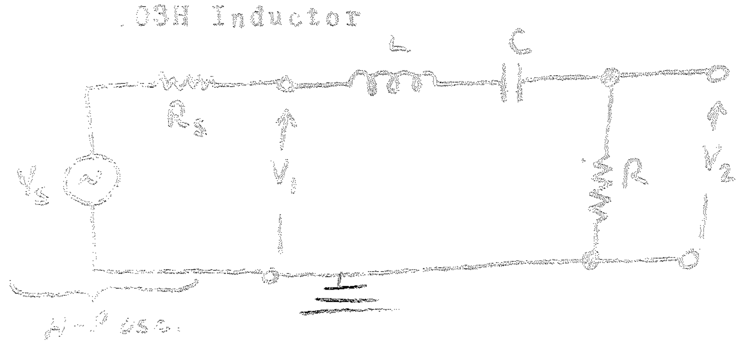
Introduction:

Resonant circuits find many practical applications in electrical engineering. In this experiment we will investigate a few of the more important characteristics of the series resonant circuit. Among these are frequency of resonance, bandwidth, and Q.



Equipment:

Audio Oscillator
Cathode-Ray Oscilloscope
Vacuum Tube Voltmeter
Laboratory Resistor Decade
Laboratory Capacitor Decade
.03H Inductor



R = 1000 ohm

C = .001 μF

L = .03 H

1. Measure the value of R_s for the H-P audio oscillator.
2. Keeping $\dot{V}_s$ constant at 10V R.M.S. plot a curves of $|\dot{V}_2|$ AND $|\dot{V}_1|$ vs. frequency which clearly shows the resonant frequency.
3. From the curve of part 2 determine the resonant frequency and the upper and lower half power frequencies.
4. Determine the bandwidth of the circuit and thus the value of Q.

5. Use the Q_o above to determine the total series resistance of the circuit. From this determine R_s and compare with result of part 1

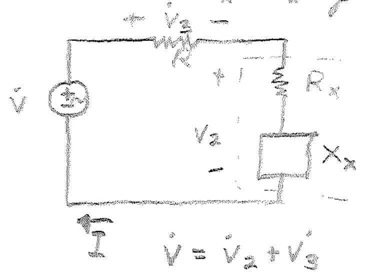
6. Connect the CRO to display $\mathring{V}_1$ on the vertical and $\mathring{V}_2$ on the horizontal axes. Describe the appearance of the Lissajons pattern as the frequency is varied through resonance.

7. Determine the frequency where the maximum voltage across the capactior occurs.

12-10-70

3 VOLTMETER METHOD OF IMPEDENCE MEASUREMENT

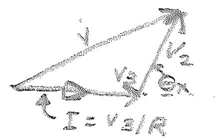
$\mathbb{Z}_x = R_x + jX_x = \mathbb{Z}_x \angle \Theta_x$



MAKE 3 VOLTAGE MEASUREMENTS

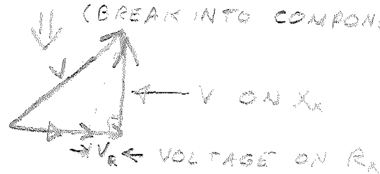
$\dot{V} = \dot{V}_2 + \dot{V}_3$

PHASOR DIAGRAM



$\therefore \mathbb{Z}_x = \frac{V_2}{I} = R_x + jX_x = \frac{V_2}{I} \angle \Theta_x$

(BREAK INTO COMPONENTS)

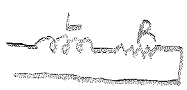


USE LAW OF COSINES TO DETERMINE Θ

$\phi = 180° - \Theta$

$V^2 = V_3^2 + V_2^2 - 2V_3V_2 \cos\phi \Rightarrow \cos\phi = \frac{V_2^2 + V_3^2 - V^2}{2V_2V_3} = -\cos\Theta$

$|\mathbb{Z}_x| = \frac{V_2}{V_3} R$

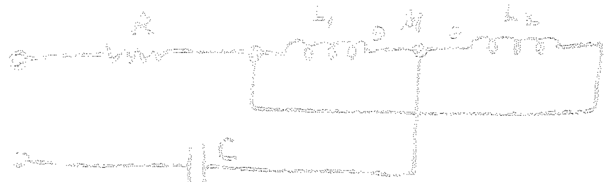


$Q = \frac{\omega L}{R}$

12-17-70

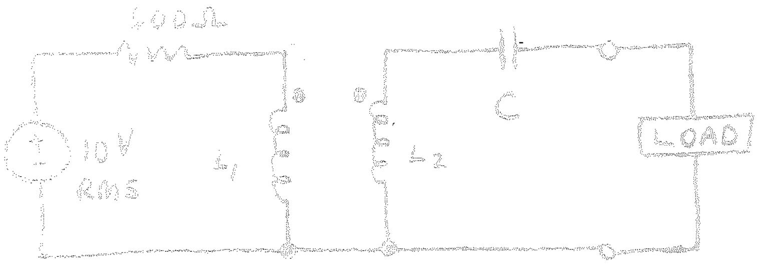
LAB PARTNERS, LN & LAWRENCE

[illegible] lower half power frequency and bandwidth for the series resonant circuit shown.



L_1 = 100 mH
L_2 = 300 mH
M = 120 mH
C = 2000 pF
R = 100 Ω

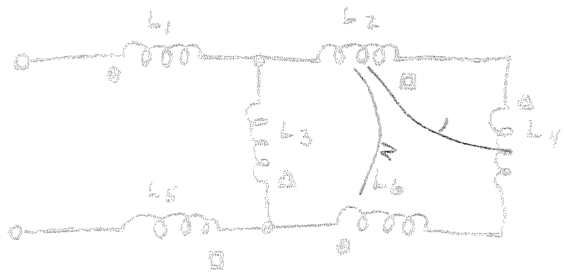
2. Find the load impedance which will extract maximum power from the source shown. How many watts is this power?



ωL_1 = 300 Ω
ωL_2 = 1000 Ω
$\frac{1}{\omega C}$ = 600 Ω
$\omega M = 500 \Omega$

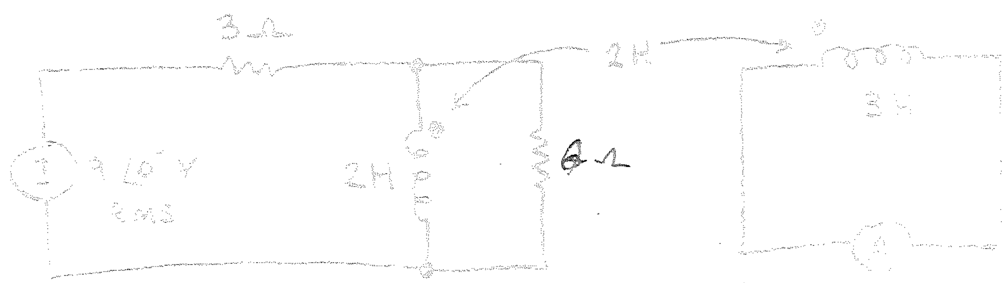
3. Two coils, L_1 and L_2, have a mutual inductance equal to M. If $M = \frac{1}{2}\sqrt{L_1 L_2}$, show and tell how you may use these two coils to obtain at least 4 different values of inductance. What are these values?

4. Use the T-equivalent circuit where possible to find the equivalent inductance of the interconnection of coils shown



L_1 = 1 H
L_2 = 2 H
L_3 = 3 H
L_4 = 4 H
L_5 = 5 H
L_6 = 6 H
[illegible]

5. Find the reading of the ideal a-c ammeter.

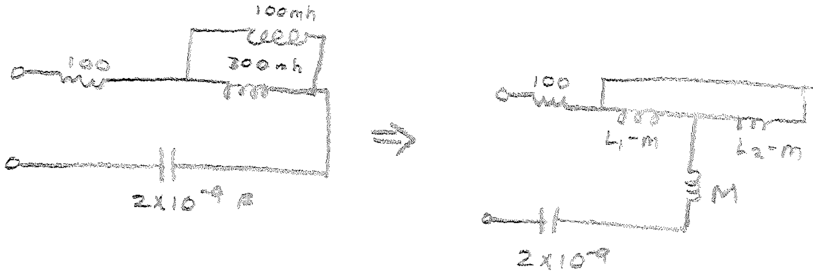


[illegible]

BOB MARKS

2

1)



$$L_{eq} = \frac{1}{\frac{1}{180} - \frac{1}{20}} + 120 = 97.5\ mh = .0975\ H$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 7.16\times10^{4}\ \text{RAD/SEC}$$

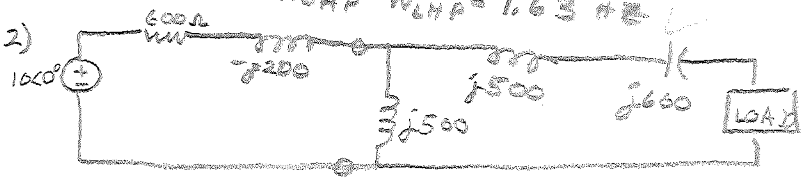
$$f = 1[illegible]\times10^{4}\ \text{HZ}$$

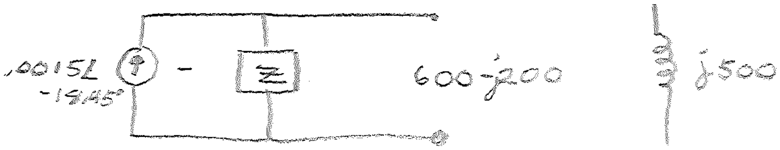
$$W_{UHP} = \omega_0\sqrt{1+\frac{1}{4Q^2}} + \frac{\omega_0}{2Q} = 7.21\times10^{4}\ \frac{\text{RAD}}{\text{SEC}} = 1.15\times10^{4}\ \text{HZ}$$

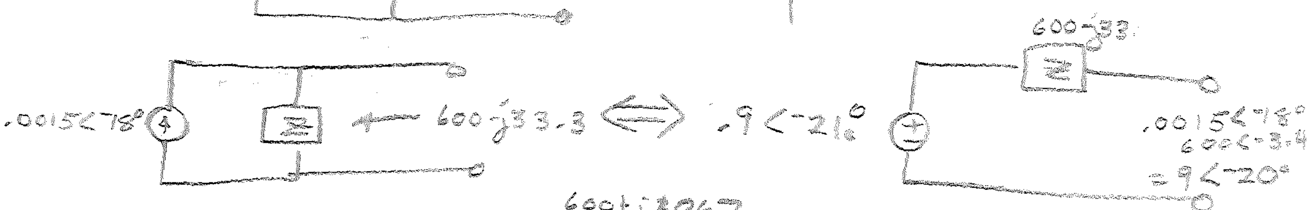
$$W_{LHP} = \omega_0\sqrt{1+\frac{1}{4Q^2}} - \frac{\omega_0}{2Q} = 7.11\times10^{4}\ \frac{\text{RAD}}{\text{SEC}} = 1.13\times10^{4}\ \text{HZ}$$

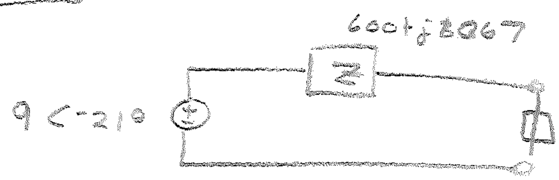
$$BW = W_{UHP} - W_{LHP} = 1.63\ \text{HZ}$$

2)





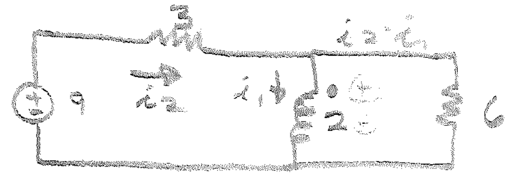


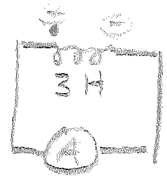


$$\text{LOAD} = (600 - 1067j)\ \Omega$$

$$\text{POWER} = \frac{V^2}{R} = \frac{9^2}{1200} = 5.75\times10^{-2}\ \text{WATTS}$$

5)





BOB MARES

$$9 = 3I_2 + j2I_1 + j2I_3$$

$$0 = 6(I_2 - I_1) + [j2I_1 - j2I_3]$$

$$= -(6+j2)I_1 + 6I_2 - j2I_3$$

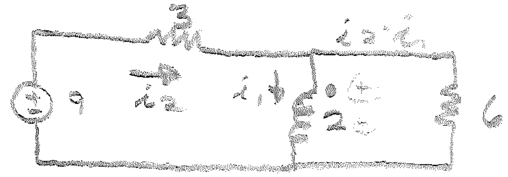
$$0 = j3I_3 + j2I_1$$

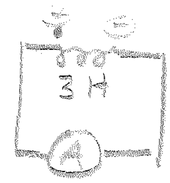
$$\left|\begin{bmatrix} j2 & 3 & j2 \\ -(6+j2) & 6 & -j2 \\ j2 & 0 & j3 \end{bmatrix}\right| \Longrightarrow -14 + j42$$

$$\left|\begin{bmatrix} j2 & 3 & 0 \\ -(6+j2) & 6 & 0 \\ j2 & 0 & 9 \end{bmatrix}\right| \Longrightarrow 54 + j126$$

$$\frac{54 + j126}{-14 + j42} = \frac{137\angle 66^\circ}{44.3\angle 110^\circ} = 3.04\angle -44^\circ$$

304 AMPS

5) 



BOB MARES

$$9 = 3I_2 + j2I_1 + j2I_3$$
$$0 = 6(I_2 - I_1) + [j2I_1 - j2I_3]$$
$$= -(6+j2)I_1 + 6I_2 - j2I_3$$
$$0 = j3I_3 + j2I_1$$

$$\left|\begin{bmatrix} j2 & 3 & j2 \\ -(6+j2) & 6 & -j2 \\ j2 & 0 & j3 \end{bmatrix}\right| \Rightarrow -14 + j42$$

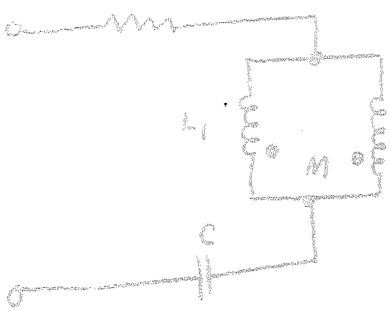
$$\left|\begin{bmatrix} j2 & 3 & 0 \\ -(6+j2) & 6 & 0 \\ j2 & 0 & 9 \end{bmatrix}\right| \Rightarrow 54 + j126$$

$$\frac{54 + j126}{-14 + j42} = \frac{137 \angle 66^\circ}{44.3 \angle 110^\circ} = 3.04 \angle -44^\circ$$

<u>3.04 AMPS</u> X

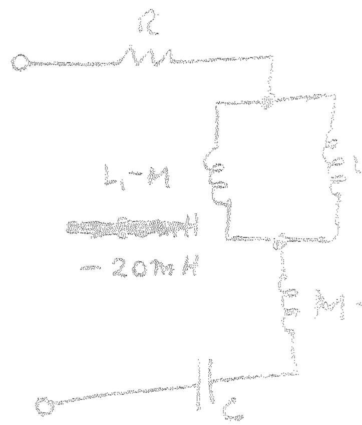
EE 208 - LABORATORY PROBLEM SESSION SOLUTIONS

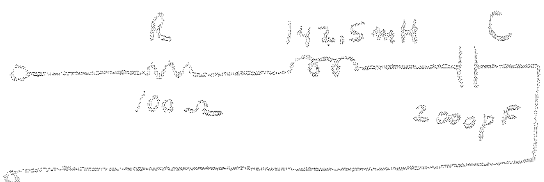
PROBLEM 1



$\Longrightarrow$

use equivalent T





$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \boxed{9427\ Hz}$$

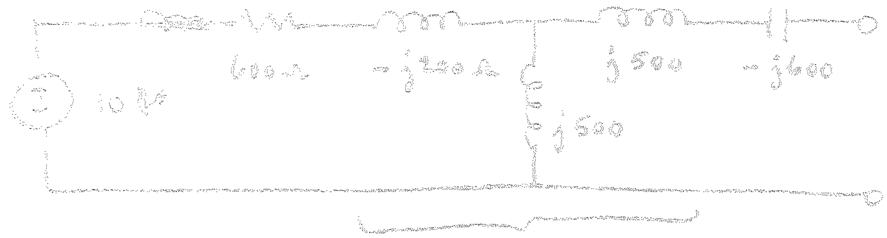
$$Q_0 = \frac{2\pi f_0 L}{R} = \boxed{84.4}$$

$$f_U = \frac{1}{2\pi}\left[\sqrt{1+\frac{1}{4Q_0^2}} + \frac{1}{2Q_0}\right]$$

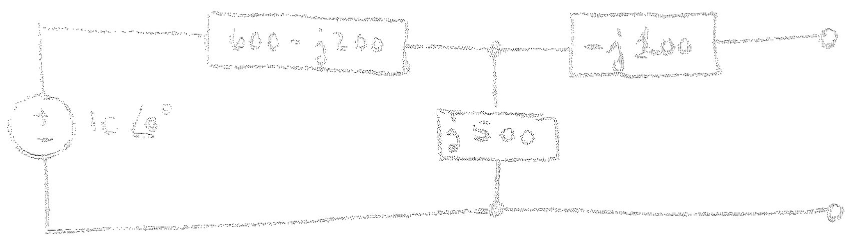
$$f_L = \frac{1}{2\pi}\left[\sqrt{1+\frac{1}{4Q_0^2}} - \frac{1}{2Q_0}\right]$$

$$BW = f_U - f_L = \boxed{111.7\ Hz}$$

Use Thevenin's theorem for the portion [illegible] the left of the load terminals



Equivalent T



$$\hat{Z}_{THEV} = -j100 + \frac{(j500)(600-j200)}{600-j200+j500} = -j100 + [illegible]$$

$$= -j100 + 10^3\left(\frac{1+j3}{6+j3}\right) = -j100 + 10^3\left([illegible] \right)$$

$$= -j100 + 1000\left(.472\angle 45^\circ\right) = [illegible]$$

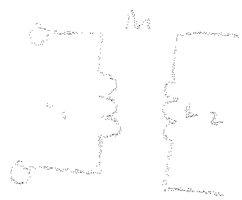
$$= \boxed{333 + j233\ \Omega}$$ make $\hat{Z}_L = [illegible]$

$$V_{THEV} = \frac{j500}{600+j300}\cdot 10\angle 0^\circ = \frac{5000\angle 90^\circ}{100(6.71\angle 26.5^\circ)} [illegible]$$

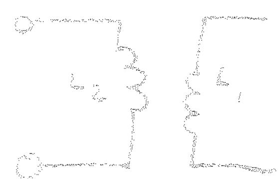
$$= [illegible]$$

Equivalent inductance

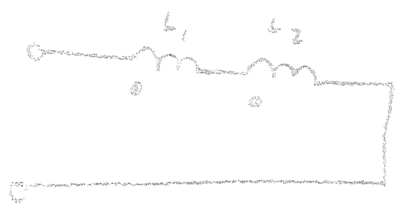
(1)



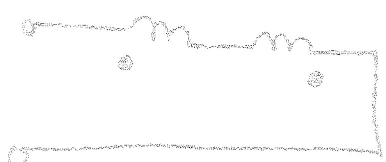
(2)



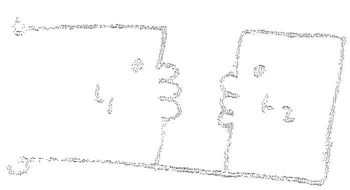
(3)



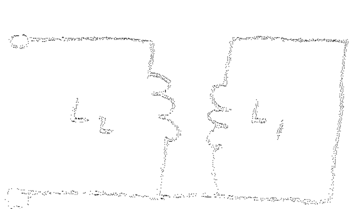
(4)

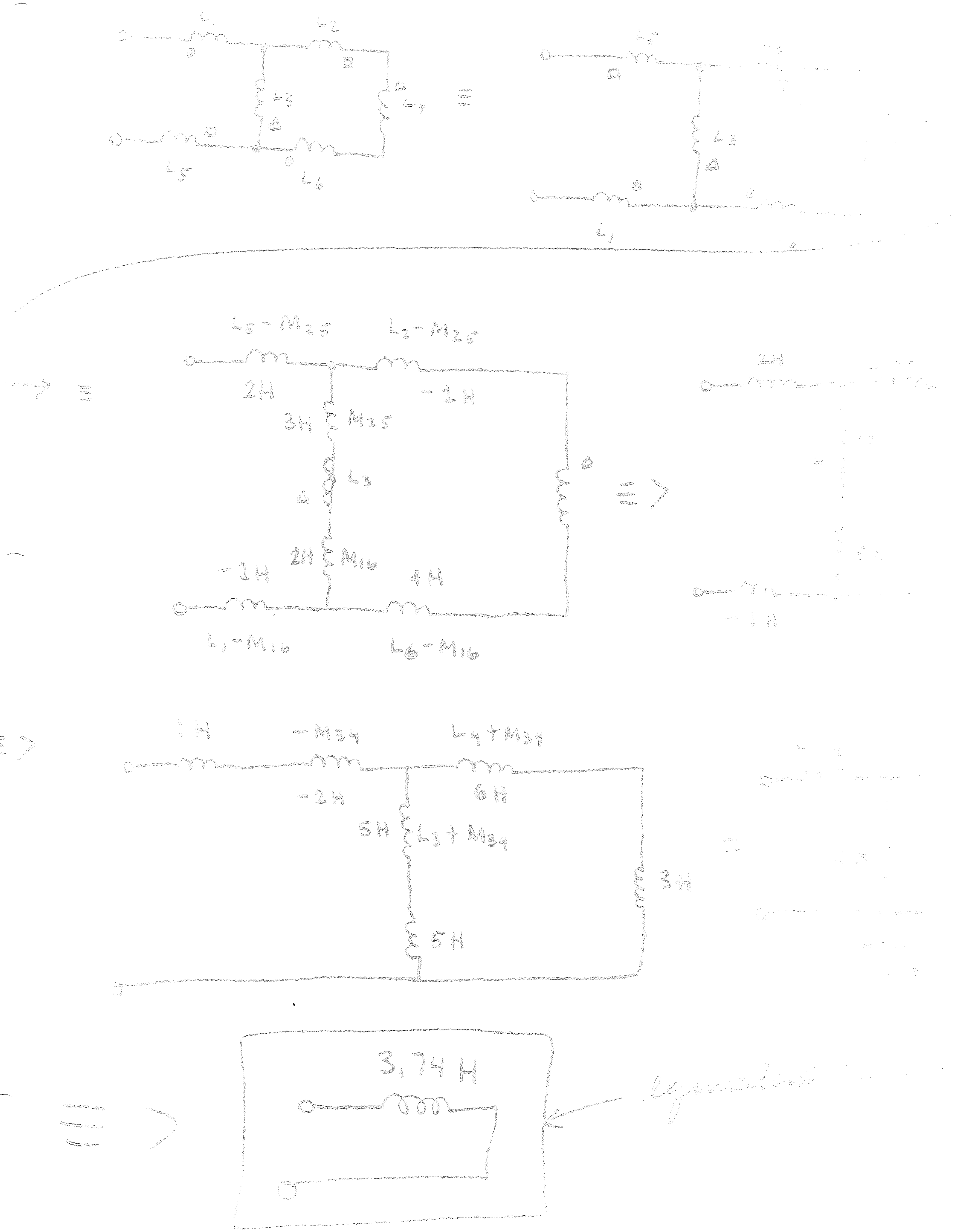


(5)

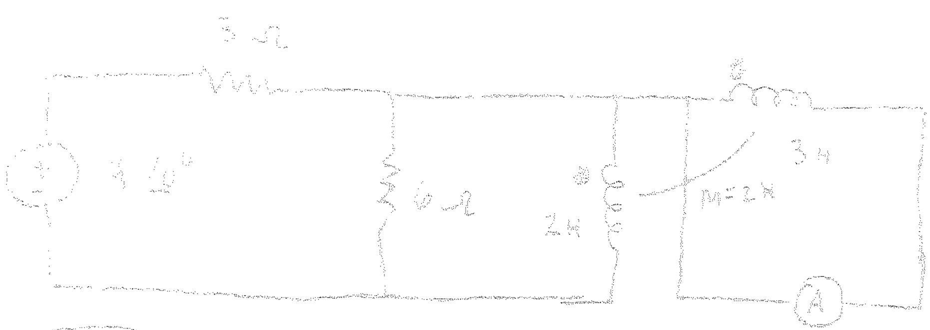


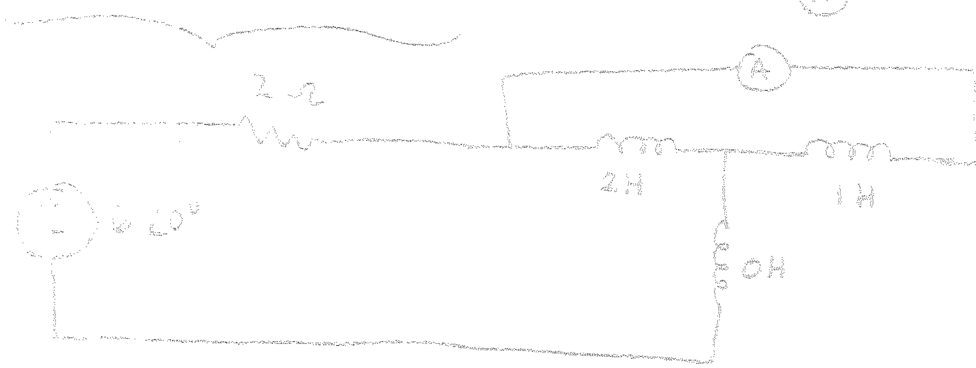
(6)

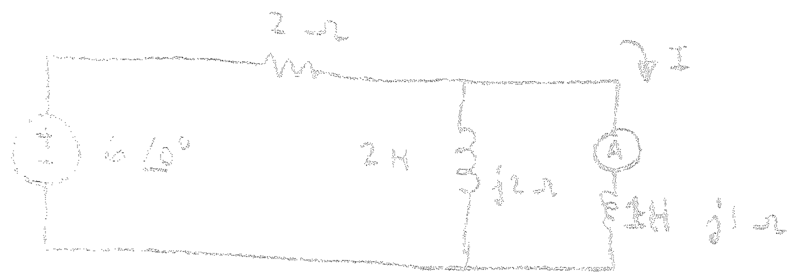




$L_5 - M_{25}$
$L_2 - M_{26}$
2H
-1H
3H
M_{35}
L_3
2H
M_{16}
-1H
4H
$L_1 - M_{16}$
$L_6 - M_{16}$
1H
$-M_{34}$
$L_4 + M_{34}$
-2H
6H
5H
$L_3 + M_{34}$
3H
5H
3,74 H



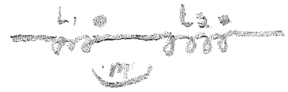




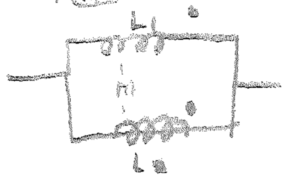
$$I = \left(\frac{6}{2 + j\frac{2}{3}} \right) \frac{j2}{j3} = \frac{4}{2 + j.67} =$$

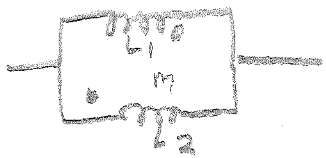
$$= \boxed{1.89 \angle -18.5^\circ \text{ A}}$$

Ammeter reads

3)  $L_{EQ} = L_1 + L_2 + \sqrt{L_1 L_2} = L_1 + L_2 + 2M$

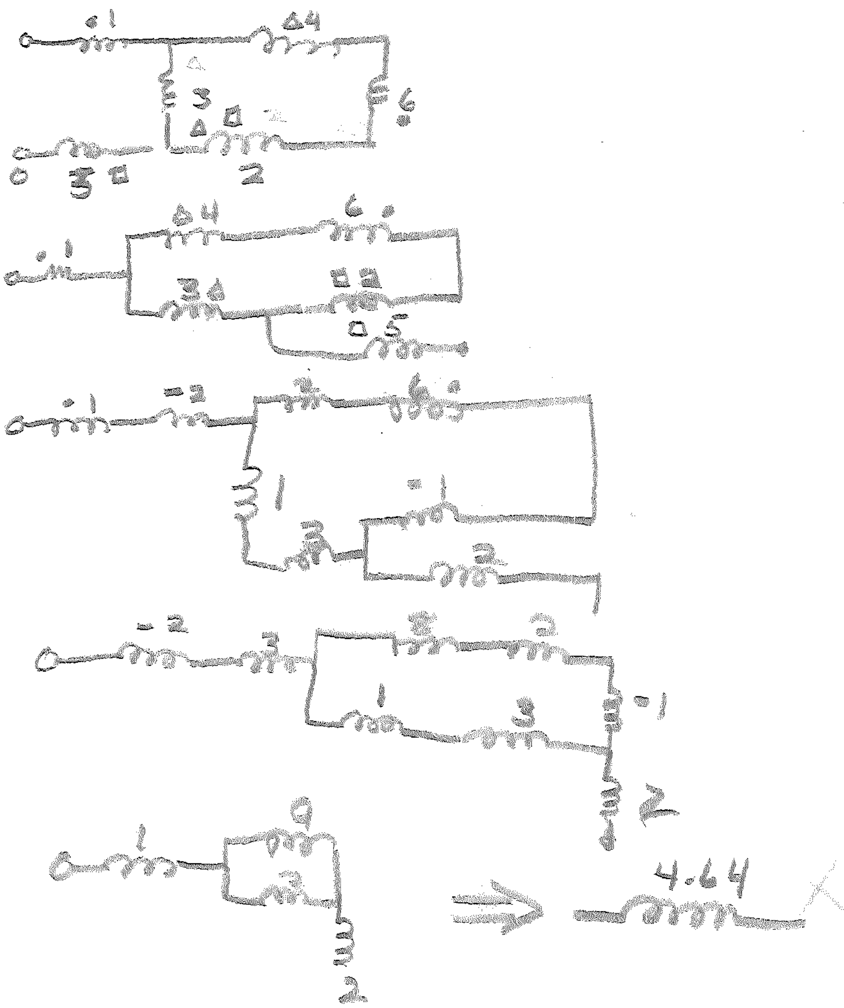
 $L_{EQ} = L_1 + L_2 - \sqrt{L_1 L_2}$

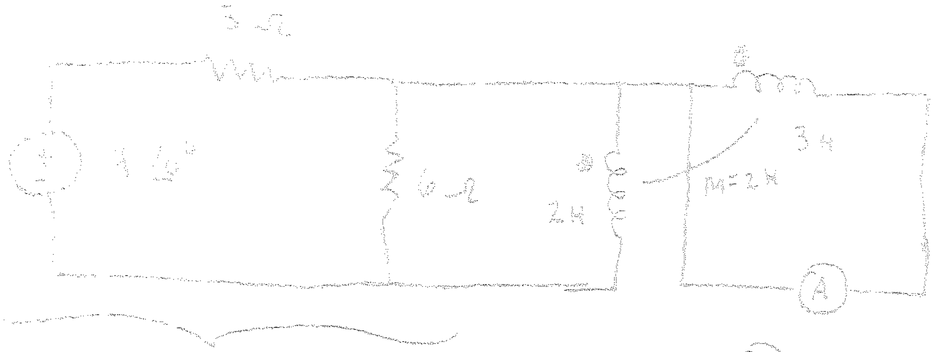
 $$L_{eq} = \frac{1}{\frac{1}{L_1 - M} + \frac{1}{L_2 - M}} + M$$

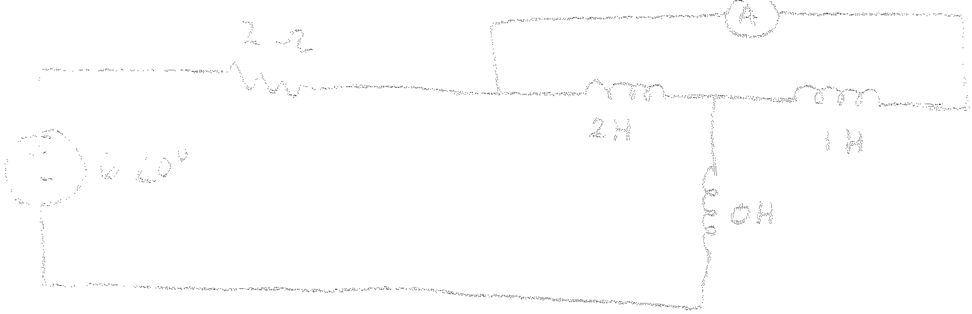
 $$L_{eq} = \frac{1}{\frac{1}{L_1 + \frac{1}{2}\sqrt{L_1 L_2}} + \frac{1}{L_2 + \frac{1}{2}\sqrt{L_1 L_2}}} + M$$

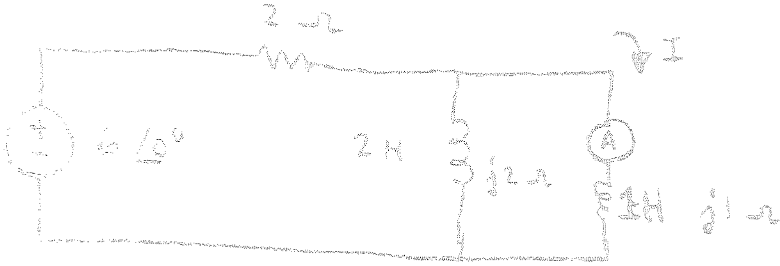
2

4)









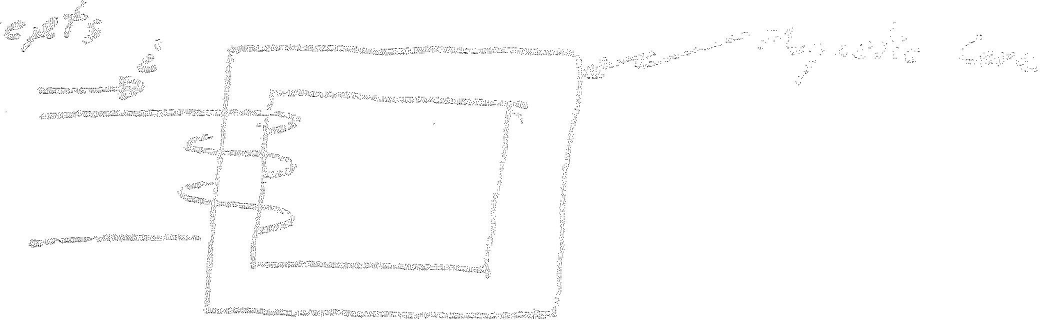
$$I = \left(\frac{6}{2 + j\frac{2}{3}}\right)\frac{j2}{j3} = \frac{4}{2 + j.67} = \ldots$$

$$= \boxed{1.89 \angle -18.5^\circ \text{ A}}$$

Ammeter reads =

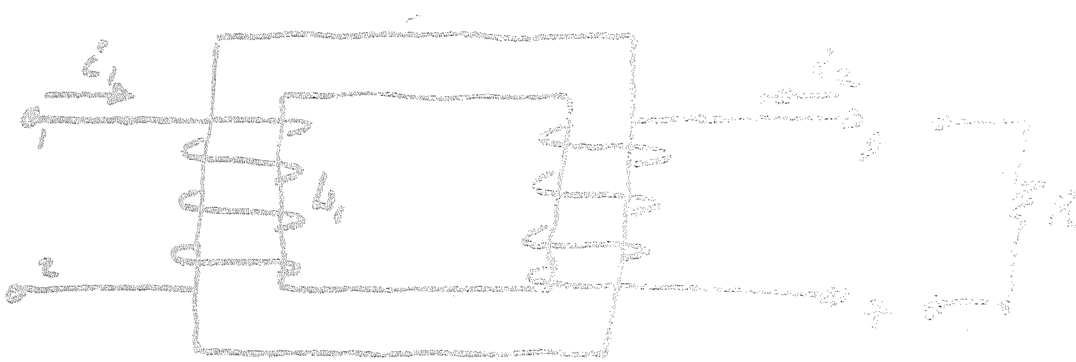
Mutual Inductance

A. Concepts



If $i > 0$, the magnetic flux ϕ is clockwise in the magnetic core (the "right-hand-rule" is applicable.)

If $\frac{di}{dt} > 0$, the top terminal is positive w.r.t. the bottom terminal and the flux in the c.w. direction is increasing.



If $i_2 = 0$ (open-circuit) and $\frac{di_1}{dt} > 0$, then terminal 4 is positive w.r.t. term. 3. Thus, when a resistive load is connected between

[illegible]

[illegible]

$$v_{12} = L_1 \frac{di_1}{dt}$$

potential of 1 w.r.t. 2

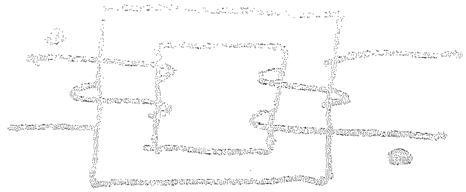
$$v_{43} = + M \frac{di_1}{dt}$$

Mutual Inductance in [illegible]

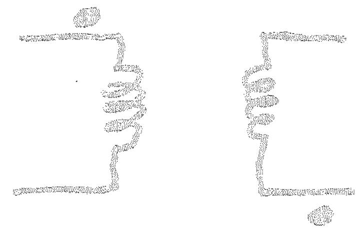
When the resistor is connected, [illegible] will cause a flux component in [illegible] direction (right-hand rule) [illegible] to decrease the flux caused [illegible] current. A changing i_2 current [illegible] will induce a negative voltage [illegible] circuit,

$$v_{12} = L_1 \frac{di_1}{dt} - [illegible]$$

where it is assumed that [illegible] same in both directions



Positive currents into the marked terminals produce additive fluxes in the core. with this convention the drawing of the core is not required.



B. Complete system of Equations for Mutual Coupling

$$\left.\begin{aligned} v_1 &= L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \\ v_2 &= -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \end{aligned}\right\} \quad (1)$$

If the positive current sense for both currents enter (or leave) at the polarity marks, then the M-term has the same sign as the L-terms; otherwise the signs are opposite.

Mutual Coupling with Steady-State Sinusoidal Signals

In the same way that

$$v = L \frac{di}{dt}$$

for the self-inductance leads to

$$V = j\omega L I$$

in phasor notation,

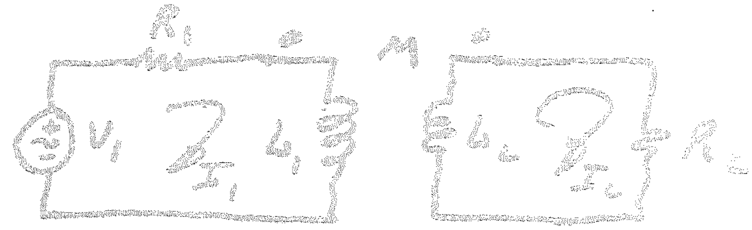
$$v_2 = M \frac{di_1}{dt} \Longrightarrow V_2 = j\omega M I_1$$

for mutual inductance.

Thus, Equations (1) become

$$\left.\begin{aligned} V_1 &= j\omega L_1 I_1 - j\omega M I_2 \\ V_2 &= -j\omega M I_1 + j\omega L_2 I_2 \end{aligned}\right\} \quad (2)$$

Example



$$V_1 = (R_1 + j\omega L_1) I_1 - j\omega M I_2$$

$$0 = -j\omega M I_1 + (R_2 + j\omega L_2) I_2$$

1. The 8 ft uniform rod AB in Figure 1 weighs 50 lb. A 30 lb weight D is attached to rod AB by a cable that passes over a smooth pulley C. DETERMINE the work done by the two weights as the rod rotates clockwise from a vertical position to the horizontal position shown as dashed lines. Pin A is smooth. (ANS. + 100 ft lb.)

2. The 17 lb block B in Figure 2 slides along a smooth vertical rod under the action of a 50 lb vertical force. [illegible] has one end fixed at A and the other end attached to B. The modulus of the spring, K, is 20 lb/ft and its unstretched length is 3 ft. DETERMINE the work done on B during a displacement from y = 2 ft to y = 4 ft. (ANS. + 29.7 ft lb)

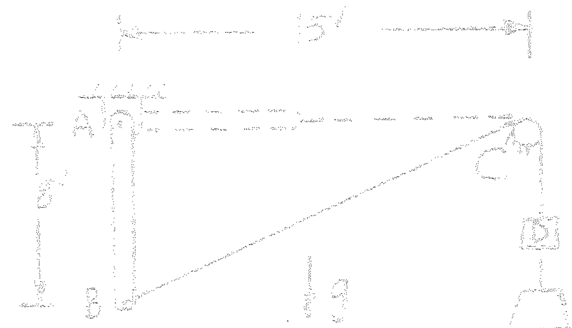


FIGURE 1

FIGURE 2

circuit shown below

cord line currents $I_A, I_B, \& I_C$

and the line-to-line voltage (Assume

balanced line voltages).

b. Record the phase current I_{BC}

c. Verify the $\sqrt{3}$ relationship

between line and phase currents

d. Calculate the total power to the load by assu...

balanced phase currents & line voltages

$$P_{Total} = 3 I_\phi V_{AB} \cos\theta$$

Calculate total power using $P_{Total} = \sqrt{3} V_L I_L \cos\theta$

and compare with results of d.

2. Assume ABC sequence for the generated voltages ... the line currents I_B and I_C to show which lamp ... be the brightest, lamp B or lamp C.

a. Which lamp was brightest in the lab circuit?

b. What phase sequence exists in the circuits lab

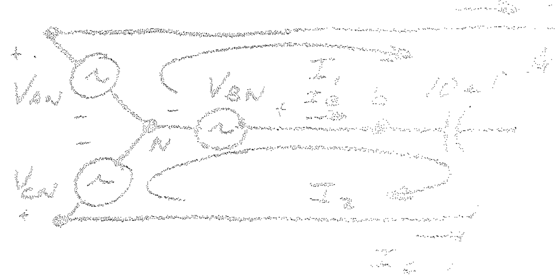
Assume the following

$V_{AB} = 208 \angle 30°$ V

$V_{AN} = 120 \angle 0°$

$V_{BN} = 120 \angle -120°$

$V_{CN} = 120 \angle +120°$



3. The two-wattmeter method was used to ... motor as shown below

FINAL REVIEW

1) $A = \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 0 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -1 \\ 1 & 0 \\ 2 & -1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \end{bmatrix}$

a) $A + B = \begin{bmatrix} 4 & 2 \\ 3 & -2 \\ 2 & 3 \end{bmatrix}$

b) $AC = \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 5 & 0 \\ -2 & 2 & 0 \\ 8 & 4 & 0 \end{bmatrix}$

c) $CA = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 4 & 4 \end{bmatrix}$

4) $$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & -1 & 4 \\ 1 & 1 & -1 & 1 & -4 \\ 1 & -1 & 1 & 1 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 & -4 \\ 0 & 0 & 2 & 0 & 4 \\ 0 & 2 & 0 & 0 & -2 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \quad \therefore \quad \begin{array}{l} x_4 = -2 \\ x_3 = 2 \\ x_2 = -1 \\ x_1 = 1 \end{array}$$

5) $$\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} F_2(-2) \Rightarrow \begin{bmatrix} 3 & 4 \\ -2 & -4 \end{bmatrix} G_{21}(+1) \Rightarrow \begin{bmatrix} 3 & 4 \\ +1 & 0 \end{bmatrix} G_{12}(-1) \Rightarrow \begin{bmatrix} 0 & 4 \\ 1 & 0 \end{bmatrix} F_1(\tfrac{1}{4})$$

$$\Rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} E_{12} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \therefore \quad \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} F_2(-2)\, G_{21}(1)\, G_{12}(-1)\, F_1(\tfrac{1}{4})\, E_{12} = I$$

6) $$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & +1 & +4 \end{bmatrix} ; \begin{bmatrix} 2 & 3 & 2 \\ 1 & 1 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 4 & 6 \\ 0 & 1 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \end{bmatrix}$$

FINAL REVIEW

1) $$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 1 & 3 & 1 \\ 2 & -1 & 2 & 2 & 6 & 2 \\ 3 & 2 & -4 & 3 & 9 & 3 \end{array}\right] \Rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 3 & 1 \\ 0 & -3 & 6 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 3 & 1 \\ 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_5 + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + x_4 + 3x_5 = 1$

$x_2 - 2x_3 = 0$

$\therefore x_1 = -x_4 - 3x_5 + 1$

$x_2 = 2x_3$

2) $x - 4y + 5z = 1$; $2x - y + 3z = 2$; $3x + 2y + z = p$

$$\left[\begin{array}{ccc|c} 1 & -4 & 5 & 1 \\ 2 & -1 & 3 & 2 \\ 3 & 2 & 1 & p \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -4 & 5 & 1 \\ 0 & 7 & -7 & 0 \\ 0 & 14 & -14 & p-3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & p-3 \end{array}\right]$$

CONSISTENT IFF $p = 3$

3) $$\begin{bmatrix} 3 & 1 & 2 & 4 \\ 5 & 2 & 3 & 6 \\ 4 & 1 & 3 & 6 \\ 5 & 1 & 4 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -2 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -2 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ RANK = 2

4) $$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 4 & 5 & 0 & 1 & 0 \\ 3 & 5 & 6 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & 0 \\ 0 & -1 & -3 & -3 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & -5 & 0 & 2 \\ 0 & 1 & 3 & 3 & 0 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -3 & 2 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

5) $\begin{vmatrix} x & 3x & 2x \\ e^x & e^x & e^x \\ 2 & 3 & -2 \end{vmatrix} = xe^x \begin{vmatrix} 1 & 3 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & -2 \end{vmatrix} = xe^x \begin{vmatrix} 0 & 2 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{vmatrix} = xe^x \begin{vmatrix} 0 & 0 & 9 \\ 1 & 0 & 5 \\ 0 & 1 & -4 \end{vmatrix}$

$= 9xe^x \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 5 \\ 0 & 1 & -4 \end{vmatrix} = 9xe^x \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 9xe^x$

6) a) $\begin{vmatrix} 2 & 2 & -2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 2\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} - 2\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} - 2\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 2(-1) + 2(+2) + 2(+1) = 4$

b) $\begin{vmatrix} 2 & 2 & -2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = -2\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} + 2\begin{vmatrix} 2 & -2 \\ 2 & 4 \end{vmatrix} - 3\begin{vmatrix} 2 & -2 \\ 1 & 3 \end{vmatrix} = -2(-2) + 2(12) - 3(8) = 4$

COF. OF $a_{12} = -\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 2$

COF. OF $a_{22} = \begin{vmatrix} 2 & -2 \\ 2 & 4 \end{vmatrix} = 12$

FINAL REVIEW

1) $\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -2 & -3 & -5 \\ 6 & 4 & 9 & 7 \\ -5 & 1 & -1 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 6 & 9 & 15 \\ 1 & 5 & 8 & 12 \\ 0 & 6 & 9 & 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 4 & 6 & 10 \\ 0 & 4 & 6 & 10 \\ 1 & 5 & 8 & 12 \\ 0 & 6 & 9 & 15 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 2 & 3 & 5 \\ 1 & 5 & 8 & 12 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow$ S IS OF SECOND DIMENSION

$\begin{bmatrix} 1 & 1 \\ 0 & -2 \\ 6 & 4 \\ -5 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \vec{a}$ & $\vec{b}$ ARE LINEARLY IND.

$\therefore$ $\vec{a}$ & $\vec{b}$ MAY BE BASIS FUNCTIONS FOR S

2) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 1 & 2 & 2 & 5 \\ 1 & 3 & -1 & -2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 2 & 2 & 2 \\ 0 & 3 & -1 & -5 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -4 & -8 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$\therefore \vec{a} = 3\vec{e}_1 - \vec{e}_2 + 2\vec{e}_3$

4) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ -1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 1 & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$ ETC

BY INSPECTION, $\vec{a} = \vec{b} - \vec{a}$

4) $A^T = \begin{bmatrix} 3 & 1 & 1 & 5 \\ 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \therefore cr_A = 2$

$A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 5 & 1 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \therefore rr_A = 2$

$A\vec{X} = 0 \Rightarrow$ DIM. SPACE $= 3 - 2 = 1$

$A^T\vec{Y} = 0 \Rightarrow$ DIM. SPACE $= 4 - 2 = 2$

5) $\begin{bmatrix} 2 & 1 & 0 & 3 & 0 & 1 \\ -1 & 2 & 0 & 0 & 1 & 2 \\ 3 & 0 & 3 & 2 & 2 & 3 \\ 0 & 6 & -1 & 1 & -2 & -1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ -\beta_1 \\ -\beta_2 \\ -\beta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -1 & 3 & -1 & 2 & 2 \\ 0 & -3 & 0 & 3 & -2 & -3 \\ 0 & 0 & 1 & -1 & 2 & 1 \\ 1 & 2 & 0 & 0 & 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & -19 & 0 \\ 0 & 1 & 0 & 0 & 29 & 1 \\ 0 & 0 & 1 & 0 & 5 & 1 \\ 0 & 0 & 0 & 1 & 3 & 0 \end{bmatrix}$

$\alpha_1 = -19\beta_2$; $\alpha_2 = +29\beta_2 + \beta_3$ $\alpha_3 = 5\beta_2 + \beta_3$; $\beta_1 = -3\beta_2$

$\vec{W} = \alpha_1 a + \alpha_2 b + \alpha_3 c = \beta_1 d + \beta_2 e + \beta_3 f$

$w = (-19\beta_2)\vec{a} + (29\beta_2 + \beta_3) b + (5\beta_2 + \beta_3) c$

$= \beta_1(0) + \beta_2(29b - 19a + 5c) + \beta_3(b + c)$

BASIS FOR $S \cap T$ (SECOND DIMENTION)

$V_1 = 29b - 19a + 5c$

$V_2 = b + c$

FINAL REVIEW

$$A = \begin{bmatrix} 0 & -2 & 2 \\ -3 & 1 & 3 \\ -1 & 1 & 3 \end{bmatrix} \quad \text{WITH EIGEN VALUES } -2 \;\; 2 \;\; 4$$

FOR $\lambda_1 = -2$

$$\begin{vmatrix} -2 & -2 & 2 \\ -3 & 3 & 3 \\ -1 & 1 & 5 \end{vmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ -1 & 1 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} z = 0 \\ x = y \end{matrix} \;\therefore \vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \alpha_1$$

FOR $\lambda_2 = 2$

$$\begin{vmatrix} -2 & -2 & 2 \\ -3 & -1 & 3 \\ -1 & 1 & 1 \end{vmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ -3 & -1 & 3 \\ 0 & 2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \therefore V_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \alpha_2$$

FOR $\lambda_3 = 4$

$$\begin{bmatrix} -4 & -2 & 2 \\ -3 & -3 & 3 \\ -1 & 1 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & -1 & 1 \\ +1 & +1 & -1 \\ 0 & 2 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \therefore V_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \alpha_3$$

$$\Rightarrow P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow P^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow \operatorname{adj} P^T = \begin{bmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

$\det P = -2$

$$\Rightarrow -\frac{1}{2} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow P^{-1} = -\frac{1}{2} \begin{bmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

$$P^{-1}AP = -\frac{1}{2} \begin{bmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 0 & -2 & 2 \\ -3 & 1 & 3 \\ -1 & +1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} 2 & 2 & -2 \\ -2 & 2 & -2 \\ 4 & -4 & -4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 \\ 1 & -1 & 1 \\ -2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

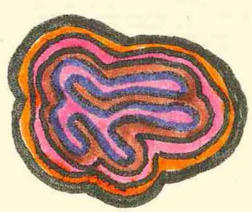
$$= \begin{bmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

E.501

II

E. SCI. II

	1 87:50	2 8:45	3 9:40	4 10:35	5 11:30	6 12:25	7 1:20	8 2:15	9 3:10
MON				PHYS V RAL C 126	ELEC. I POS D04		HUMAN	. VI AUD	E. SCI. II →
TUES							HUM VI DV A 205		E. SCI. II · D-132
WED	ELEC	TRONICS I LAB	 D01		ELEC. I POS D04	E. SCI. II D04	HUM VI DV A 205		E. SCI. II A 121
THU				PHYS V RAL C 126	E. SCI. SOME- TIMES		HUM VI DV A 205	PHYS PRM	V LAB A-205
FRI				PHYS V RAL C 126	ELEC. I POS D04				



4-1-70

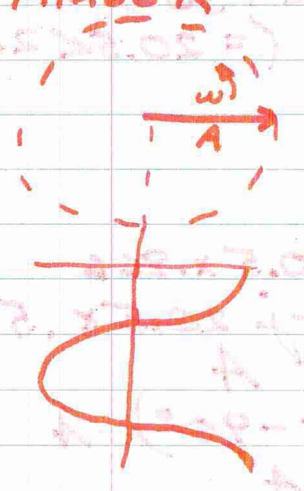
I METHODS OF REPRESENTING SINUSOIDS

PROPERTIES:

1) AMPLITUDE
2) FREQUENCY
3) PHASE

LET FREQUENCY BE UNDERSTOOD

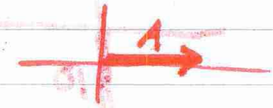
II) PHASOR



SIDE YIELDS SIMPLE HARMONIC MOTION

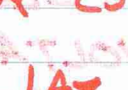
PHASOR CAN REPRESENT SINUSOID:

$v(t) = A\cos\omega t$



$v(t) = A\cos(\omega t + \phi)$

III) A) TIME FUNCTION → PHASOR (STILL)
PHASOR → COMPLEX #

B) $v(t) = A\cos(\omega t + \phi) \longrightarrow$ 

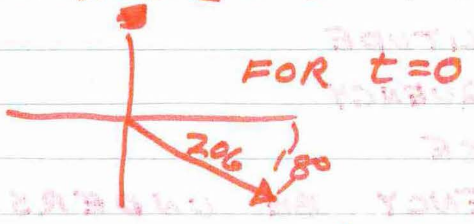
$\longrightarrow A\angle\phi = A\cos\phi + jA\sin\theta$

EXAMPLE:

1) $v(t) = 206 \cos(377t - 18^\circ)$ V

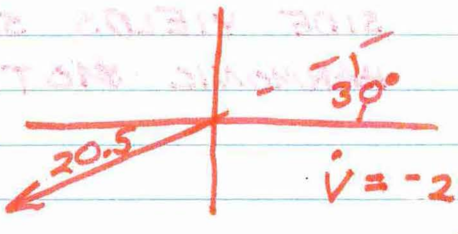
$\dot{V} = 206 \angle -18^\circ$ V

FOR $t=0$



2) $v(t) = -20.5 \cos(\omega t + 30^\circ)$ V

$\dot{V} = -20.5 \angle 30^\circ$ V $(= 20.5 \angle 210^\circ$ V$)$

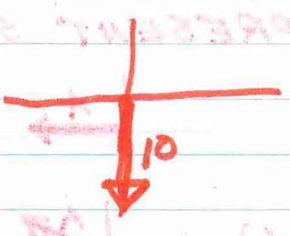


$\dot{V} = -20.5 \times .866$
$\quad - j\, 20.5 \times .5$

3) $i(t) = 10 \sin 10t$ A

$= 10 \cos(10t - 90^\circ)$ A

$\dot{I} = 10 \angle -90^\circ$ A

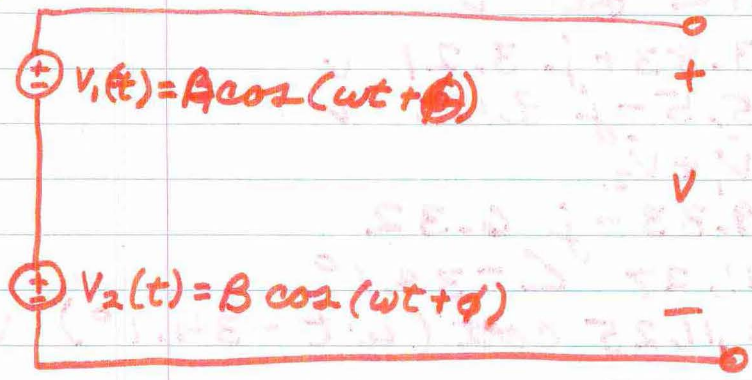


1) CURRENT PHASORS HAVE CLOSED HEADS
2) VOLTAGE PHASORS HAVE OPEN HEADS
3) INCLUDE UNITS ON EQUATIONS
4) PHASORS CAPATILIZED WITH DOTS ON 'EM

$x = A\sin(\omega t + \phi) = A\cos(\omega t + \phi - 90^\circ)$

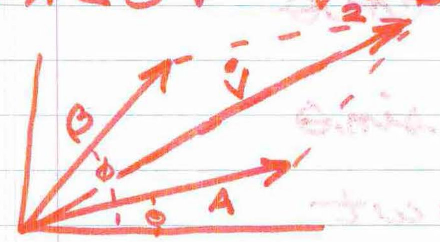
$\dot{X} = A\angle\phi - 90^\circ$

EX)



$\dot{V}_1 = A\angle\theta$ V $\quad \dot{V}_2 = B\angle\phi$ V

1)



2) $\dot{V}_1 = A\angle\theta;\ \dot{V} = A\cos\theta + jA\sin\theta$

$\dot{V}_2 = B\angle\phi;\ \dot{V} = B\cos\phi + jB\sin\phi$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 = A\cos\theta + B\cos\phi + j(A\cancel{\cos}\theta + B\sin\phi) = C\angle\gamma$$

$$C\angle\phi = \sqrt{(A\cos\theta + B\cos\phi)^2 + j^2(A\cancel{\cos}\theta + B\sin\phi)^2}$$

$$\angle\tan^{-1}\ (\text{MESS})\ \frac{A\sin\theta + B\sin\phi}{A\cos\theta + B\cos\phi}$$

EXAMPLE

$$\begin{cases} v_1(t) = 5\cos(\omega t + 40^\circ)\ V \\ v_2(t) = 11\sin(\omega t + 30^\circ)\ V \end{cases}$$

$$\begin{cases} \dot{V}_1 = 5 \angle 40^\circ\ V \\ \dot{V}_2 = 11 \angle -60^\circ\ V \end{cases}$$

$$\begin{cases} \dot{V}_1 = 3.83 + j\,3.21\ V \\ \dot{V}_2 = 5.5 - j\,9.53\ V \end{cases}$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2$$

$$= 9.33 - j\,6.32$$

$$= 11.27 \angle -34.1^\circ$$

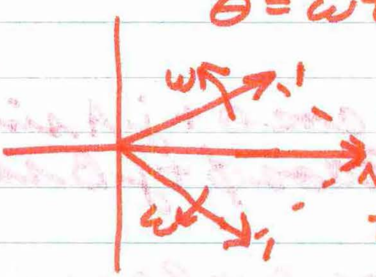
$$v(t) = 11.25\cos(\omega t - 34.1^\circ)\ V$$

$$\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta$$

$$\frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin\theta$$

$$\theta = \omega t$$

cos



NO COMPEX TIME FUNCTION

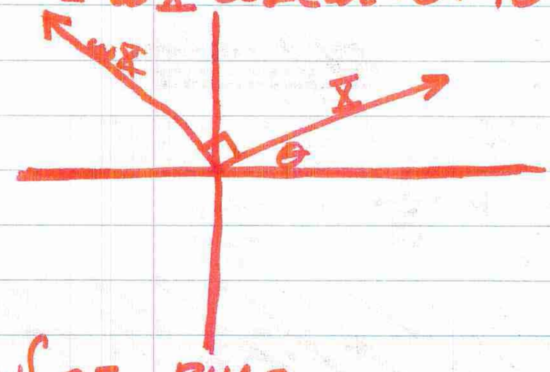
SUMMATION IS ALWAYS REAL
GOES BACK AND FORTH

I) DIR. OF PH.

$x(t) = X\cos(\omega t + \theta) \Rightarrow \dot{X} = X\angle\theta$

$x'(t) = -\omega X \sin(\omega t+\theta)$ $\quad \dot{X}' =$

$= \omega X \cos(\omega t+\theta+90°) \Rightarrow \omega X\angle\theta+90°$

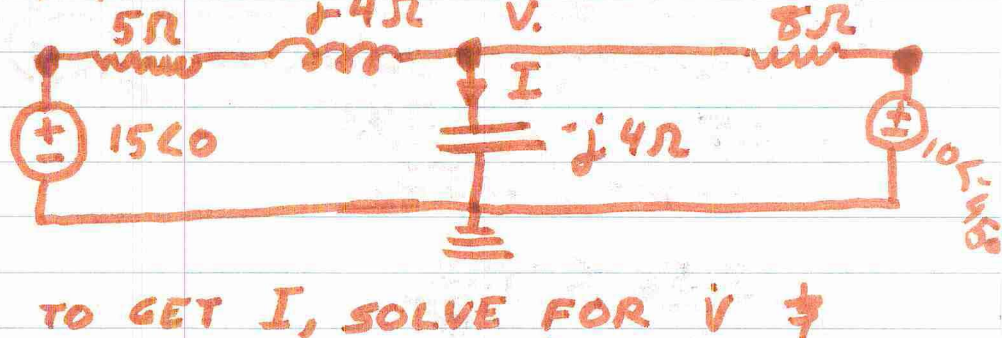


II) ∫ OF PHAS

$x(t) = A\cos(\omega t+\theta)$

$\int_{-\infty}^{t} x(\lambda)d\lambda = A/\omega \cos(\omega t+\theta-90°)$

4-7-70



TO GET I, SOLVE FOR $\dot{V}$ & USE OHM'S LAW

$$\frac{15\angle 0° - V}{5+j4} + \frac{10\angle -45° - V}{8} + \frac{0-V}{-4j} = 0$$

$$\frac{15\angle 0°}{5+j4} + \frac{10\angle 45°}{8} = V\left(\frac{1}{5+4j} + \frac{1}{8} - \frac{1}{4j}\right)$$

CONT

$$\dot{V} = \frac{\dfrac{15\angle 0^\circ}{5+4j} + \dfrac{10\angle -45}{8}}{\dfrac{1}{5+4j} + \dfrac{1}{8} - \dfrac{1}{4j}}$$

$$= \frac{\dfrac{15\angle 0^\circ}{6.4\angle 38.6} + 1.25\angle -45^\circ}{\dfrac{1}{6.4\angle 38.6} + \dfrac{1}{8} - \dfrac{1}{4j}}$$

$$\cancel{= 2.34\angle -39^\circ + 1.25\angle -45^\circ}$$

$$\cancel{\dot{V} =}$$

$$\dot{V} = \frac{(1.86 - j1.46) + (.885 - j\,.885)}{(.0103 - j.0982) + (.125 + j.25)}$$

$$= \frac{2.745 - j\,2.345}{.228 + j\,.152}$$

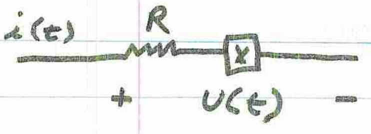
$$= \frac{3.56\angle -41.2}{.274\angle 33.7}$$

$$= 13\angle -74.9$$

$$\dot{I} = \frac{13\angle -74.9}{4\angle -90}$$

$$= 3.25\angle 15.1^\circ \text{ AMPS}$$

4-8-70



$v(t) = V\cos\omega t$

$\dot{Z} = R + jX = Z\angle\theta_Z$

$v(t) = V\cos\omega t \Rightarrow \dot{V} = V\angle 0°$

$\dot{I} = \dot{V}/\dot{Z} = I\angle -\theta_Z$, WHERE $I = V/Z$

$i(t) = I\cos(\omega t - \theta_Z)$

POWER $p(t) = v(t)i(t) = VI\cos\omega t \cos(\omega t - \theta_Z)$

$\dot{P} \Leftrightarrow$ NEVER

$p(t) = VI\cos\omega t(\cos\omega t\cos\theta_Z + \sin\omega t\sin\theta_Z)$

TO FIND AVERAGE POWER

$$P_{AVE} = \int_{(\omega t)=0}^{2\pi} p(\omega t)\, d(\omega t)$$

$$P_{AVE} = \frac{VI}{2\pi}\int_0^{2\pi}\cos\theta_Z\cos^2\omega t\, d(\omega t)$$

$$+ \frac{VI}{2\pi}\int_0^{2\pi}\sin\theta_Z\sin\omega t\cos\omega t\, d(\omega t)$$

$$= \frac{VI\cos\theta_Z}{2\pi}\int_0^{2\pi}\frac{1+\cos 2\omega t}{2}\, d(\omega t)$$

$$+ \frac{VI\sin\theta_Z}{2\pi}\int_0^{2\pi}\underbrace{\frac{\sin 2\omega t}{2}\, d(\omega t)}_{}^{=0}$$

$$= \frac{VI\cos\theta_Z}{2\pi}\int_0^{2\pi}\frac{1+\cos 2\omega t}{2}\, d(\omega t)$$

$$= \frac{VI\cos\theta_Z\,\cancel{2\pi}}{2\,\cancel{2\pi}} \Rightarrow P_{AV} = \frac{VI\cos\theta_Z}{2}$$

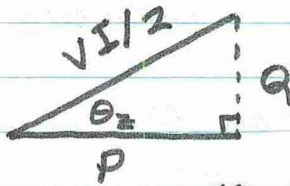
(CONT →)

$\cos\theta_Z \Rightarrow$ POWER FACTOR

$$P = \frac{V}{\sqrt{2}}\,\frac{I}{\sqrt{2}}\cos\theta_Z$$
$$= V_{EFF}\, I_{EFF} \cos\theta_Z$$

$$V\, I \cos\theta_Z \Rightarrow V_{RMS},\ I_{RMS}$$
$$\frac{V\, I \cos\theta_Z}{2} \Rightarrow V_{MAX},\ I_{MAX}$$

THE POWER TRIANGLE

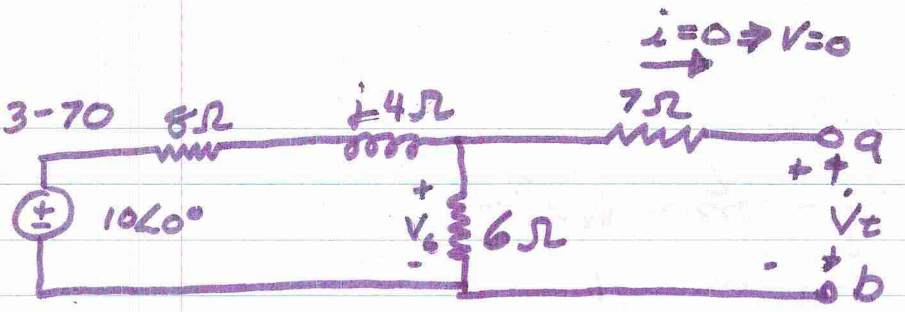


$\Rightarrow p = \frac{VI}{2}\cos\theta_Z$ WATTS

$$Q = \frac{VI}{2}\sin\theta_Z$$

Q IS CALLER REACTIVE POWER, MEASURED IN VARS (VOLT AMPERE REACTIVE)

4-13-70



$$V_6 = \frac{6}{6+j4+8}\cdot 10\angle 0^\circ$$

$$= \frac{60\angle 0^\circ}{14+j4} = \frac{60\angle 0^\circ}{1455\angle 15.95^\circ}$$

$$= 4.13\angle -15.95^\circ \text{ V}$$

REPLACE V.S. BY SHORT CIRCUIT

$$Z_T = 7 + \frac{1}{\frac{1}{6}+\frac{1}{8+j4}}$$

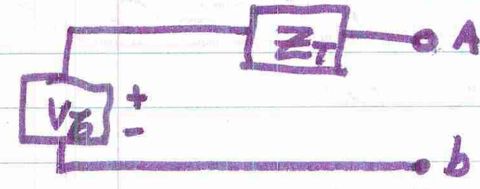
$$= 7 + \frac{6(8+j4)}{14+j4}$$

$$= 7 + \frac{6(8.93\angle 26.6)}{14.55\angle 15.95}$$

$$= 7 + 3.68\angle 10.65^\circ$$

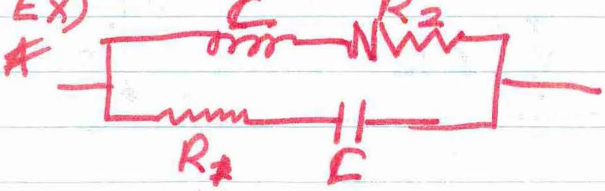
$$= 7 + 3.62 + j.68$$

$$= 10.62 + j.68$$



4-15-70

EX)



FIND ω SUCH THAT Z_{eq} IS REAL

A) $$Z = \frac{\left(R_1 + \frac{1}{j\omega C}\right)(R_2 + j\omega L)}{R_1 + R_2 + j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{(1 + j\omega R_1 C)(R_2 + j\omega L)}{1 - \omega^2 LC + j\omega C(R_1 + R_2)}$$

$$= \frac{R_2 - \omega^2 R_1 CL + j(\omega R_1 R_2 C + \omega L)}{1 - \omega^2 LC + j\omega C(R_1 + R_2)}$$

WILL BE REAL WHEN

$\angle$ NUM = $\angle$ DEN

$\theta_N = \theta_D$

$\tan \theta_N = \tan \theta_D$

$$\underbrace{\frac{\omega R_1 R_2 C + \omega L}{R_2 - \omega^2 R_1 CL} = \frac{\omega C(R_1 + R_2)}{1 - \omega^2 LC}}$$

SOLVE FOR ω

B) $R_1 C$ ADMITTANCE

$$\frac{1}{R_1 + \frac{1}{j\omega C}} = \frac{j\omega C}{1 + j\omega C R_1}\left(\frac{1 + j\omega R_1 C}{1 - j\omega R_1 C}\right)$$

$$= \frac{\omega^2 R_1 C^2 + j\omega C}{1+\omega^2 R_1^2 C^2}$$

L.L ADMITTANCE

$$\frac{R_2 - j\omega L}{R_2^2+\omega^2 L^2}$$

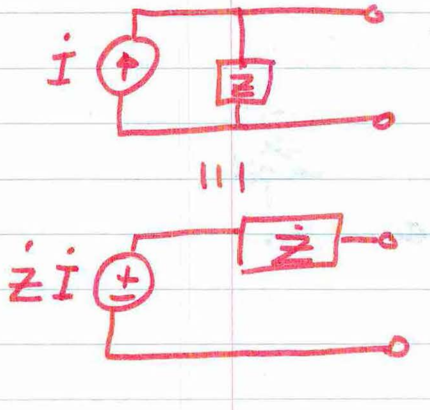
$$Y_{eq} = \text{sum} = G + j\underline{B} \leftarrow \text{SUSCEPTANCE}$$

WANT $B = 0$

$$\frac{\omega C}{1+\omega^2 R_1^2 C^2} = \frac{\omega L}{R_2^2+\omega^2 L^2}$$

$$\omega^2 = \frac{L - CR_2^2}{CL^2 - LR_1^2 C^2}$$

POWER (SEE HANDOUT)

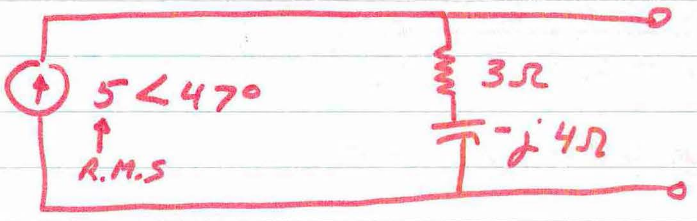


$$P_{MAX} = \frac{|\dot{Z}_g \dot{I}|^2}{4R_g}$$

(I IS RMS)

(OVER)

EX)



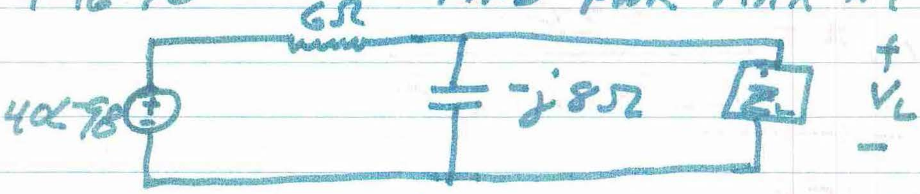
WHAT IS P_{MAX}

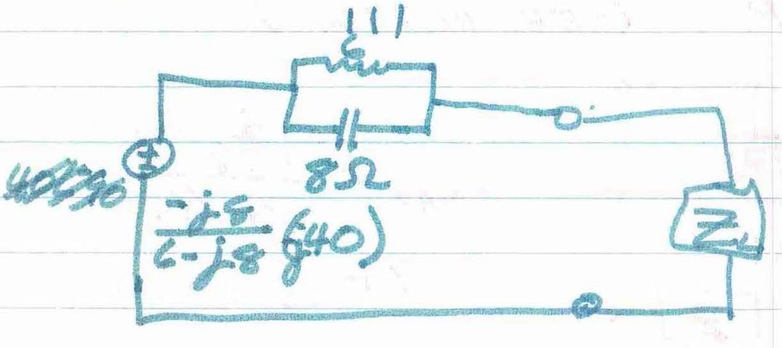
$$\cancel{P_{MAX} = \frac{(3-j4)^2\left(\frac{5}{\sqrt{2}}\right)^2}{(4)(3)}}$$

$$|Z_g| = 5$$

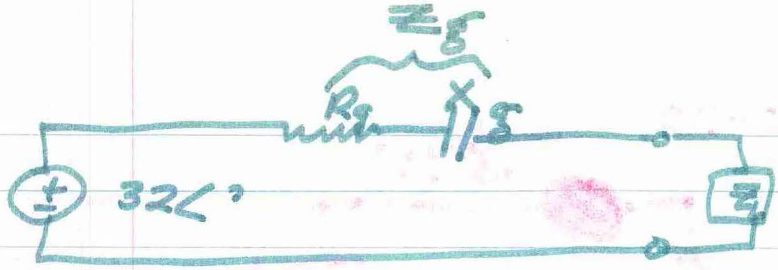
$$\therefore\ P_{MAX} = \frac{(5\cdot 5)^2}{4\cdot 3} = 32.08 \text{ WATTS}$$

4-16-70 FIND PWR MAX AT $\dot{Z}_L$





$$\frac{-j8}{6-j8}(j40)$$



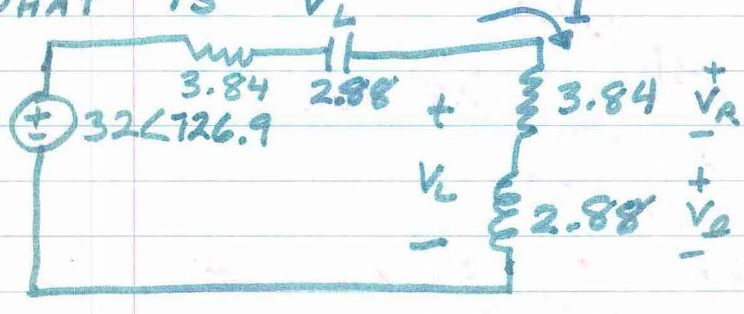
$$Z_g = \frac{6(-j8)}{6-j8} = \frac{48\angle -90^\circ}{10\angle -53.1}$$

$$= 4.8\angle -36.9^\circ$$
$$= 3.84 - j\,2.88$$

$\therefore \quad Z_L = 4.8\angle 36.9^\circ = 3.84 + j\,2.88\,\Omega$

$$P_{MAX} = \left.\frac{|\dot V_g|^2}{4R_g}\right|_{|V_{g_{RMS}}} = \frac{32^2}{4\times 3.84} = 66.7 \text{ WATTS}$$

WHAT IS V_L

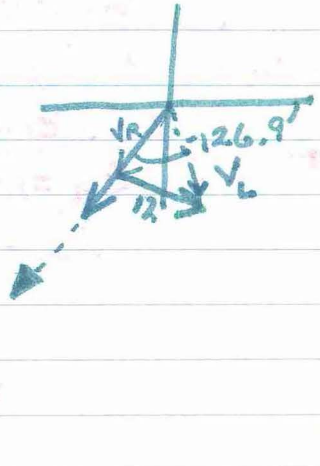


$$\dot V_L = \dot I\, Z_L$$

$I = \frac{32}{7.68} = 4.17$

$V_\ell = 2.88 \times 4.17 = 12$

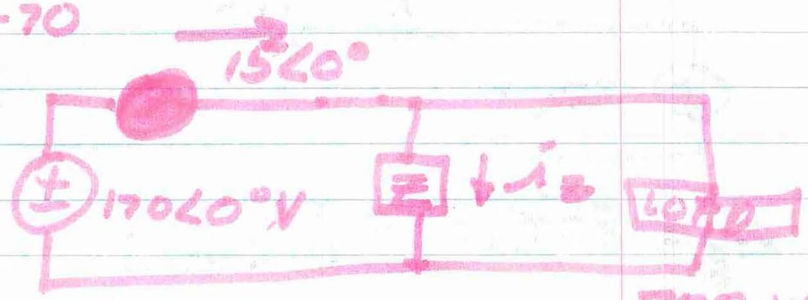
$V_R = 3.84 \times 4.17 =$



$V_L = 20\angle -90^\circ$ ✓

4-20-70

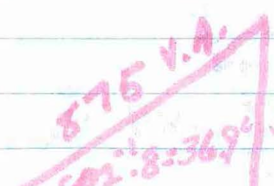
X-12)



700 WATTS
.8 POWER
FACTOR
(LAGGING
→ INDUCTIVE)

$P_{GEN} = (170) \cdot 15 = 255$ WATT

POWER Δ FOR THE LOAD

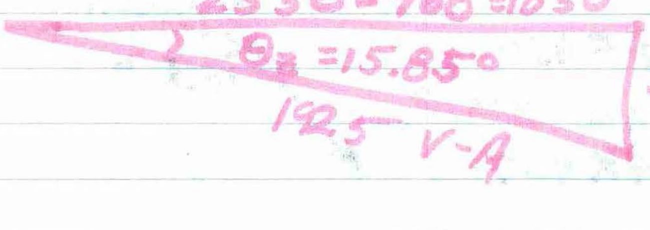


POWER Δ FOR GEN

2550 W

POWER Δ FOR Z

2550 - 700 = 1850



(CONT)

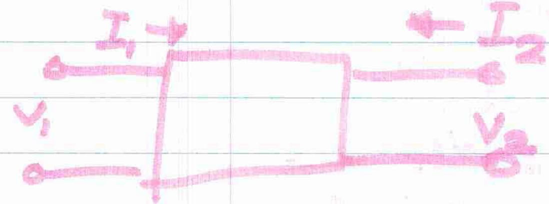
$P = \frac{VI}{2}\cos\theta_Z$

$I_Z = \frac{1850}{170 \times .963} = 11.3\ A$

$I_Z = 11.3\angle 15.85^\circ$

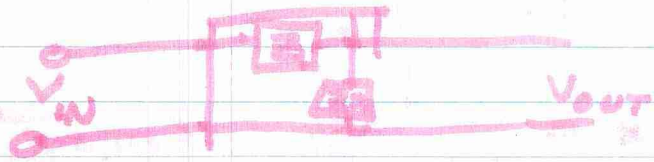
$Z = V/I = 170/11.3 = 15.05\,\Omega$

$= 15.05\angle 15.85^\circ$

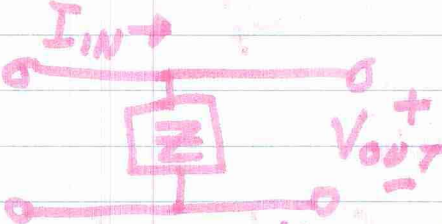


$\dot V_1 = Z_1 I_1 + \dot Z_2 I_2$

$\dot V_2 = V_2 I_1 + Z_2 \dot I_2$



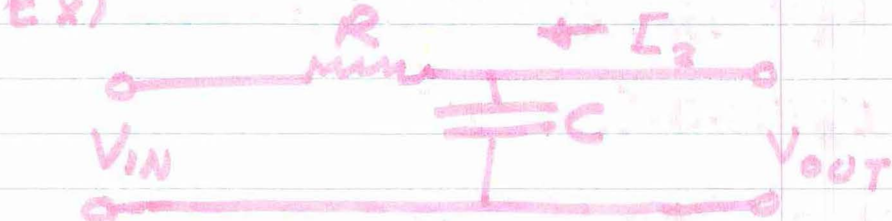
$\frac{V_{OUT}}{V_{IN}} = \dot H = \frac{Z_2}{Z_1 + Z_2} = F(f)\ (= F(\omega))$



$\dot H(\omega) = \frac{V_{OUT}}{I_{IN}} = \dot Z(\omega)$

THINK OF INPUT & OUTPUT

Ex)



$$\dot{H}(\omega) = \frac{V_{IN}}{V_{OUT}}\Big|_{I_2=0}$$

$$\dot{H}(\omega) = \frac{\dot{Z}_C}{Z_R + Z_C}$$

$$= \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$= \frac{1}{1 + j\frac{\omega}{\omega_0}} \quad \left(\omega_0 = \frac{1}{RC}\right)$$

$$|\dot{H}(\omega)| = \frac{1}{\sqrt{1+\left(\omega/\omega_0\right)^2}}$$

$$\theta_H = \tan^{-1}\frac{\omega}{\omega_0}$$

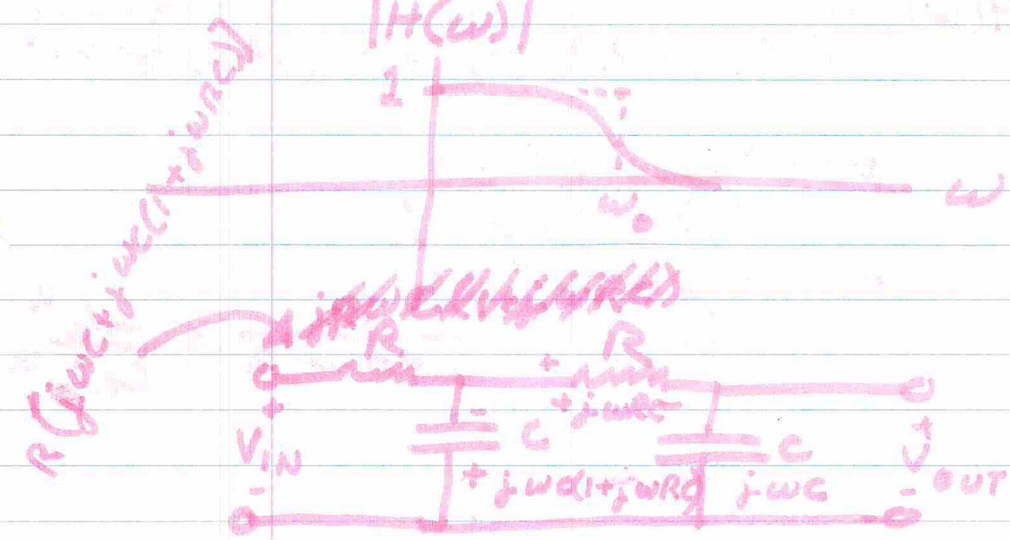
FOR LOW ω (SAY $\omega < .1\omega$)

$$\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2} \approx 1 \;;\; \theta_H \approx \frac{\omega}{\omega_0}$$

HIGH FREQ. RES
(SAY $\omega > 10\,\omega_0$)

$$|H(\omega)| \approx \frac{1}{\frac{\omega}{\omega_0}} = \frac{\omega_0}{\omega}$$

HAVE INVENTED THE
'LOW PASS FILTER'



ASUME V_{OUT} & WORK BACK

$V_{OUT} = (1\angle 0^\circ)$

$\cancel{V_{in} = [(1+j\omega RC) + j\omega C(1+j\omega RC)R]\ 1\angle 0^\circ}$

$\cancel{\frac{V_o}{V_{IN}}}$

(OVER)

$V_{IN} = 1 + j\omega RC + R(j\omega 2C + R(j\omega C)^2)$

$$\frac{\dot{V}_{OUT}}{V_{IN}} = \frac{1}{(j\omega)^2 (RC)^2 + j\omega\, 3RC + 1}$$

$$= \dot{H}(\omega)$$

$$|\dot{H}(\omega)| = \frac{1}{\sqrt{(1-\omega^2R^2C^2)^2 + 9\omega^2R^2C^2}}$$

FOR 'SMALL' ω

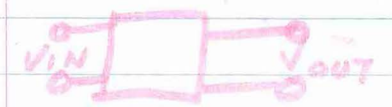
$|\dot{H}(\omega)| \simeq 1$

FOR 'LARGE' ω

$|\dot{H}(\omega)| \simeq \frac{1}{\omega^2R^2C^2} = \frac{\omega_0^2}{\omega^2}$

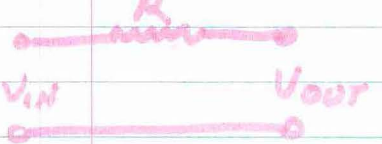
4-21-70

$H(\omega)$

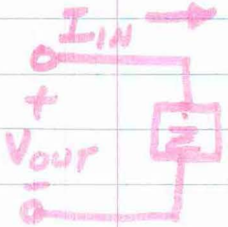


$V_{OUT} = H \quad V_{IN}$

H IS THE SYSTEM FUNCTION



$$\frac{V_{OUT}}{V_{IN}} = H(\omega) = 1$$

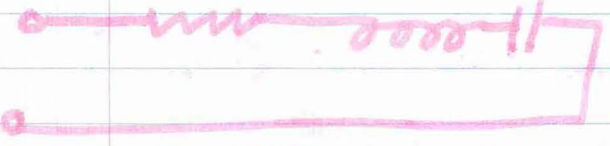


$$\frac{V_{OUT}}{V_{IN}} = Z = H$$

(CALLED DRIVING POINT IMPEDENCE)

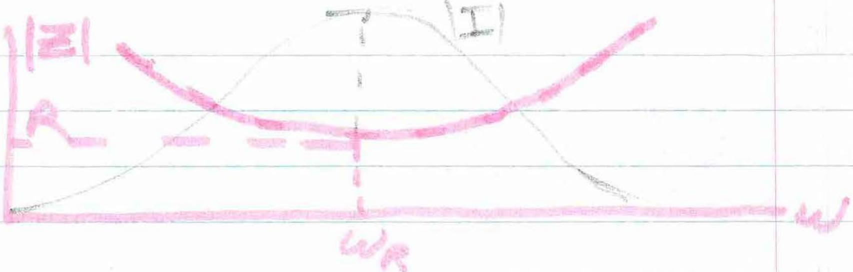
AT RESONANT FREQUENCY
$\Theta = 0$, OR Z IS REAL

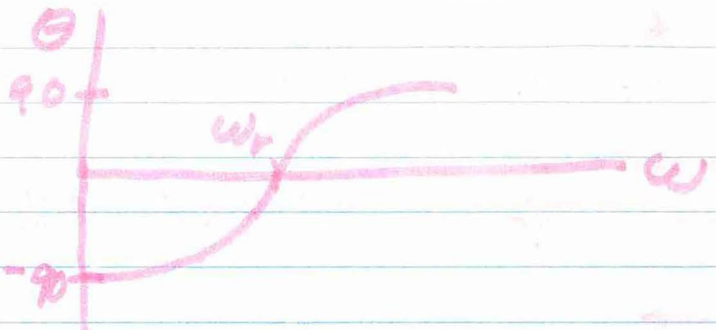
FOR RLC NETWORK



$\omega = \sqrt{\frac{1}{LC}}$ AT RESONANCE

WHEN $Z = R$





$$P = |I|^2 R = \left|\frac{V}{Z}\right|^2 R$$

WHEN $|I| = \frac{1}{\sqrt{2}} I_{MAX}$, $P = \frac{1}{2} P_{MAX}$

THEN $|Z| = \frac{1}{.707} Z_{MIN} = 1.414 R$
(Z AT ½ POWER FREQUENCY)

SOLVE FOR ½ PWR ω

$$2R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$R^2 = \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$R = \left(\omega L - \frac{1}{\omega C}\right)$$

(CONT)

$$\omega^2 LC - \omega RC - 1 = 0$$

$$\omega = \frac{RC \pm \sqrt{R^2C^2 + 4LC}}{2LC}$$

4-22-70

$$\omega L - \frac{1}{\omega C} = \pm R$$

USING +

1) $\omega = \frac{R}{2L} \pm \sqrt{\frac{1}{LC} + \left(\frac{R}{2L}\right)^2}$

USING +

2) $\omega_U = \frac{R}{2L} + \sqrt{\frac{1}{LC} + \left(\frac{R}{L2}\right)^2}$

$$\omega = \pm \frac{R}{2L} + \sqrt{\frac{1}{LC} + \left(\frac{R}{2L}\right)^2}$$

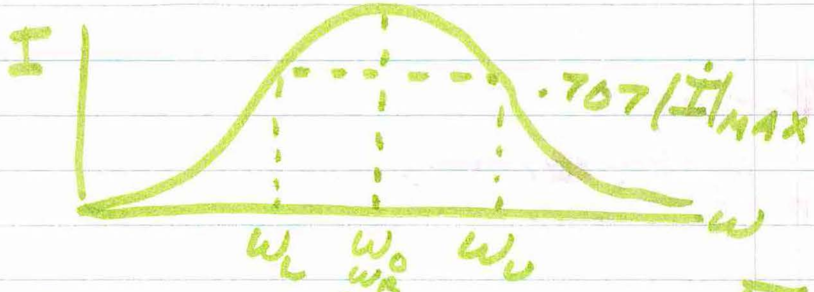
ω_U IS UPPER $\frac{1}{2}$ PWR FREQ

ω_L IS LOWER " " "

let $\frac{R}{2L} = a$; $\frac{1}{LC} = \omega_0^2$

$$\omega_L = \omega_0 \sqrt{1 + \left(\frac{a}{\omega_0}\right)^2} - a$$

$$\omega_U = \omega_0 \sqrt{1 + \left(\frac{a}{\omega_0}\right)^2} + a$$



DEFINITION BANDWIDTH $\overline{BW} = \omega_U - \omega_L$

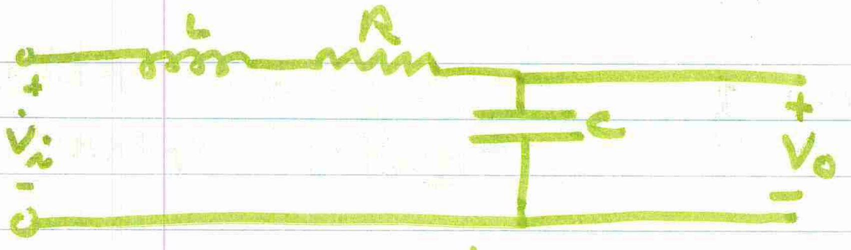
$\overline{BW} = 2a = R/L$

FOR THIS CIRCUIT

$Q_0 = \dfrac{\omega_r L}{R}$

$\therefore \overline{BW} = \dfrac{\omega_r}{Q_0} \Longleftarrow$

$\dfrac{a}{\omega_r} = \dfrac{R/2L}{1/\sqrt{LC}} = \dfrac{R}{2}\sqrt{\dfrac{C}{L}}$



$$\frac{V_o}{V_i} = H(\omega) = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{1}{1 - \omega^2 LC + j\omega RC}$$

$$|H(\omega)| = \frac{1}{\sqrt{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}}$$

$$\angle H = -\tan^{-1} \frac{\omega RC}{1 - \omega^2 LC}$$

WHERE IS $|H(\omega)|$ MAX

WANT TO MINIMIZE DEN

$$\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}$$

OR

$$(1-\omega^2 LC)^2 + \omega^2 R^2 C^2$$

$$0 = \frac{d}{d\omega} = 2(1-\omega^2 LC)(-2\omega LC) + 2\omega R^2 C^2$$

$$\omega_p^2 = \frac{1}{LC} - \frac{1}{2}\left(\frac{R}{L}\right)^2$$

(ω FOR PEAK) CONT.

CASE 1 $\quad \frac{1}{LC} > \frac{1}{2}\left(\frac{R}{L}\right)^2$ DEFINITE PEAK

CASE II $\quad \frac{1}{LC} \leq \frac{1}{2}\left(\frac{R}{L}\right)^2$ "PEAK" AT 0

$$\omega_p = \sqrt{1/LC - \frac{1}{2}(R/L)^2}$$

$$\Rightarrow \omega_p = \omega_r \sqrt{1 - 1/2Q_0^2} \Leftarrow$$

IF Q_0 IS LARGE ENOUGH:

$\omega_p = \omega_R$

CONSIDER $H(\omega_R)$

$$|H(\omega_r)| = 1/\omega_r RC$$

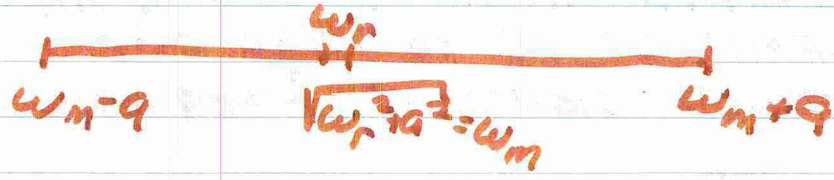
$$= \frac{L/R}{\frac{\omega L}{R} \cdot RC} = \frac{1}{Q_0 R C} = \frac{\sqrt{LC}}{RC}$$

Q_0

PEAK FREQUENCY

$$Q_0 = \frac{\omega_r L}{R} = \frac{1}{\omega_r CR}$$

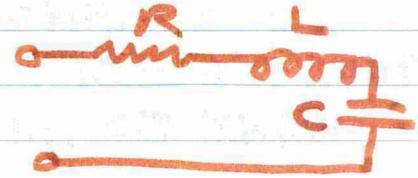
4-23-70



$$\omega_m = \omega_r \sqrt{1 + \left(\frac{a}{\omega_r}\right)^2}$$

$$= \omega_r \sqrt{1 + \left(\frac{BW}{2\omega_r}\right)^2}$$

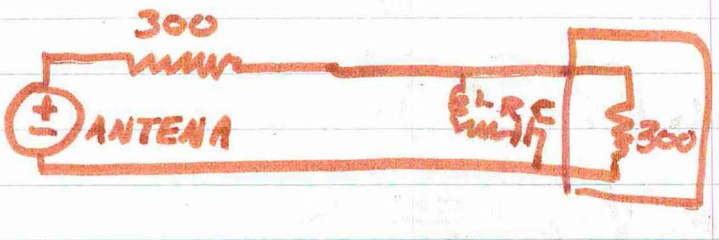
$$= \omega_r \sqrt{1 + \left(\frac{1}{2Q_0}\right)^2}$$



$$BW = \frac{R}{L}$$

$$\omega_r = \sqrt{1/CL}$$

DESIGN A TRAP CIRCUIT TO USE AT THE TERMINALS OF TV TO TRAP OUT INTERFERING SIGNAL AT $f = 89$ MHz



AT $f = f_1$, TRAP IMPEDENCE = R
CAN GET COILS AT Q=200
AT 89 MHz
MUST SELECT L & C
WANT TO MAKE R LOW (10Ω)

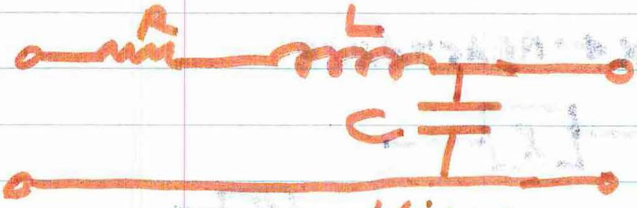
$$\frac{\omega L}{R} = Q = 200 \Rightarrow L = \frac{200 \cdot 10}{6.28 \cdot 89 \cdot 10^6}$$

$$= 3.58\ \mu H$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{\omega_r^2 L} = \frac{10^6}{6.28^2 \times 89^2 \times 10^{12} \times 3.58}$$

$$= .894\ pF$$



$$H(\omega) = \frac{1/j\omega C}{R + j(\omega L - 1/\omega C)}$$

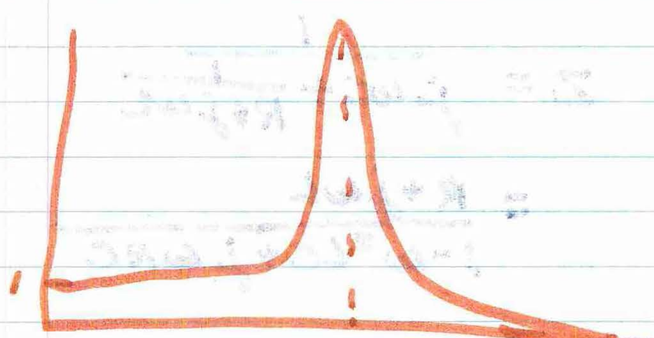
$$= \frac{1}{1 - \omega^2 LC + j\,\omega RC}$$

$$\omega_p = \sqrt{\frac{1}{LC} - \frac{1}{2}\left(\frac{R}{L}\right)^2}$$

$$= \sqrt{\omega_r^2 - \frac{1}{2}a^2}$$

$$= \omega_r \sqrt{1 - \frac{1}{2}\left(\frac{a}{\omega_r}\right)^2}$$

$$= \omega_r \sqrt{1 - 1/2Q_0^2}$$



4-27-70

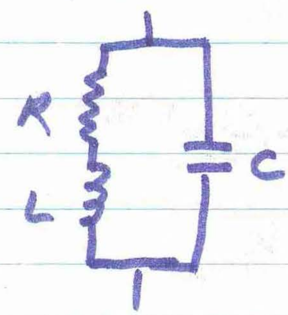
$\dot{Z} = R + jX$ ← REACTANCE

$\dot{Y} = G + jB$

CONDUCTANCE ↗ ↑ SUSCEPTANCE

+SUS ⇒ − REACTANCE

X-1(b)



$L = 30\ mH$
$R = 10\,\Omega$
$C = .5\ \mu F$

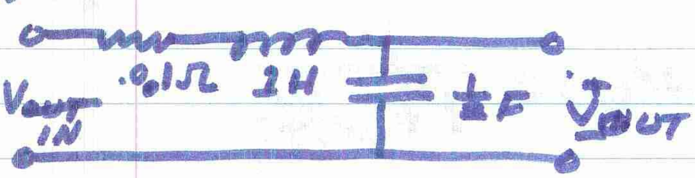
$$Z = \frac{1}{j\omega C + \frac{1}{R + j\omega L}}$$

$$= \frac{R + j\omega L}{1 - \omega^2 LC + j\omega RC}$$

DRIVING PT. IMPEDENCE:

$\dot{Z} = \dot{V}/\dot{I}$

X-17)



$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1-\omega^2 LC + j\omega RC}$$

MINIMIZE

$$\frac{d}{d\omega}\left[(1-.5\omega^2)^2 + 25\times10^{-4}\omega^2\right] = 0$$

$$2(1-.5\omega^2)(-\omega) + 50\times10^{-4}\omega = 0$$

$$\omega^2 = 2$$

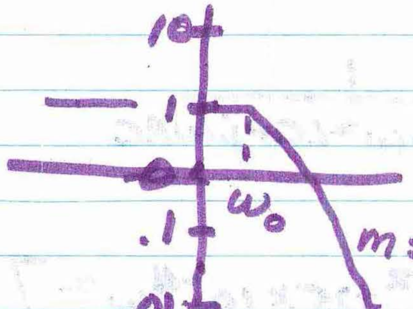
$$\omega = \sqrt{2}\ \frac{RAD}{SEC}$$

4-30-70

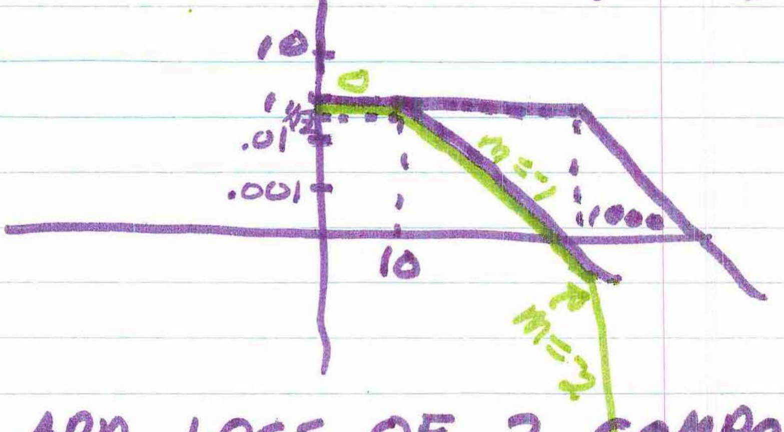
LOG-LOG PLOT $|H(\omega)|$ & $\angle H(\omega)$

$$H(\omega) = \frac{(1+j\frac{\omega}{\omega_1})(1+j(\frac{\omega}{\omega_2}))(1+j\frac{\omega}{\omega_3})}{(1+j\frac{\omega}{\omega_a})(1+j\frac{\omega}{\omega_b})\cdots(j\frac{\omega}{\omega_x})}$$

$$H(\omega) = \frac{1}{1+j\frac{\omega}{\omega_0}}$$



$$H(\omega) = \frac{1}{(1+j\frac{\omega}{10})(1+j\frac{\omega}{1000})}$$

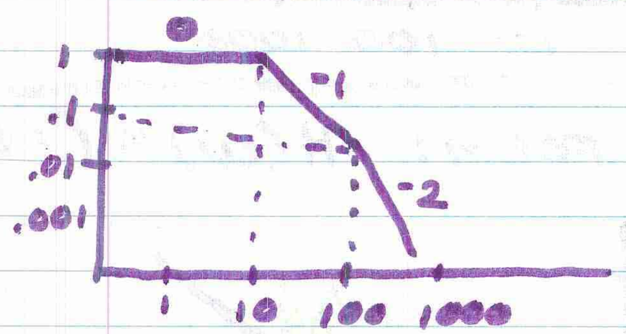


ADD LOGS OF 2 COMPONENT CURVES

FOR REAL CURVE, ROUND OFF CORNERS

$$H(\omega) = \frac{10^3}{(10+j\omega)(100+j\omega)}$$

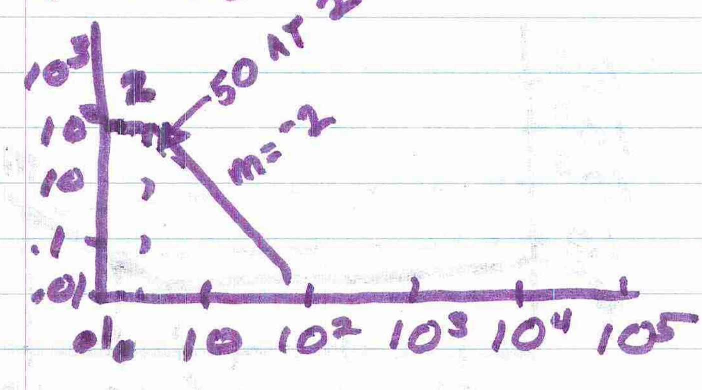
$$= \frac{1}{\left(1+j\frac{\omega}{10}\right)\left(1+j\frac{\omega}{100}\right)}$$



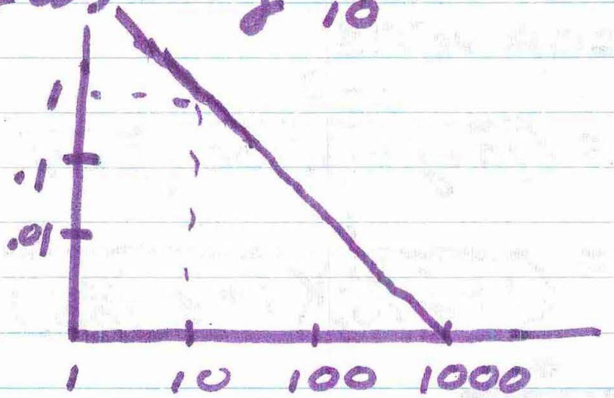
$$H(31.6) = \frac{1}{\sqrt{(1+3.16^2)(1+.316)^2}}$$

$$= \sqrt{1/12.1} = .287$$

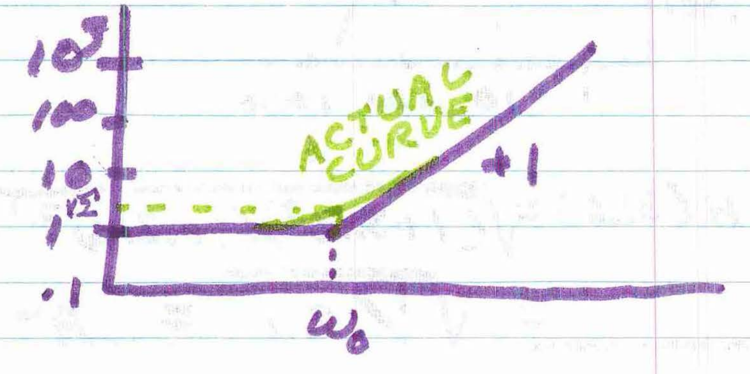
$$H(\omega) = \frac{100}{\left(1+j\frac{\omega}{2}\right)^2}$$



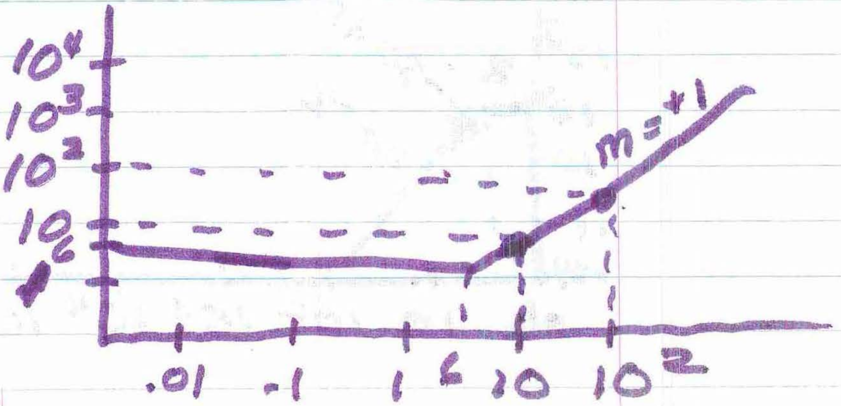
$$H(j\omega) = \frac{1}{j\frac{\omega}{10}}$$



NEW FORM: $H(\omega) = \left(1 + j\frac{\omega}{\omega_0}\right)$

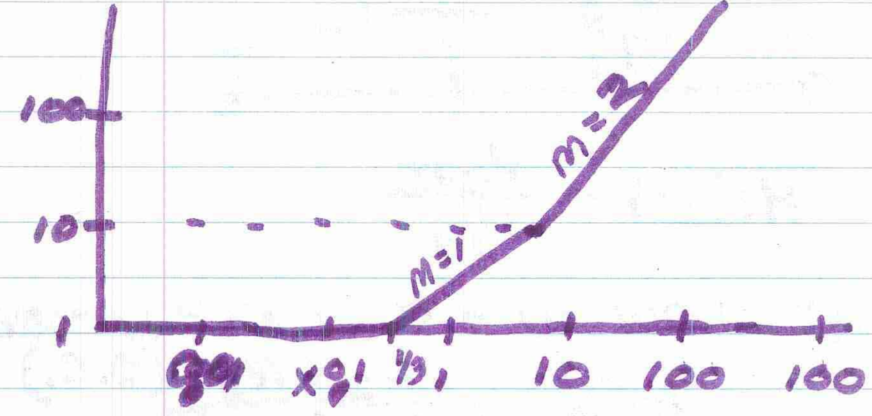


$$H(\omega) = 6 + j\omega = 6\left(1 + j\frac{\omega}{6}\right)$$

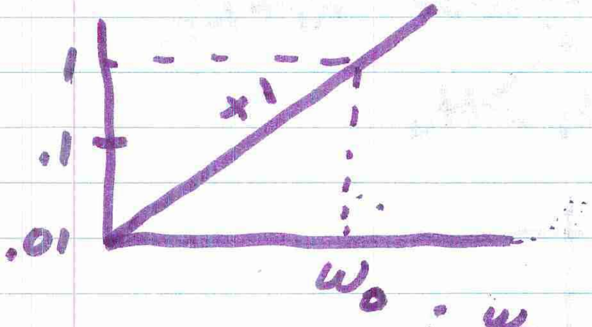


$$H(\omega) = (1 + j\,3\omega)(1 + j\,.1\omega)$$

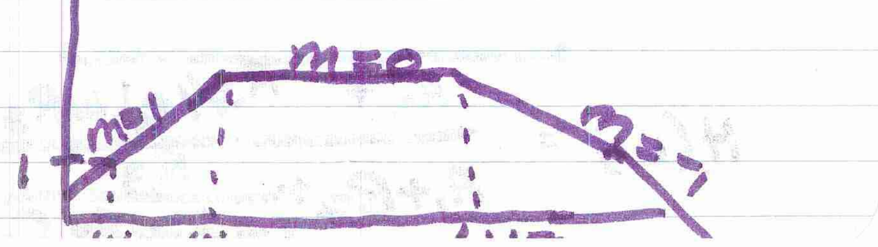
$$= \left(1 + j\frac{\omega}{1/3}\right)\left(1 + j\frac{\omega}{10}\right)$$

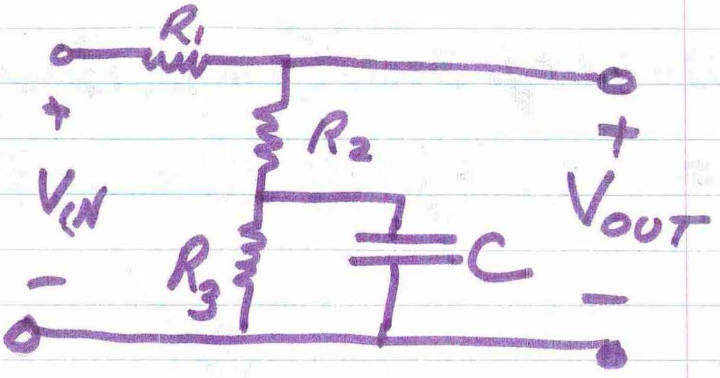


NEW FORM: $H(\omega) = j\frac{\omega}{\omega_0}$



$$H(\omega) = \frac{j\frac{\omega}{\omega_1}}{\left(1 + j\frac{\omega}{\omega_2}\right)\left(1 + j\frac{\omega}{\omega_3}\right)} \qquad \omega_1 < \omega_2 < \omega_3$$





$$H(\omega) = \frac{V_{OUT}}{V_{IN}}$$

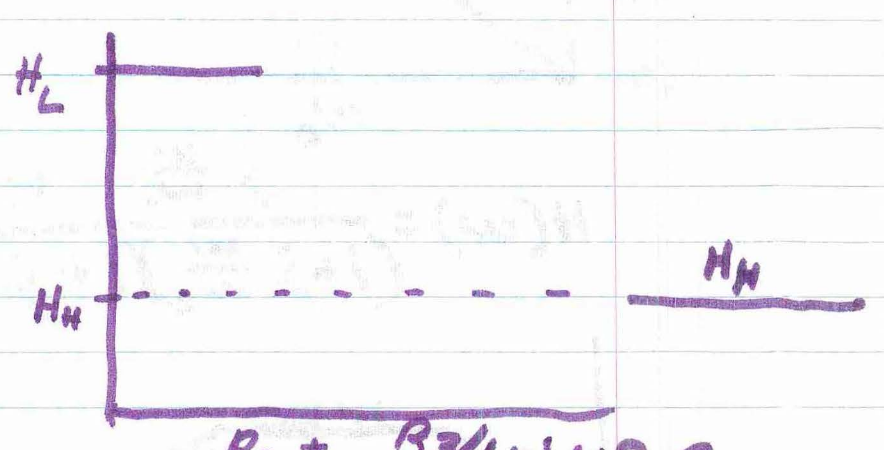
AT LOW ω - C IS NEGLIGIBLE

$$H_L(\omega) = \frac{R_2 + R_3}{R_1 + R_2 + R_3} \quad \text{(O.C.)}$$

AT HIGH ω; C IS SHORTED

$$H_H(\omega) = \frac{R_2}{R_1 + R_2}$$

$$H_L > H_H$$



$$H(\omega) = \frac{R_2 + R_3/(1+j\omega R_3 C)}{R_1 + R_2 + \frac{R_3}{1 + j\omega R_3 C}}$$

(CONT.)

$$H(\omega) = \frac{R_3 + R_2 + j\omega R_2 R_3 C}{R_3 + R_2 + R_1 + j\omega R_3 (R_1 + R_2) C}$$

$$= \frac{R_2 + R_3 \left(1 + j\omega C \frac{R_2 R_3}{R_2 + R_3}\right)}{R_1 + R_2 + R_3 \left(1 + j\omega C \frac{(R_1 + R_2) R_3}{R_1 + R_2 + R_3}\right)}$$

$$= H_L \frac{1 + j \frac{\omega}{\omega_N}}{1 + j \frac{\omega}{\omega_D}}$$

$$\omega_N = \frac{1}{C R_2 R_3 / (R_2 + R_3)}$$

$$\omega_D = \frac{1}{C R_3 (R_1 + R_2) / (R_1 + R_2 + R_3)}$$

$$\frac{H_L}{H_H} = \frac{\omega_N}{\omega_D} \Rightarrow H_H = H_L \frac{\omega_D}{\omega_N}$$

5-4-70

5-5-70

$$\dot{H}(\omega) = \frac{\left(1+j\frac{\omega}{\omega_1}\right)\left(1+j\frac{\omega}{\omega_2}\right)\cdots j\frac{\omega}{\omega_L}}{\left(1+j\frac{\omega}{\omega_A}\right)\left(1+j\frac{\omega}{\omega_B}\right)\cdots\left(1+j\frac{\omega}{\omega_m}\right) j\frac{\omega}{\omega_Z}}$$

ONE LINEAR FACTOR

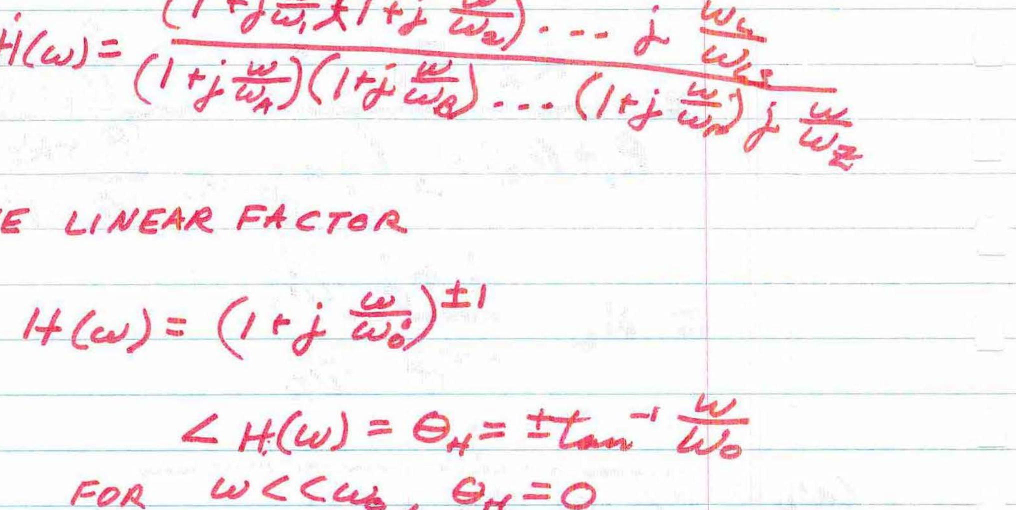
$$H(\omega) = \left(1+j\frac{\omega}{\omega_0}\right)^{\pm 1}$$

$$\angle \dot{H}(\omega) = \theta_H = \pm\tan^{-1}\frac{\omega}{\omega_0}$$

FOR $\omega \ll \omega_0$, $\theta_H = 0$

FOR $\omega \gg \omega_0$, $\theta_H = \pm 90°$

FOR $\omega = \omega_0$, $\theta_H = \pm 45°$



ω	θ_H
$.1\omega_0$	$5.47°$
$.316\omega_0$	$17.55°$
$1.00\,\omega_0$	$45°$
$3.16\omega_0$	$72.45°$
$10\omega_0$	$84.26°$
$.017\omega_0$	$1°$

FOR ASYMYTOTE, TAKE TANGENT AT 45°

$$\theta = \tan^{-1} \frac{\omega}{\omega_0}$$

$$\theta = \tan^{-1} \log \frac{\omega}{\omega_0}$$

$$\omega = \omega_0 \Rightarrow \theta = 45^\circ$$
$$\omega = 1.01\omega_0 \Rightarrow \theta = 45.3^\circ$$

$$\text{slope} = \frac{.3}{.01} \quad \frac{}{(\text{\# DECADE})}$$

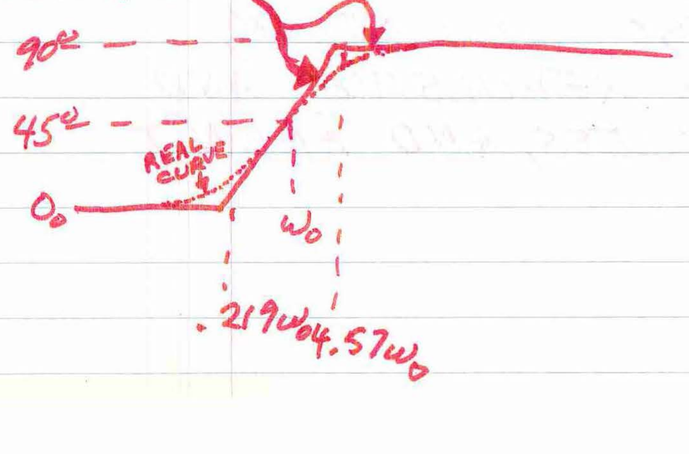
$$(1.01)^x = 10 \Rightarrow X = 231.$$

$$\text{slope} = \frac{.30^\circ}{(1/231)\text{ DECADES}} = 69.3^\circ/\text{DECADE}$$

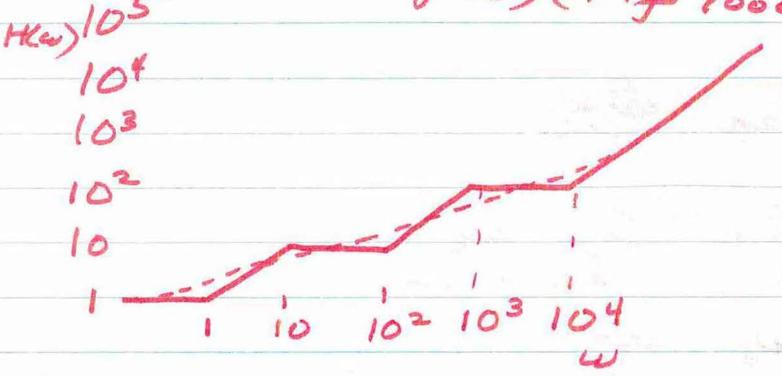
DECADES FROM $45^\circ - 90^\circ = \frac{45}{69.3}$ DECADES
$= .659$ DECADES

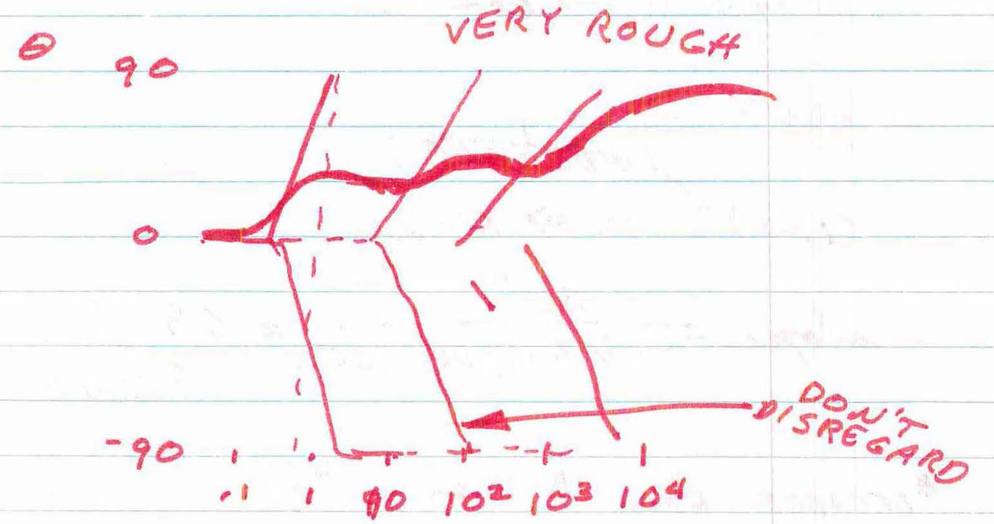
AT $\theta = 90^\circ$; $\omega = 4.57$

ASYMYTOTE PLOT OF θ_H



$$H(\omega) = \frac{(1+j\omega)(1+j\frac{\omega}{100})(1+j\frac{\omega}{10^4})}{(1+j\frac{\omega}{10})(1+j\frac{\omega}{1000})}$$





ASYMYTOTIC PLOTS FOR PHASE ANGLES ARE WORTHLESS, AND STINK, AND ARE REPULSIVE, AND ARE USELESS, AND AREN'T SO NICE.

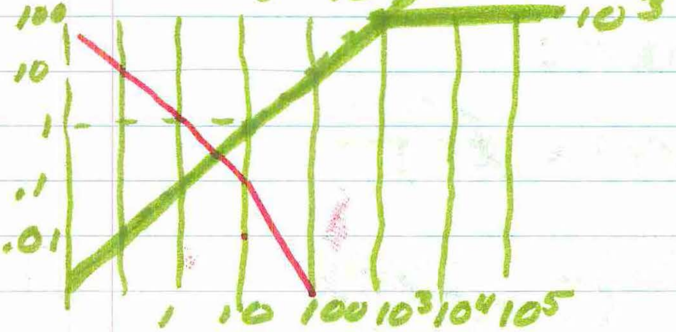
COMPUTOR DO $\Rightarrow$

PLOT MAG. & PHASE FOR

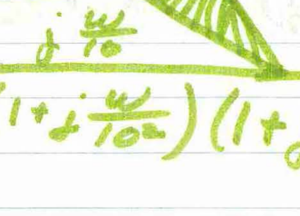
$$H(\omega) = \frac{(1+j\omega)\left(1+j\frac{\omega}{100}\right)\left(1+j\frac{\omega}{10^4}\right)}{\left(1+j\frac{\omega}{10}\right)\left(1+j\frac{\omega}{10^2}\right)\left(1+j\frac{\omega}{10^5}\right)}$$

5-6-70

$$H(\omega) = \frac{j\frac{\omega}{10}}{1+j\frac{\omega}{10^3}}$$



$$H(\omega) = \frac{1}{j\omega\left(1+j\frac{\omega}{10}\right)}$$



$$H(\omega) = \frac{j\frac{\omega}{10}}{\left(1+j\frac{\omega}{10^2}\right)\left(1+j\frac{\omega}{10^3}\right)}$$

$$H(s) = \frac{s+10}{(s+100)(s+1000)}$$

$$s = j\omega$$

$$H(\omega) = \frac{j\omega+10}{(j\omega+100)(j\omega+10^3)}$$

$$= 10^{-4}\,\frac{\left(1+j\frac{\omega}{10}\right)}{\left(1+j\frac{\omega}{100}\right)\left(1+\frac{\omega}{10^3}\right)}$$

CONSIDER:

$$H(s) = \frac{Y(s)}{X(s)}$$

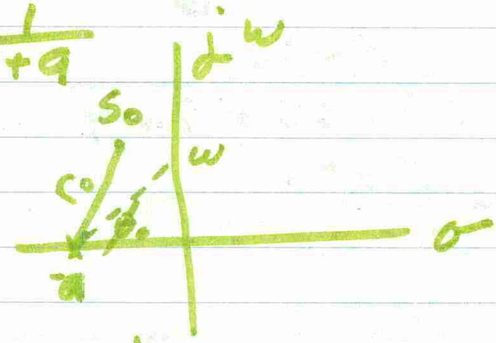
$$Y(s) = \mathcal{L}\{y(t)\}$$
$$X(s) = \mathcal{L}\{x(t)\}$$

NO INITIAL CONDITIONS

$$H(s) = \frac{P(s)}{Q(s)}$$ WHERE BOTH ARE POLYNOMIALS

$s = \sigma + j\omega$

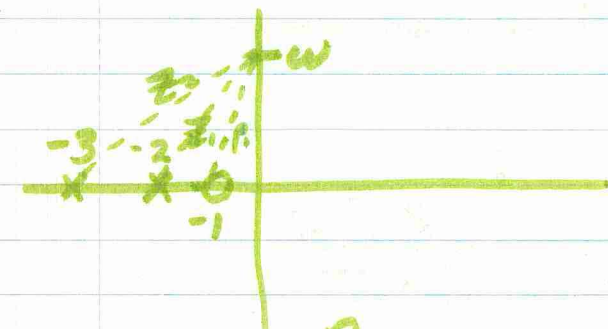
$H(s) = \frac{1}{s+a}$



$|H(s_0)| = 1/r_0$

$\theta_H(\omega_0) = -\phi_0$

$H(s) = \frac{s+1}{s^2+5s+6} = \frac{s+1}{(s+3)(s+2)}$



$|H(\omega_0)| = \frac{p_1}{z_1 \cdot z_2}$

$\angle H(\omega_0) = \theta_H(\omega_0)$

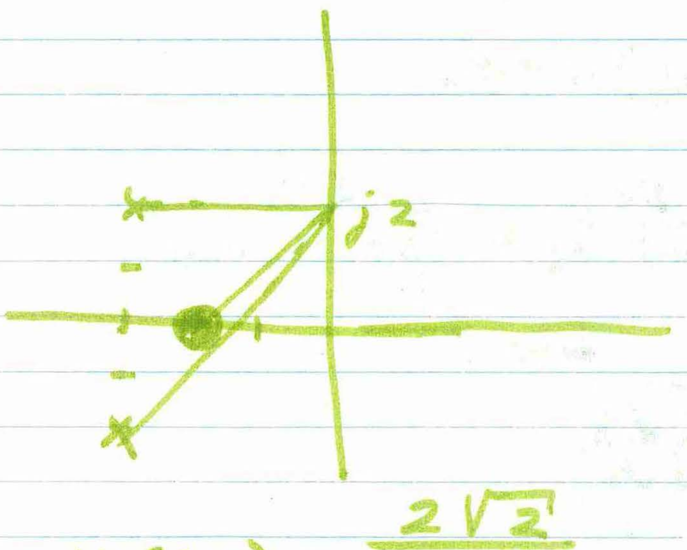
$= \phi_1 - \phi_2 - \phi_3$

$$H(s) = \frac{s+2}{s^2+6s+\cancel{}13}$$

$$= \frac{s+2}{s^2+6s+9+4}$$

$$= \frac{s+2}{(s+3)^2+2^2}$$

$$= \frac{s+2}{(s+3+j2)(s+3-j2)}$$



$$H(j2) = \frac{2\sqrt{2}}{3 \cdot 5}$$

$$\theta_H(j2 = 45° - 0° - 53.9°$$

$$= -8.9°$$

5-11-70

$Z = X + jY = |Z| \angle \theta$

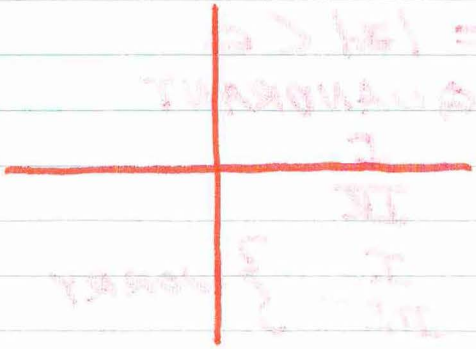
X	Y	QUANDRANT
+	+	I
+	−	IV
−	+	II
−	−	III

II, III } WORRY

$\tan^{-1}\left(\frac{Y}{X}\right)$ ON IBM RANGES ONLY FROM −90 TO 0 TO +90

~~SUB~~

```
      FUNCTION ATAN2(Y,X)
      ATAN2=ATAN(Y/X)
      IF X 1,1,2
1     ATAN2=ATAN2+3.1415926537
2     RETURN
      END
```



$$H(s) = \frac{s+2}{s^2+6s+13} = \frac{s+2}{s^2+6s+9+13-9}$$

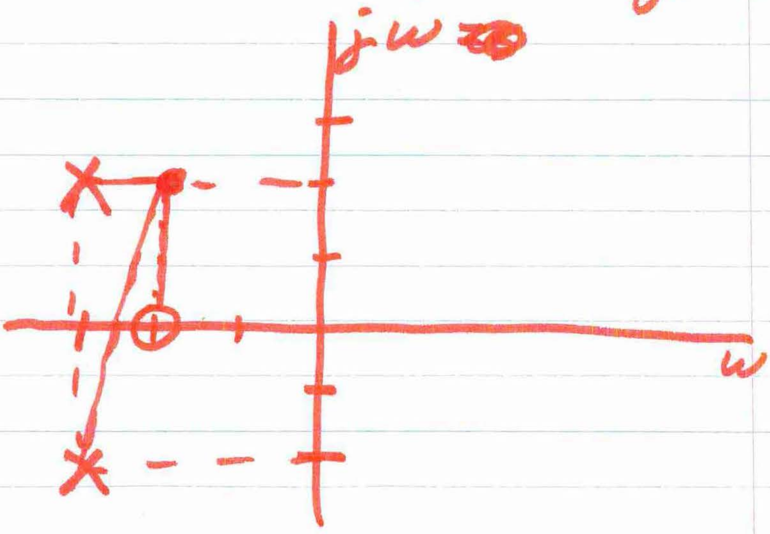
$$= \frac{s+2}{(s+3)^2+2^2}$$

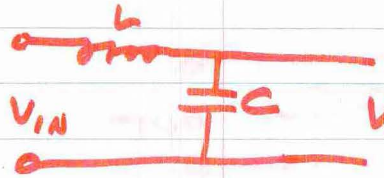
$$= \frac{s+2}{(s+3+j2)(s+3-j2)}$$

O AT $s = -2$

POLES AT $s = -3-j2$

$s = -3+j2$





$$H = \frac{V_o}{V_{IN}} = \frac{\frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{1}{1+s^2LC} = \frac{1/LC}{s^2 + 1/LC}$$

$$= \frac{1/LC}{\left(s+j\sqrt{\frac{1}{LC}}\right)\left(s-j\sqrt{\frac{1}{LC}}\right)}$$

$$H(-2+j2) = ?$$

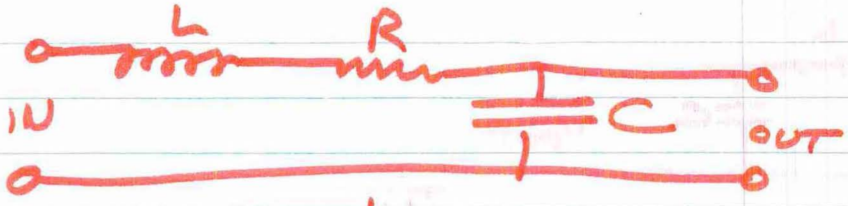
$$|H(-2+j2)| = \frac{2}{1\sqrt{17}}$$

$$= .486$$

$$\approx 75.95^\circ$$

$$\angle H(-2+j2) = 90 - 0 - \tan^{-1} 4$$

$$= 14.05^\circ$$



$$\frac{V_o}{V_i} = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}}$$

$$= \frac{1}{LCs^2 + RCs + 1}$$

$$= \frac{1}{LC} \cdot \frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$= \frac{1}{LC} \cdot \frac{1}{s^2 + \frac{R}{L}s + \left(\frac{R}{2L}\right)^2 + \frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$H(s) = \frac{1}{LC} \cdot \frac{1}{\left(s + \frac{R}{2L}\right)^2 + \left[\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\right]^2}$$

$$= \frac{1}{LC} \cdot \frac{1}{\left(s + \frac{R}{2L} + j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\right)}$$

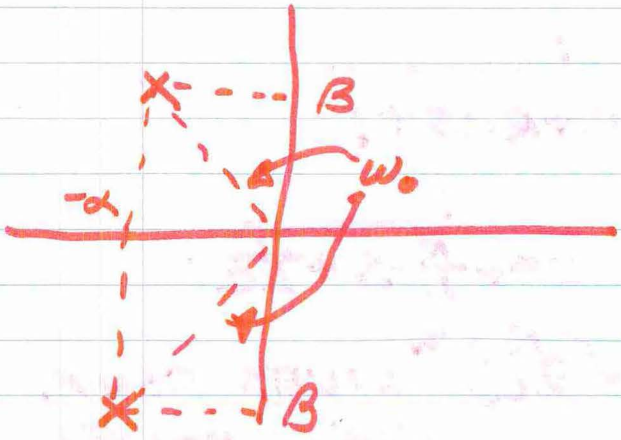
$$\cdot \frac{1}{s + \frac{R}{2L} - j\underbrace{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}}_{\beta}}$$

LET $\frac{1}{LC} = \omega_0^2$

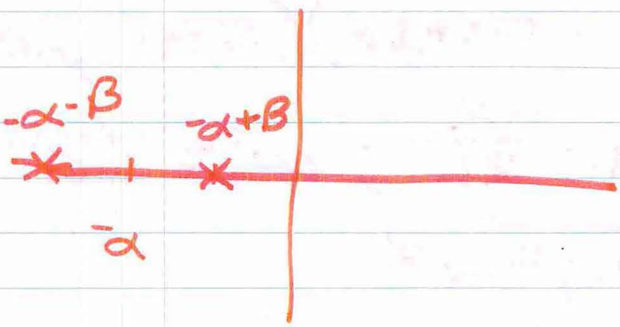
AND $\frac{R}{2L} = \alpha$

$$H(s) = \frac{\omega_0^2}{(s+\alpha+j\beta)(s+\alpha-j\beta)}$$

LET $\beta^2 > 0 \Rightarrow \beta$ IS REAL

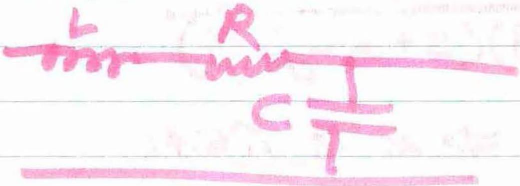


IF $\beta^2 \leq 0 \Rightarrow \beta$ IS IMAGINARY



1) MAKE R LARGER, POLES SWEEP A COICLE
2) VARY C, POLES GO UP & DOWN
3) VARY L, GOES HAY WIRE

5-12-70



$$H(S) = \frac{1}{S^2LC + RCS + 1}$$

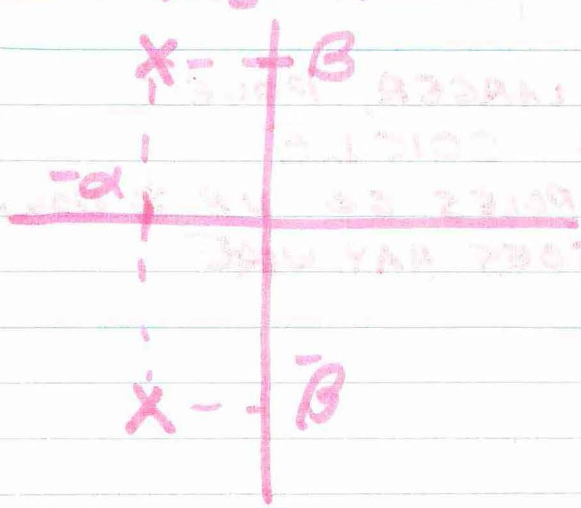
$$= \frac{1}{LC} \frac{1}{S^2 + \frac{R}{L}S + \frac{1}{LC}}$$

$\frac{1}{LC} > \left(\frac{R}{2L}\right)^2$ GIVES COMP. CONJ. POLES

$$H(S) = \frac{1}{LC} \frac{1}{(S+\alpha+j\beta)(S+\alpha-j\beta)}$$

WHERE $\alpha = \frac{R}{2L}$; $\omega_0^2 = \frac{1}{LC}$

AND $\beta = \sqrt{\omega_0^2 - \alpha^2}$



$$\beta = \sqrt{\omega_0^2 - \alpha^2} = \omega_0\sqrt{1 - \left(\frac{\alpha}{\omega_0}\right)^2}$$

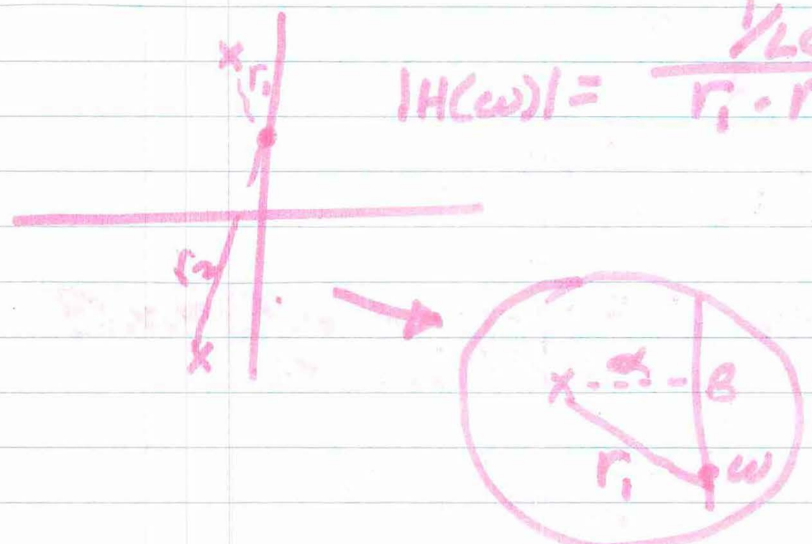
WHERE $\frac{\alpha}{\omega_0} = \frac{R\sqrt{LC}}{2L}$

$$= \frac{R}{2L\omega_0} = \frac{1}{2Q_0}$$

$$\left(Q_0 = \frac{\omega_0 L}{R} = \frac{\text{REACTANCE}}{\text{RESISTANCE}}\right)$$

$$\beta = \omega_0\left(1 - \frac{1}{4Q^2}\right)^{\frac{1}{2}}$$

$$|H(\omega)| = \frac{1/LC}{r_1 \cdot r_2}$$



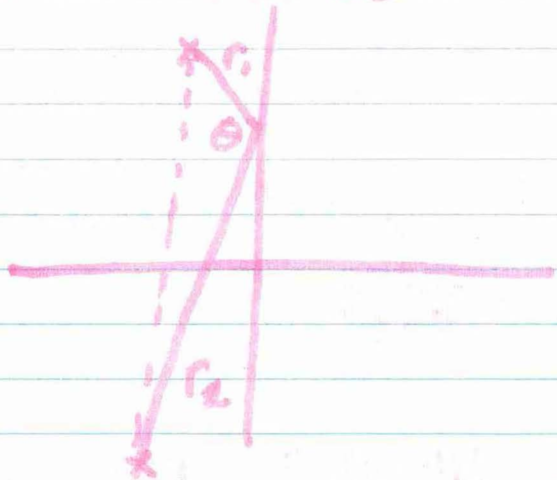
r_2 ALMOST CONSTANT IN VICINITY; $r_2 \simeq 2\beta$

$$\therefore |H(\omega)| = \frac{1/LC}{2\beta r_1}$$

(CONT.)

$$|H(\omega)|_{MAX} \cong \frac{1}{2BLC} \quad \frac{1}{\alpha}$$

$$\cong Q_0$$



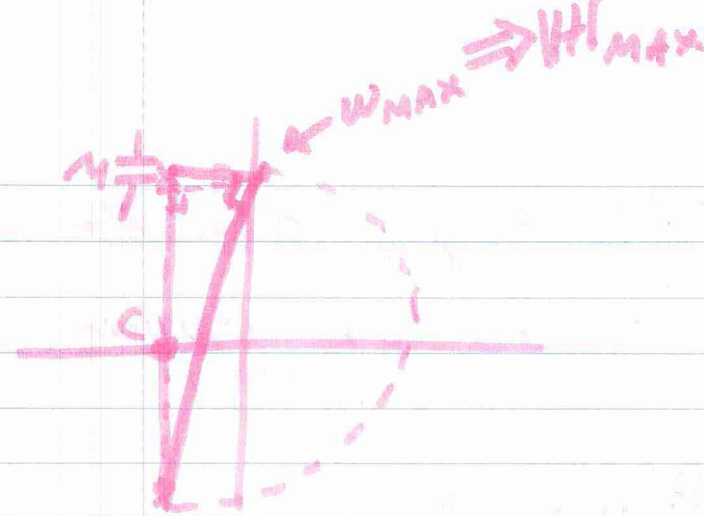
$$|H| = \frac{1/LC}{r_1 \cdot r_2}$$

AREA OF $\Delta = r_1 \cdot r_2 \cdot \sin\theta = \alpha\beta$

$\therefore r_1 r_2 = \alpha\beta / \sin\theta$

$$\Rightarrow |H| = \frac{1}{LC\alpha\beta} \sin\theta$$

$$|H|_{MAX} = \frac{1}{LC\alpha\beta} \qquad (\theta = 90^\circ)$$



$$\omega_{MAX} \simeq \beta$$
$$= \beta - m$$

$$\frac{m}{\alpha} = \frac{\alpha}{2\beta - m}$$

$$0 = (m - \beta - \sqrt{\beta^2 - \alpha^2})(m - \beta + \sqrt{\beta^2 - \alpha^2})$$

$$m = \beta \pm \sqrt{\beta^2 - \alpha^2}$$

$$\therefore \omega_{MAX} = \sqrt{\beta^2 - \alpha^2}$$

ALSO $\omega_{MAX} = \omega_0 \sqrt{1 - \frac{2\alpha^2}{\omega_0^2}}$

$$= \omega_0 \sqrt{1 - \frac{1}{2Q_0^2}}$$

COMPARE WITH:

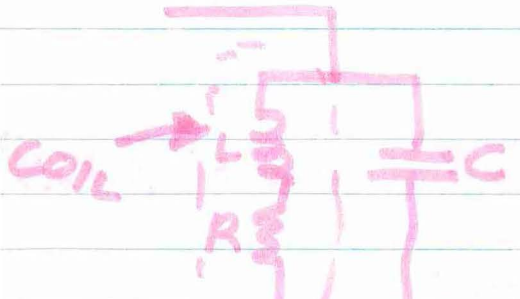
$$\beta = \omega_0 \sqrt{1 - \frac{1}{4Q_0^2}}$$

$|H(\omega)| = |H|_{MAX} \cancel{\sin} \, \theta$

FIND 1/2 POWER VALUES
(AT 45° & 135°)

$\omega_L \cong \omega_0 - \alpha$
$\omega_U \cong \omega_0 + \alpha$
$BW = 2\alpha = \frac{R}{L}$

5-14-70
PRACTICAL ANTI-RESONANT CIR



$$Z(s) = \frac{\frac{1}{sC}(R+sL)}{R+sL+1/sC}$$

$$= \frac{R+sL}{s^2LC + RsC + 1}$$

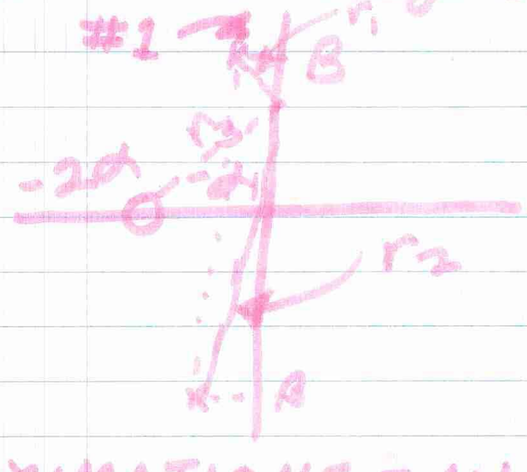
$$= \frac{1}{C}\,\frac{s + R/L}{s^2 + R/L\, s + 1/LC}$$

ROOTS OF DEM. ARE COMP. CONJ.

$$Z(s) = \frac{1}{C}\,\frac{s + R/L}{(s+\alpha)^2 + \beta^2}$$

$$\begin{cases} \alpha = R/2L \\ \beta = \sqrt{\omega_0 - \alpha^2} \\ \omega_0^2 = 1/LC \end{cases}$$

$$Z(s) = \frac{1}{C}\,\frac{s + 2\alpha}{(s+\alpha+j\beta)(s+\alpha-j\beta)}$$



APROXIMATIONS - $\omega_0 >> \alpha$

~~IN VICINITY OF POLE #1~~

$$|Z(\omega)| = \frac{1}{C}\cdot\frac{r_3}{r_1 r_2}$$

IN VICINITY OF POLE 1

$$r_2 \doteq 2\beta;\ r_3 \doteq \beta$$

$$|Z(\omega)| \doteq \frac{1}{C}\,\frac{\beta}{\beta\, 2r} \cong \frac{1}{2Cr_1}$$

$$Z(\omega)_{MAX} \doteq \frac{1}{2\alpha C}$$

$$= \frac{1}{2C\frac{R}{2L}} = \frac{L}{RC}$$

$$\frac{L}{RC} = \frac{\omega_0 L}{\omega_0 RC} = \frac{Q_0}{\omega_0 C} = Q_0 ?$$

$$= \frac{\omega_0 L R}{\omega_0 R^2 C} = \left(\frac{\omega_0 L}{R}\right)\left(\frac{1}{\omega_0 RC}\right)R$$

$$= Q_0^2 R$$

$Q_0 = \frac{1}{\omega_0 RC}$ IS SERIES

$Q_0 \cong \frac{1}{\omega_0 RC}$ IN PARALLEL

$$\omega_{MAX} \doteq \sqrt{1/LC} = \omega_0$$

$$\omega_U \doteq \omega_0 + \alpha$$

$$\omega_P \doteq \omega_0 - \alpha$$

$$BW \doteq 2\alpha$$

$$\angle Z(\omega_M) \cong 0$$

FIND $Z(\omega)$ MAX

$$Z(\omega) = \frac{R + j\omega L}{1-\omega^2 LC + j\omega RC}$$

$$|Z|^2 = \frac{R^2+\omega^2L^2}{(1-\omega^2LC)^2+\omega^2R^2C^2}$$

$x = \omega^2$

$$|Z|^2 = \frac{R^2+xL^2}{(1-xLC)^2+xR^2C^2}$$

$$\cancel{= \frac{R^2+xL}{1-2LCx+L^2C^2x^2+RCx}}$$

$$\cancel{= \frac{R^2+L^2x}{LCx^2+(RC-2LC)x+1}}$$

$$\cancel{= \frac{\frac{R^2}{LC}+\frac{L}{C}x}{x^2+\left(\frac{R}{L}-2\right)x+\frac{1}{LC}}}$$

$$= \frac{R^2+L^2x}{L^2C^2x^2+(R^2C^2-2LC)x+1}$$

$$= \frac{1}{C^2}\,\frac{x+\left(\frac{R}{L}\right)^2}{x^2+\left(\frac{R^2}{L^2}-\frac{2}{LC}\right)x+\frac{1}{L^2C^2}}$$

DIFFERINTIATE &
SET TO 0

(CONT)

$$w_M = \sqrt{x}$$

$$x = \sqrt{\left(\frac{b}{2}\right)^2 - E} - \frac{b}{2}$$

HAIRY!!

5-18-70

FOURIER SERIES

$$x(t) = \sum_{n=N_1}^{N_2} \alpha_n \underbrace{\phi_n(t)}_{\text{BASIS FUNCTIONS}}$$

FOR FOURIES SERIES, $\phi_n(t)$ ARE SINUSOIDS

$$\therefore\ x(t) = \frac{C_0}{2} + \sum_{n=1}^{\infty} c_n \sin(n\omega_0 t + \theta_N)$$

$$t_1 < t < t_2$$

D.C. VALUE

(C_n) ARE THE COEFFICIENS WHICH MUST BE DETERMINED

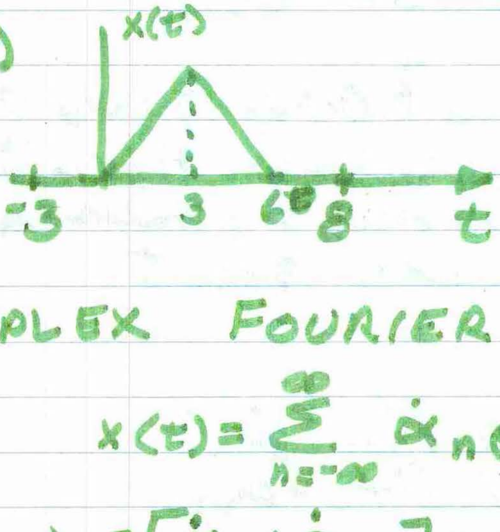
ANOTHER REPRESENTATION:

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t$$
$$+ \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$
$$t_1 < t < t_2$$

ω_0 IS FUNDIMENTAL FREQ OF F.S.

$$\omega_0 = 2\pi f_0$$
$$f_0 = \frac{1}{t_2 - t_1} = \frac{1}{T} = \frac{1}{\text{PERIOD}}$$

Ex)



$T = 11$ SEC

COMPLEX FOURIER SERIES

$$x(t) = \sum_{n=-\infty}^{\infty} \dot{\alpha}_n e^{jn\omega_0 t}$$
$$t_1 < t < t_2$$

$$(\ \ ) = [\dot{\alpha}_n + \dot{\alpha}_{-n}] \cos n\omega_0 t$$
$$+ j[\dot{\alpha}_n - \dot{\alpha}_{-n}] \sin n\omega_0 t$$

$\dot{\alpha}_n + \dot{\alpha}_{-n}$ MUST BE REAL

$\dot{\alpha}_n - \dot{\alpha}_{-n}$ MUST BE IMAG.

(CONT.)

$$Re(\alpha_n) + j\,Im(\alpha_n) + Re(\alpha_{-n}) + j\,Im(\alpha_{-n}) = \text{REAL}$$

$$Re(\alpha_n) + Re(\alpha_{-n}) + j\left[Im(\alpha_{-n}) + Im(\alpha_n)\right]$$

$$\therefore Im(\alpha_n) = -Im(\alpha_{-n})$$

$$\cancel{Re}\ \alpha_n - \alpha_{-n} = \cancel{\text{real}}\ \text{imag}$$

$$Re(\alpha_n) = R(\alpha_{-n})$$

ERGO $\dot{\alpha}_n$ IS COMPLEX CONJUGATE OF $\dot{\alpha}_{-n}$

$$(\ ) = 2Re\{\alpha_n\}\cos n\omega_0 t - 2\,Im\{\alpha_n\}\sin n\omega_0 t$$
$$= a_n \cos n\omega_0 t + \cancel{2\,Im(\alpha_n}$$
$$- b_n \sin n\omega_0 t$$

CONCESION:

$$a_n = 2\,Re\{\dot{\alpha}_n\}$$
$$b_n = 2\,Im\{\dot{\alpha}_n\}$$

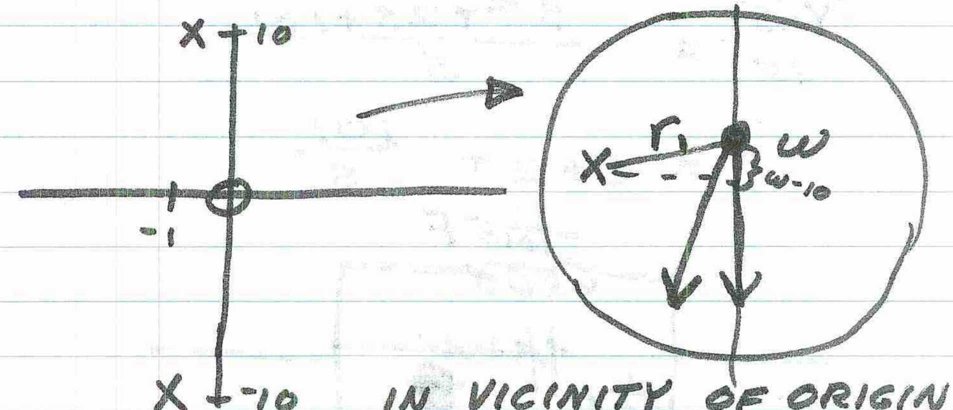
$$a_n = \frac{2}{T}\int_{t_1}^{t_2} x(t)\cos n\omega_o t\,dt$$

$$b_n = \frac{2}{T}\int_{t_1}^{t_2} x(t)\sin n\omega_o t\,dt$$

$$\alpha_n = \frac{1}{T}\int_{t_1}^{t_2} x(t)\,e^{-jm\omega_o t}\,dt$$

5-20-70

18.34) $Z(s) = \frac{1}{10}\,\frac{s}{(s+1+j10)(s+1-j10)}$



IN VICINITY OF ORIGIN

a) $H(\omega) = \frac{1}{10}\;\frac{10}{20\cdot r_1}$

$= .05/r_1$

$= .05/\sqrt{1+(\omega-10)^2}$

Z(S) IN RESONANCE WHEN

$\angle$NUM = $\angle$DEN OR WHEN FUNC. PEAKS (TWO DEF.)

(CONT.)

b) $Z(s) = \frac{1}{10} \frac{s}{(s+1+j10)(s+1-j10)}$

$= \frac{.1s}{s^2+2s+101}$

$\angle Z(\omega) = 0$ AT $\omega = \sqrt{101}$

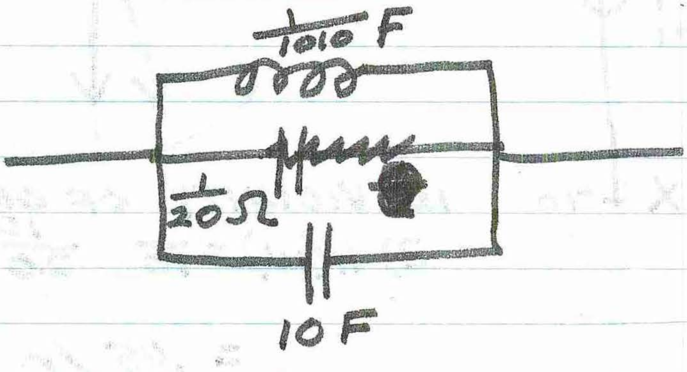
c) $Z(j\sqrt{101}) = \frac{j \cdot .01 \times \sqrt{101}}{j\,2 \cdot \sqrt{101}}$

$= .05\,\Omega$

HOW DRAW CIRCUIT FROM

$\frac{Y(s)}{10} = \frac{s^2+2s+101}{s}$

$= s + 2 + \frac{101}{s}$



MORE ON SWELL FOURIER SERIES

$$x(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{jn\omega_0 t}$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$\alpha_n = \frac{1}{T}\int_T x(t) e^{-jn\omega_0 t} dt$$

$$a_n = 2\,\mathrm{Re}\{\alpha_n\} = \frac{2}{T}\int_T x(t)\cos n\omega_0 t\, dt$$

$$b_n = -2\,\mathrm{Im}\{\alpha_n\} = \frac{2}{T}\int_T x(t)\sin n\omega_0 t\, dt$$

Let $x(t)$ BE AN EVEN FUNCTIOT

$$\int_{-T/2}^{T/2}$$

$$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \underbrace{\underbrace{x(t)}_{\text{EVEN}}\ \underbrace{\sin n\omega_0 t}_{\text{ODD}}}_{\text{ODD}}\, dt = 0$$

$$\Rightarrow \alpha_n \text{ IS REAL } \{\mathrm{Im}\{\alpha_n\} = 0\}$$

$$\alpha_n = \frac{1}{T}\int_T x(t)\left[\cos n\omega_0 t - j\sin n\omega_0 t\right] dt$$

$$= \frac{1}{T}\int_T x(t)\cos n\omega_0 t\, dt$$

$$- j\frac{1}{T}\int_T x(t)\sin n\omega_0 t\, dt$$

1) IF $x(t)$ IS EVEN

$$b_n = 0$$
$$Im\{\alpha_n\} = 0$$
$$\alpha_n = \frac{1}{T}\int_T x(t)\cos n\omega_o t\,dt$$

2) IF $x(t)$ IS ODD

$$a_n = 0$$
$$Re\{\alpha_n\} = 0$$
$$\alpha_n = \cancel{\frac{1}{T}}\int_T x(t)\sin n\omega_o t\,dt$$

SHIFTED TIME FUNCTIONS

$$f(t-t_o)$$

$$\alpha_n = \frac{1}{T}\int_{t_2}^{t_1} f(t-t_o)e^{-jn\omega_o t}dt$$

Let $t - t_o = U$

$$\alpha_{n_S} = \frac{1}{T}\int_{t_1-t_o}^{t_2-t_o} f(U)\,e^{-jn\omega_o u} \cdot e^{-jn\omega_o t_o}\,du$$

$$\alpha_{n_S} = e^{-jn\omega_o t_o}\left[\frac{1}{T}\int_{t_1'}^{t_2'} f(U)e^{-jn\omega_o u}\,dU\right]$$

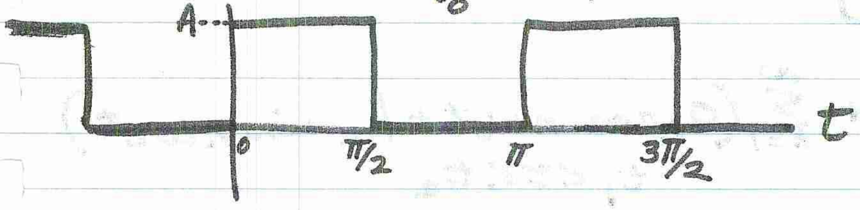
$$\alpha_{nS} = e^{-jn\omega_o t_o}\alpha_n$$

$e^{jx} = \cos x + j\sin x$

$\left| e^{-jn\omega_0 t_0} \right| = 1$ (CHANGING MAGNITUDE OF $\angle$)

EXAMPLE

$t_0 = +\pi/4$



$\alpha_{n_0} = \frac{A}{2}\left(\sin\frac{n\pi}{2}\right)/\,n\pi/2$

$\alpha_n = \frac{A}{2}\;\frac{\sin\frac{n\pi}{2}}{\frac{n\pi}{2}} \cdot e^{-jn\overbrace{\left(\frac{2\pi}{T}\right)}^{\omega_0}\frac{T}{4}}$

$= \frac{A}{2}\;\frac{\sin\frac{n\pi}{2}}{\frac{n\pi}{2}}\, e^{-j\frac{n\pi}{2}}$

$n = 0,\; \alpha_n = \frac{A}{2}$

$n = 1;\; \alpha_n = \frac{A}{2}\frac{2}{\pi}\cdot j$

$n = 2;\; \alpha_n = (\qquad) - 1$

$n = 3;$

$$\frac{\sin \frac{n\pi}{2}}{\frac{n\pi}{2}}$$ IS NEAT

5-21-70

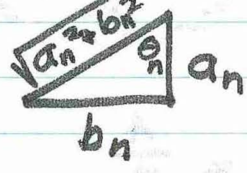
$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$t_1 < t < t_2$$

LOOK AT

$$a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$\sqrt{a_n^2+b_n^2} \left(\frac{a_n}{\sqrt{a_n^2+b_n^2}} \cos n\omega_0 t + \frac{b_n}{\sqrt{a_n^2+b_n^2}} \sin n\omega_0 t \right.$$



$$= \sqrt{a_n^2+b_n^2} \left(\sin\theta_n \cos n\omega_0 t + \cos\theta_n \sin n\omega_0 t \right.$$

$$= \sqrt{a_n^2+b_n^2} \sin(n\omega_0 t + \theta_n)$$

$$= c_n \sin(n\omega_0 t + \theta_n)$$

$$f(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{jn\omega_0 t}$$

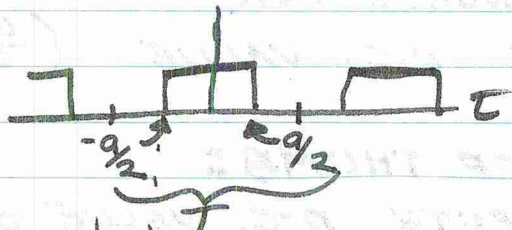
$$f(t-t_0) = \sum_{n=-\infty}^{\infty} (\alpha_n \underbrace{e^{-jn\omega_0 t_0})}_{\text{CHANGES PHASE OF THE COMPONENT}} e^{jn\omega_0 t}$$

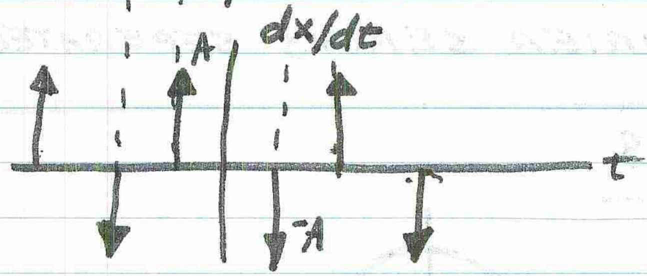
$$f(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{jn\omega_0 t}$$

$$\frac{d}{dt} f(t) = \sum_{n=-\infty}^{\infty} \alpha_n \, jn\omega_0 \, e^{jn\omega_0 t}$$

THE COEFFICIENTS OF df/dt ARE $j\alpha_n n\omega_0$

Ex)





COMPUTE FOURIER COEFFICIENTS FOR $\frac{dx}{dt}$ ALONE

$$\alpha_n' = \frac{1}{T}\int_{-T/2}^{T/2}\left[A\delta\left(t+\tfrac{a}{2}\right) - A\delta\left(t-\tfrac{a}{2}\right)\right]e^{-jn\omega_0 t}$$

$$= \frac{A}{T}\left[e^{-jn\omega_0 \frac{-a}{2}} - e^{-jn\omega_0 \frac{a}{2}}\right]$$

FOR x-t, $\alpha_n = \alpha_n' / jn\omega_0$

$$\alpha_n = \frac{A}{jn\omega_0 T}\left(e^{jn\omega_0 a/2} - e^{-jn\omega_0 a/2}\right)$$

SIFTING PROPERTY OF DELTA FUNCTION

$$\int_{t_1}^{t_2} \delta(t-x) f(t)\,dt = f(x)$$

$$\alpha_n = \frac{2A}{n\omega_0 T} \sin \frac{n\omega_0 a}{2}$$

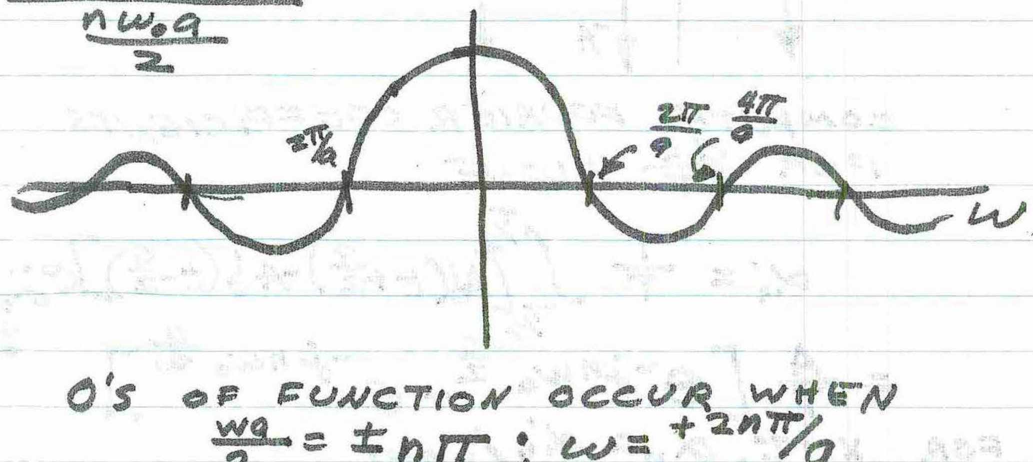
$$= \frac{Aa}{T} \frac{\sin n\omega_0 a/2}{n\omega_0 a/2}$$

MUST COMPENSATE FOR FUNCTION'S D.C. VALUE $\left(\frac{a_0}{2}\right)$

GOOD RULE OF THUMB:

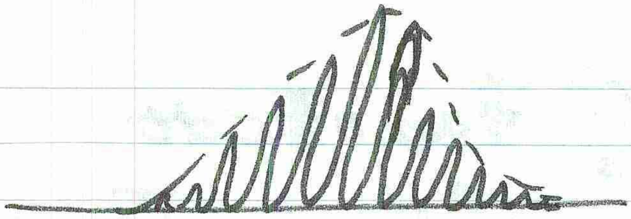
ALWAYS COMPUTE D-C. VALUE OF A FOURIER SERIES SEPERATELY

$$\frac{\sin \frac{n\omega_0 a}{2}}{\frac{n\omega_0 a}{2}}$$



0'S OF FUNCTION OCCUR WHEN

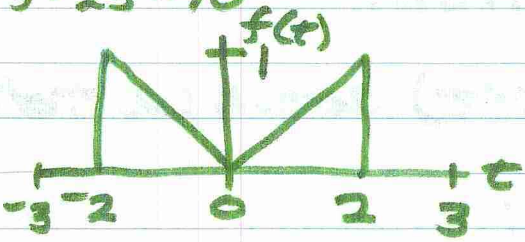
$\frac{\omega a}{2} = \pm n\pi$; $\omega = {}^{+}2n\pi/a$



WITH PULSE TRAIN

INCREASE T, INCREASE LENGTH

5-25-70



FIND FOURIER EXPANSION

$\cancel{f(t) = \sum_{n=1}}$

NEED TO FIND $\{a_n\}$ & $\{b_n\}$

EVEN FUNC. $\Rightarrow$ ALL $b_n = 0$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega_0 t \, dt$$

$$T = 6 \Rightarrow \omega_0 = \frac{\pi}{3}$$

$$a_n = \frac{1}{3}\int_{-3}^{3} f(t)\cos n\frac{\pi}{3}t \, dt$$

$$= \frac{1}{3}\Big[\int_{-2}^{0} -\frac{t}{2}\cos n\frac{\pi}{3}t \, dt$$

$$+ \int_{0}^{2} \frac{t}{2}\cos n\frac{\pi}{3}t \, dt\Big]$$ (CONT.)

$$a_n = \frac{1}{3}\left[\int_{-2}^{0} \frac{-t}{2}\cos n\frac{\pi}{3}t\,dt + \int_{0}^{2}\frac{t}{2}\cos n\frac{\pi}{3}t\,dt\right]$$

$$= \frac{2}{3}\int_{0}^{2}\frac{t}{2}\cos n\frac{\pi}{3}t\,dt$$

FOR EVEN FUNCTION:

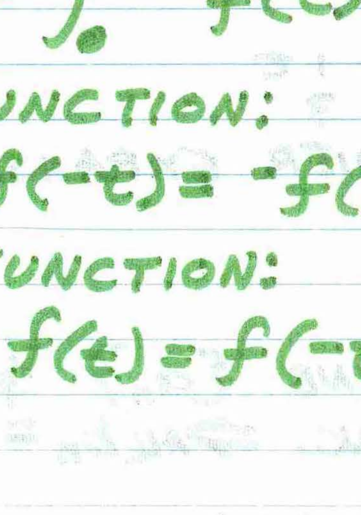
$$a_n = \frac{4}{T}\int_{0}^{\frac{T}{2}} f(t)\cos n w_0 t\,dt$$

ODD FUNCTION:

$$f(-t) = -f(t)$$

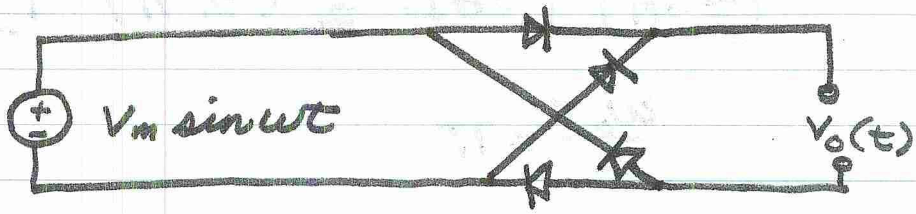
EVEN FUNCTION:

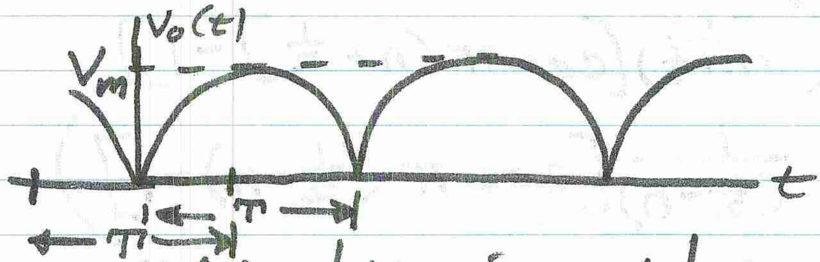
$$f(t) = f(-t)$$



5-26-70

$$a_0 = \frac{2}{T}\int_0^T f(t)\,dt$$





$V_0(t) = |V_m \sin \omega t|$

EVEN FUNC ⇒ NO SIN TERMS (all $b_n = 0$)

$$a_n = \frac{4}{T}\int_0^{\frac{T}{2}} \sin \omega t \cos n\omega_0 t\,dt$$

$$\omega_0 = 2\omega \;;\; T = \frac{2\pi}{\omega_0}$$

$$a_n = \frac{2}{T}\int_0^{\frac{T}{2}} \sin(\omega + n\omega_0)t\,dt$$

$$+ \frac{2}{T}\int_0^{\frac{T}{2}} \sin(\omega - n\omega_0)t\,dt$$

$$= \frac{2}{T}\left(-\frac{\cos(2\omega + n\omega_0 t)\Big|_0^{\frac{T}{2}}}{\omega + n\omega_0} - \frac{\cos(\omega - n\omega_0)t\Big|_0^{\frac{T}{2}}}{\omega - n\omega_0} \right)$$

(CONT.)

$$a_n = \frac{2}{T\omega_0}\left\{\frac{-1}{(n+\frac{1}{2})}\left[\cos\frac{\omega_0 t}{2}\left(\frac{1}{2}+n\right)-1\right]\right.$$

$$\left.-\frac{1}{(\frac{1}{2}-n)}\left[\cos\frac{\omega_0 t}{2}\left(\frac{1}{2}-n\right)-1\right]\right\}$$

$$\frac{\omega_0 T}{2} = \pi$$

$$a_n = \frac{V_m}{\pi}\left(\frac{-1}{(n+\frac{1}{2})}\left[\cos\pi(n+\frac{1}{2}\right]-1\right]$$

$$\left.-\frac{1}{(\frac{1}{2}-n)}\left[\cos\pi\left(\frac{1}{2}-n\right)-1\right]\right)$$

$$a_n = \frac{V_m}{\pi}\left\{\frac{2}{2n+1}\left[1-\cos\pi\left(n+\frac{1}{2}\right)\right]\right.$$

$$\left.+\frac{2}{1-2n}\left[1-\cos\pi\left(n-\frac{1}{2}\right)\right]\right\}$$

1) $n=0$
$a_0 = \frac{4V_m}{\pi} \Rightarrow$ d.c. VALUE $= \frac{2V_m}{\pi}$

2) $n=1 \Rightarrow a_1 = \frac{V_m}{\pi}\frac{4}{3}$

FIND RATIO OF a_1 TO d-c:

$$\frac{\frac{4}{3\pi}V_m}{\frac{2}{\pi}V_m} = \frac{2}{3}$$

IF $V_m = 100$

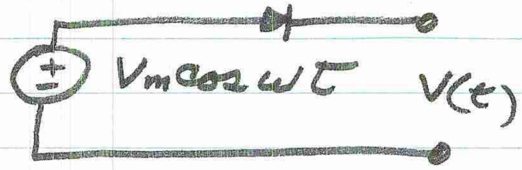
$d.c. = 63.7\ V$

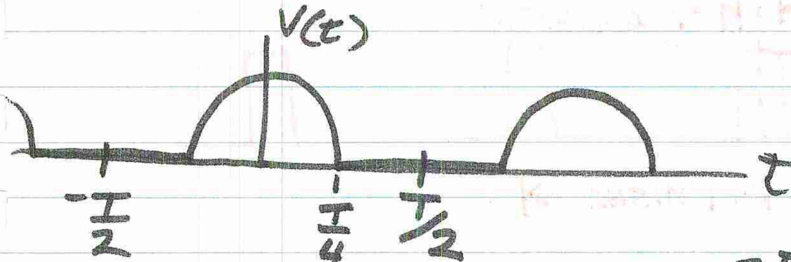
~~FOR~~ a_n MAY BE SIMPLIFIED

$$a_n = \frac{2V_m}{\pi}\left\{\frac{1}{2n+1} + \frac{1}{1-2n}\right\}$$

$$= \quad a_n = \frac{4V_m}{\pi(1-4n^2)}$$

HALF WAVE RECTIFIER





$$T = \frac{2\pi}{\omega}\ ;\ \omega = \omega_0$$

$$a_n = \frac{4}{T}\int_0^{\frac{T}{4}} \cos \omega t \cos n\omega t\, dt$$

$$= \frac{2}{T}\int_0^{\frac{T}{4}}\left[\cos(n+1)\omega t + \cos(n-1)\omega t\right]dt$$

$$= \frac{2}{T}\left[\frac{\sin(n+1)\omega t\Big|_0^{\frac{T}{4}}}{(n+1)\omega} + \frac{\sin(n-1)\omega t\Big|_0^{\frac{T}{4}}}{(n-1)\omega}\right]$$

(CONT.)

$$\cancel{a_n = \frac{2}{T}\left\{\frac{\sin(n+1)\frac{\omega T}{4}}{(n+1)\omega} + \frac{\sin(n-1)\frac{\omega T}{4}}{(n-1)\omega}\right\}}$$

$$\cancel{\frac{\omega T}{4} = \frac{\pi}{2}}$$

$$\underline{a_n = \frac{1}{\pi}\left\{\frac{\sin(n+1)\frac{\pi}{2}}{n+1} + \frac{\sin(n-1)\frac{\pi}{2}}{n-1}\right\}} \quad n \neq 1$$

$$a_n = 2\,R_e\{\alpha_n\}$$
$$b_n = -2\,\mathscr{I}m\{\alpha_n\}$$

$$\omega_0 T = 2\pi$$

6-1-70

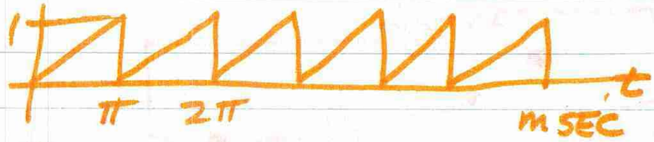
X-35)

$$a_n = \frac{2Aa}{T}\;\frac{\sin\frac{n\omega_0 a}{2}}{\frac{n\omega_0 a}{2}}$$

$A = 10$
$a = 10^{-6}$
$T = 10^{-3}$
$\omega_0 = \frac{2\pi}{T} = 2000\pi$

PLUG AND CHUG

X-36,37)



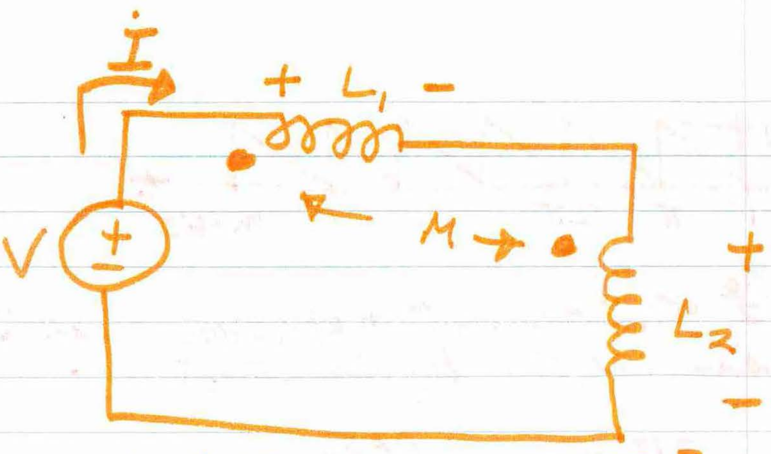
$$f(t) = \frac{a_0}{2} + a_1 \cos w_0 t + a_2 \cos 2 w_0 t \ldots + b_1 \sin w_0 t + b_2 \sin 2 w_0 t \ldots$$

$$w_0 = \frac{2\pi}{T} = 2000 \frac{RAD}{SEC}$$

$$f(t) \longrightarrow \boxed{\begin{array}{l} \text{LOPASS} \\ w_{co} = 3 \times 10^3 \frac{RAD}{SEC} \end{array}}$$

only let

$$f_F(t) = \frac{a_0}{2} + a_1 \cos 1000t + b_1 \sin 1000t$$



WHAT ARE POLARITIES?

USING KIRCHHOFF'S LAW

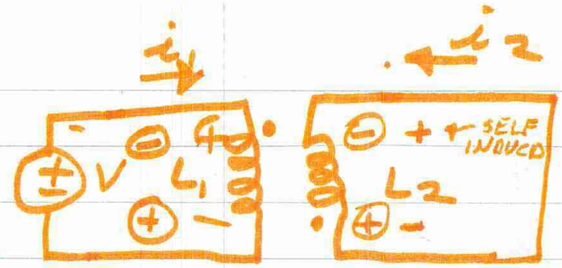
$$V = \underbrace{I(j\omega L_1)}_{\text{SELF}} + \underbrace{I(j\omega M)}_{\text{MUT. INDUCED}} + \underbrace{I(j\omega L_2)}_{\text{SELF}} + \underbrace{I(j\omega M)}_{\text{MUT. INDUCED}}$$

$$\frac{\bar{V}}{\bar{I}} = \dot{Z}_{eq} \quad L_1 + L_2 + 2M = L_{eq}$$

$$\dot{Z}_{eq} = j\omega L_{eq}$$

SERIES CONNECTION OF TWO COILS WITH INDUCTUNCES L_1 & L_2 AND MUTUAL INDUCTANCE OF M, GIVEN EQUIVILENT INDUCTANCE OF $L_{eq} = L_1 + L_2 \pm 2M > 0$

$$\rightarrow \therefore M \leq \frac{L_1 + L_2}{2}$$



$\rightarrow L_{eq}$ $\qquad V = L_{eq}\frac{di_1}{dt}$

$$V - L\frac{di_1}{dt} + M\frac{di_2}{dt} = 0$$

$$L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = 0$$

SOLVING YIELDS:

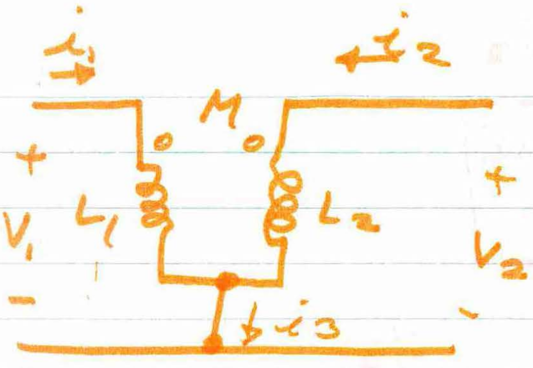
$$V - L\frac{di}{dt} + \frac{M^2}{L_2}\frac{di}{dt}$$

OR

$$V = \left(L - \frac{M^2}{L_2}\right)\frac{di_1}{dt}$$

$$\therefore L_{eq} = L - \frac{M^2}{L_2} \geq 0$$

$$\rightarrow M \leq \sqrt{L_1 L_2}$$



$$V_1 = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} + M\frac{di_1}{dt}$$

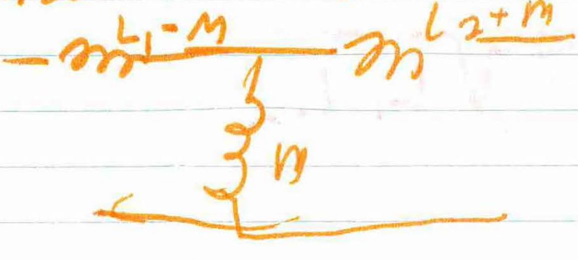
$$V_1 = L_1 \frac{di_1}{dt} + M \frac{d}{dt}(i_3 - i_1)$$

$$V_2 = L_2 \frac{di_2}{dt} + M \frac{d}{dt}(i_3 - i_2)$$

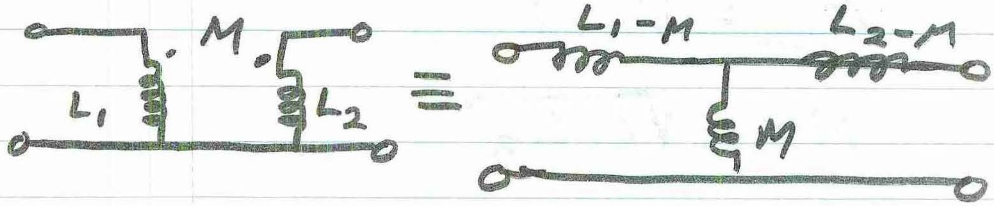
$$V_1 = L_1 - M \ \frac{di_1}{dt} + M\frac{di_3}{dt}$$

$$V_2 = (L_2 - M| \ \frac{di_2}{dt} + M\frac{di_3}{dt}$$

ANALOGOUS TO



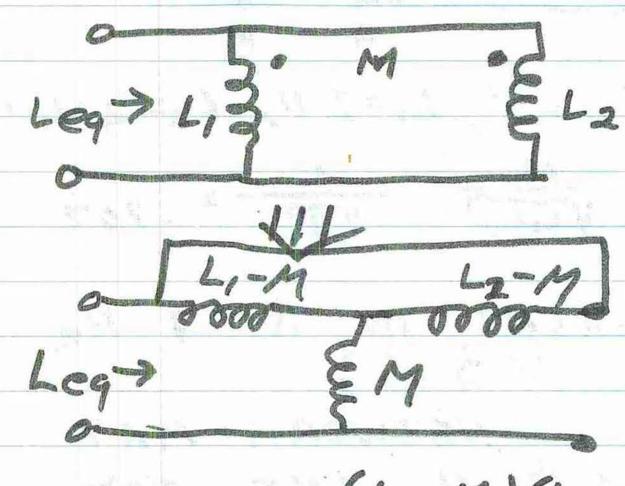
6-2-70



CHANGE DOT POLARITY ON LEFT CIRCUIT, (ONE DOT) CHANGE ALL SIGNS ON RT. CIRCUIT.

AS AN OPEN CIRCUIT, DON'T KNOW

TWO COILS IN //



$$L_{eq} = M + \frac{(L_1 - M)(L_2 - M)}{L_1 + L_2 - 2M}$$

$$= \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

$$M \leq \sqrt{L_1 L_2}$$

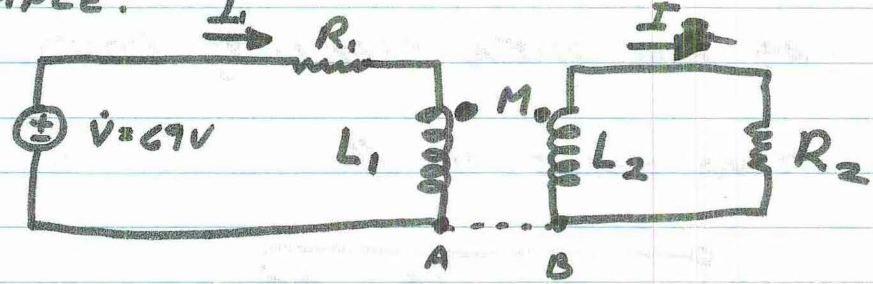
$$M_{MAX} = \sqrt{L_1 L_2}$$

$$M = k\sqrt{L_1 L_2}$$

k = COEFFICIENT OF COUPLING

$$k \leq 1$$

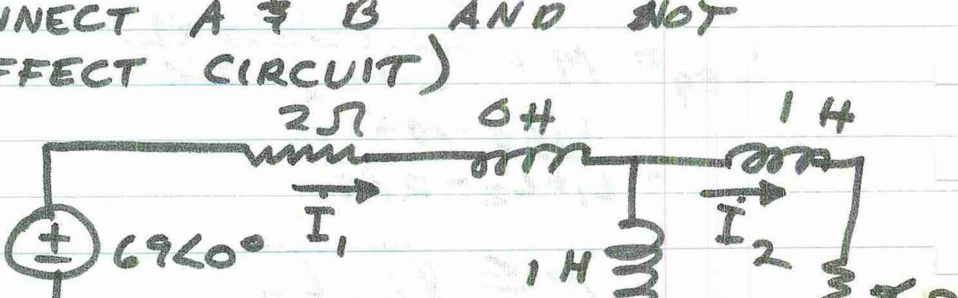
EXAMPLE:

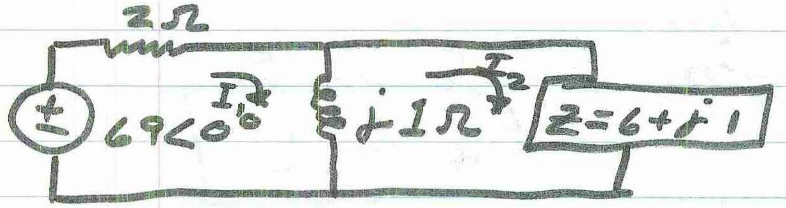


$\omega = 1$: $R_1 = 2\Omega$; $R_2 = 6\Omega$, $L_1 = 1\ H$; $L_2 = 2H$; $M = 1H$

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{1}{\sqrt{2}} = .707$$

FIND CURRENT IN R_1 & R_2
(ASIGN V & i)
(FIND EQ T NETWORK - MAY CONNECT A & B AND NOT EFFECT CIRCUIT)





MESH EQUATIONS YIELD

$$69 \angle 0° = (2 + j1)\, I_1 - j\, I_2$$
$$0 = -j\, I_1 + (6 + j2)\, I_2$$

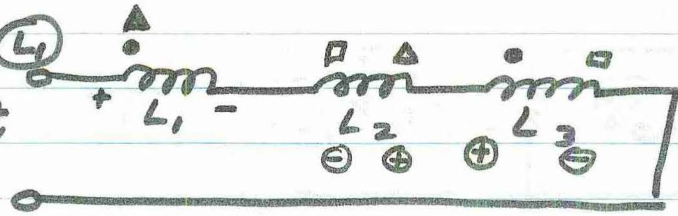
$$I_1 = \frac{6 + j2}{j}\, \dot{I}_2 = (2 - j6)\, \dot{I}_2$$

$$\cancel{69 = (2 + j1)\, \dot{I}_1 - j(2 - j6)\, I_1}$$

$$\dot{I}_2 = \frac{69}{10 - j11} = \frac{69 \angle 0°}{14.2 \angle -45.5}$$
$$= 4.85 \angle 45.5°$$

ETC.

EX) (L1)

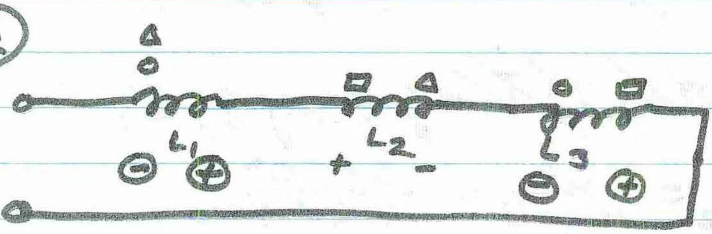


$M_{12}, M_{23}, M_{13} \neq 0$

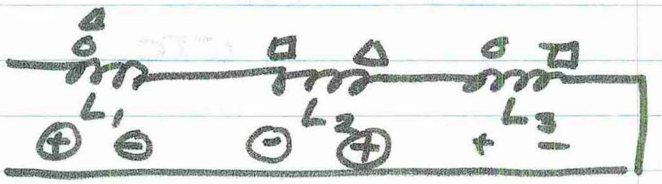
FOR L_1, L_2, L_3:

$$V = L_1 \frac{di}{dt} + M_{12}\frac{di}{dt} + M_{13}\frac{di}{dt} \Rightarrow L_1$$
$$+ L_2 \frac{di}{dt} - M_{12}\frac{di}{dt} - M_{23}\frac{di}{dt} \Rightarrow L_2$$
$$+ M_{13}\frac{di}{dt} - M_{23}\frac{di}{dt} + L_3 \frac{di}{dt}$$

(L2)



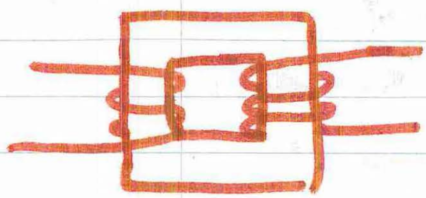
(L3)



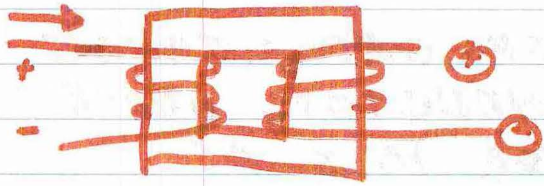
ALL SIGNS ON M_{XY} ARE SAME ALWAYS.

$$L_{eq} = L_1 + L_2 + L_3 - 2M_{12} + 2M_{13} - 2M_{23}$$

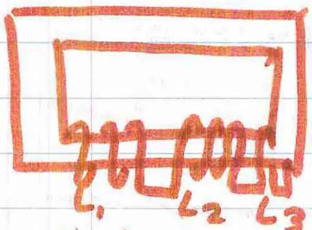
6-3-70



WHERE DO DOTS GO?



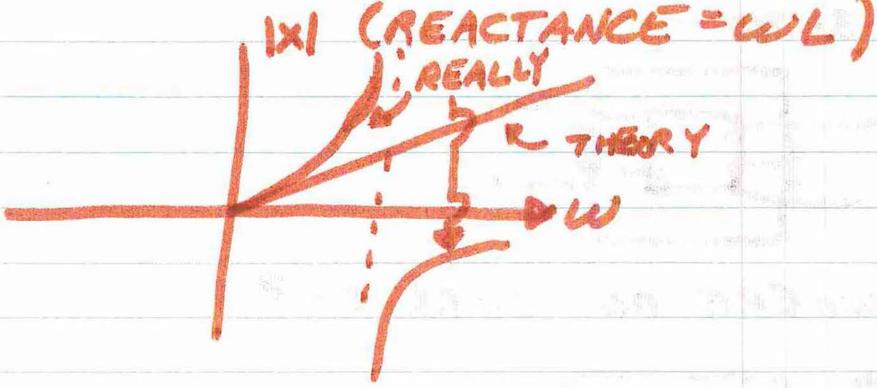
USING LENZ'S LAW & RT HAND RULE



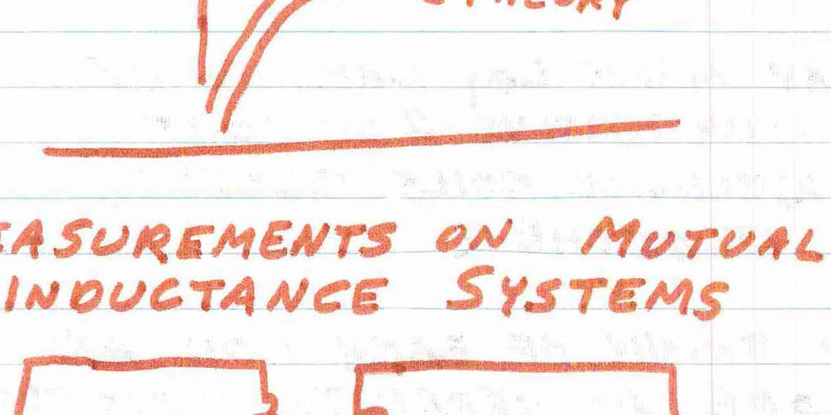
$$L_{EQ} = L_1 + L_2 + L_3 + 2M_{12} + 2M_{23} + 2M_{13}$$

MAY MAKE L_{eq} LARGER BY
1) INCREASING # OF COILS
2) SLIDING COILS CLOSER TOGETHER

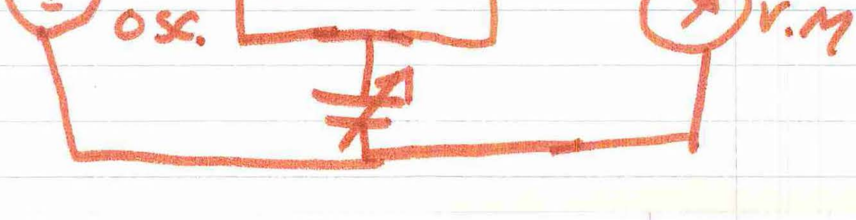
MAY THINK OF EACH COIL, OR ½ CORE AS SEPARATE INDUCTOR.



BECAUSE OF CAPACITANCE IN THE INDUCTOR ABOVE ω_0, INDUCTOR ACTS AS ~~a~~ STRANGE CAPACITOR.



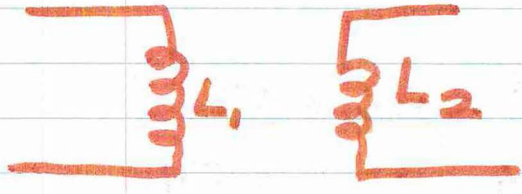
MEASUREMENTS ON MUTUAL INDUCTANCE SYSTEMS



VARY C UNTIL VOLTS ON
VOLT METER IS ~~MAXIMUM~~
MINIMIZES

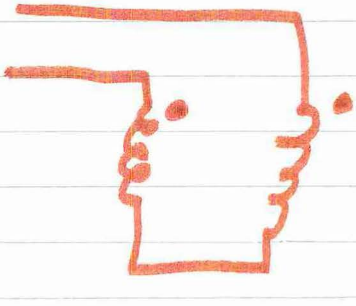
$M = \frac{1}{\omega^2 C}$ $\left(\omega M = \frac{1}{\omega C}\right)$
CIRCUIT AS RESONANCE.

MEASURE INDUCTANCE & M
USING INDUCTION METER



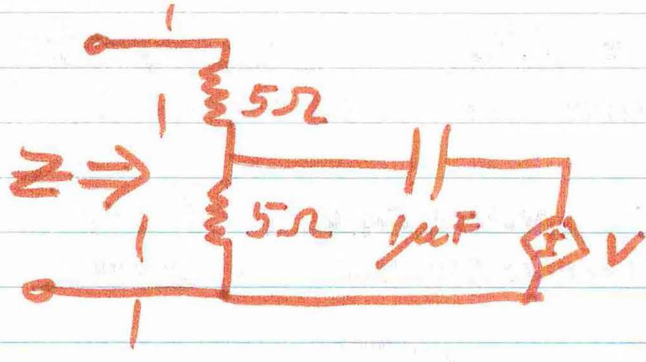
MEASURE L_{eq} OF TWO
INDUC. IN // OR SERIES
FOR SER.
$L_{eq} = L_1 + L_2 \pm 2M$

$L_1 = 2$; $L_2 = 5$; $L_{eq} = 1$
$\therefore M = 3$. CIRCUIT IN
SERIES

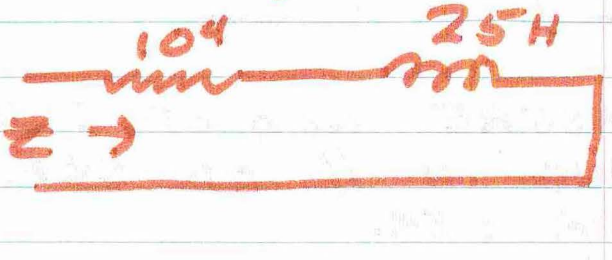


INDUCTORS LESS IDEAL THAN RESIS., CAPAC., TRANSISTOR, ETC.

MUST SIMULATE L:



$Z = 10^4 + j\omega 25$

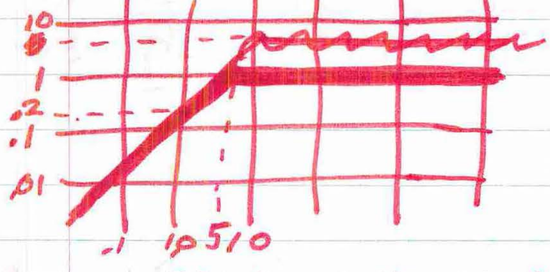


6-4-70

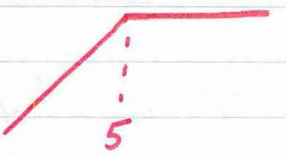
6-8-70

REVIEW

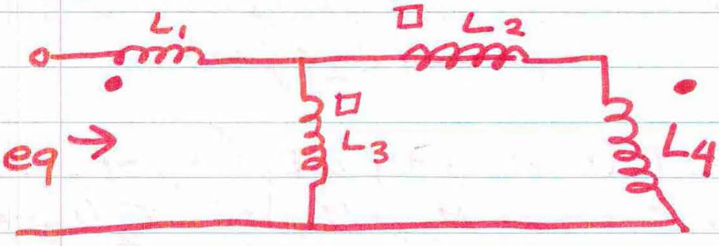
$$H(j\omega) = \frac{j\omega}{5+j\omega} = .2\,\frac{j\omega}{1+j\frac{\omega}{5}}$$



KNOW

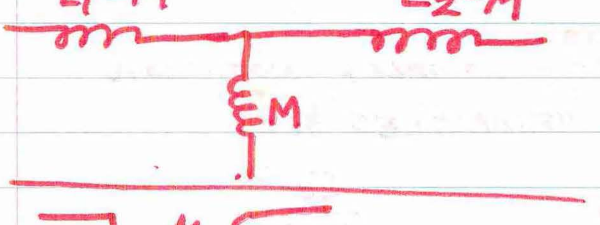


PICK A FREQ, LIKE $\omega = 1$

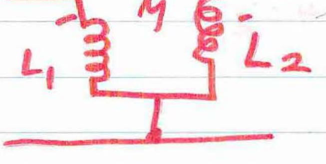


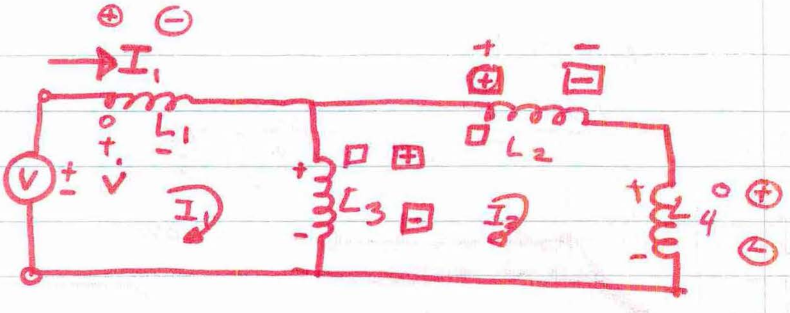
FIND L_{eq}

USE:



FROM:





$$\frac{V}{I} = j\omega L(eq)$$

$$V = I_1 j\omega L_1 + j\omega M_{14} I_2 + j\omega L_3 (I_1 - I_2) + j\omega M_{23} I_2$$

$$0 = -j\omega L_3 (I_1 - I_2) - j\omega M_{23} I_2 + j\omega L_2 I_2 + j\omega M_{23} (I_1 - I_2) + j\omega L_4 I_2 + j\omega M_{14} I_1$$

$$V = j\omega I_1 (L_1 + L_3) + j\omega I_2 (M_{14} - L_3 + M_{23})$$

$$0 = j\omega I_1 (-L_3 + M_{23} + M_{14}) + j\omega I_2 (L_3 - M_{23} + L_2 - M_{23} + L_4)$$

~~COVERED~~
~~I A.C. ANALYSIS~~
A) ~~PHASORS - COMPLEX~~ NOTATION
POWER (TRIANGLES & ST

COV $P_{MAX_{RMS}} = \frac{E_g^2}{4R_g}$

COVERED

I) A-C ANALYSIS

A) PHASORS

B) COMPLEX REPRESENTATION

1) RECT. FORM

2) POLAR FORM

II) POWER IN A-C CIRCUITS

A) EQUATIONS FOR

B) MAXIMUM POWER & IMPEDENCE MATCHING

→ III) TRANSFER FUNCTIONS

A) BODE DIAGRAM

1) MAGNITUDE PLOTS (LOG-LOG)

2) PHASE PLOTS (SEMI-LOG)

B) POLE-ZERO PATTERNS

1) NARROW BAND NETWORKS

2) HOW TO FIND MAGNITUDE }

3) HOW TO FIND PHASE } FROM

IV) FOURIER ANALYSIS

A) COMPLEX COEFFICIENTS

B) SIN-COSINE COEFFICIENTS

C) PROPERTIES

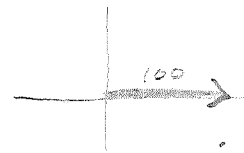
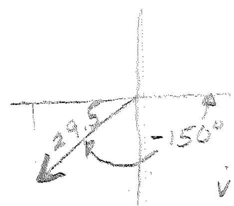
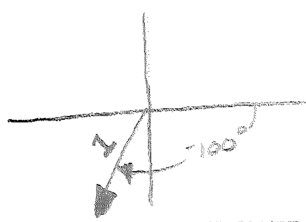
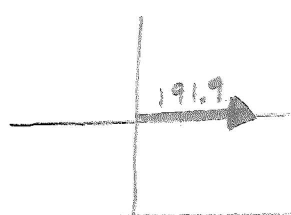
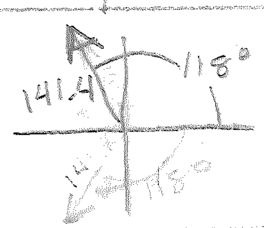
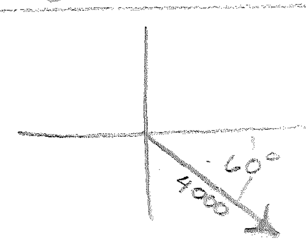
1) SHIFTING

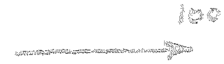
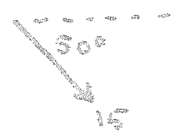
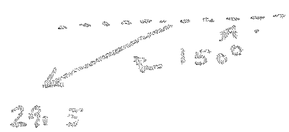
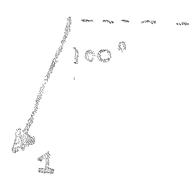
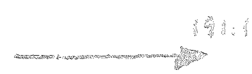
2) EVEN & ODD FUNCTIONS

V) MUTUAL INDUCTANCE

A) DOT POLARITY & HOW TO COPE WITH IT

B) EQUIVALENT T CIRCUIT

Time function	phasor	complex notation
$v(t) = 100 \cos 377t$ V.	 $\dot{V} = 100\angle 0°$ V	100 V
$v(t) = 15 \cos(100t - 50°)$ V.	 $\dot{V} = 15\angle 50°$ V	$-15j \sin -50° + 15 \cos -50°$
$v(t) = 29.5 \cos(2\pi t - 150°)$ V	 $\dot{v} = 29.5\angle -150°$ V	$-29.5 j \sin -150$
$v(t) = 90 \sin(10t + 50°)$ V.		$\dot{V} = 90\angle 50°$ V
$i(t) = \cos(5t - 100°)$ A.		$\dot{I} = 1\angle -100°$ A
$i(t) = 191.1 \cos(2\pi \cdot 10^6 t)$ A.		$\dot{I} = 191.1\angle 0°$ A
$v(t) = 141.4 \sin(40t + 208°)$ V $\frac{-90}{118}$		$\dot{V} = 141.4\angle 118$
		$\dot{X} = 4000\angle -60°$

[illegible]	[illegible]	[illegible]
$v(t) = 100 \cos 377t$ V.		$\mathring{V} = 100 \angle 0^\circ$ V.
$v(t) = 15 \cos(100t - 50^\circ)$ V.		$\mathring{V} = 15 \angle -50^\circ$ volts
$v(t) = 29.5 \cos(2\pi t - 150^\circ)$ V.		$\mathring{V} = 29.5 \angle -150^\circ$ volts
$v(t) = 90 \sin(10t + 50^\circ)$ V.		$\mathring{V} = 90 \angle -40^\circ$ V.
$i(t) = \cos(5t - 100^\circ)$ A.		$\mathring{I} = 1 \angle -100^\circ$ A.
$i(t) = 191.1 \cos(2\pi \cdot 10^6 t)$ A.		$\mathring{I} = 191.1 \angle 0^\circ$ V.
$v(t) = 141.4 \sin(90t + 208^\circ)$ V.		$\mathring{V} = 141.4 \angle +118^\circ$ V.
[illegible] $\ldots 30^\circ)$ V.		$\mathring{X} = 4000 \angle 60^\circ$ V.

Change to polar form

	angle	magnit.	polar form
$j6$	$90°$	6	$6 \angle 90°$
$1 - j2$	$-63.4°$	2.24	$2.24 \angle -63.4°$
$-1 - j2$	180 63.4 -116.6	2.24	$2.24 \angle -116.6°$
$3 + j4$	$53.2°$	5	$5 \angle 53.2°$
$4.7 + j6.3$	$53.3°$	7.81	$7.8 \angle 53.5°$
$-11.3 + j4$	$70.5°$	12.0	$12.0 \angle 70.5°$

Change to polar form

	Angle	Magnit.	polar form
$j6$	90°	6	$6\angle 90^\circ$
$1-j2$	-63.4°	2.23	$2.23\angle -63.4^\circ$
$-1-j2$	243.4°	2.23	$2.23\angle 243.4^\circ$
$3+j4$	53.1°	5	$5\angle 53.1^\circ$
$4.7+j6.3$	53.3°	7.86	$7.86\angle 53.3^\circ$
$-11.3+j4$	170.5°	12	$12\angle 170.5^\circ$

$$\begin{array}{r} 180 \\ 63.4 \\ \hline 243.4 \end{array} \qquad \begin{array}{r} 180.0 \\ -\ 19.5 \\ \hline 170.5 \end{array}$$

4-8-70

Note the second integral is zero because the integrand, $\frac{1}{2}\sin(2\omega t)$, is just a sinusoid and it is integrated over two periods.

So

$$P_{avg} = \frac{VI\cos\theta_z}{4\pi}\int_0^{2\pi} d(\omega t) + \frac{VI\cos\theta_z}{4\pi}\int_0^{2\pi}\cos(2\omega t)\, d(\omega t)$$

The second integral here is also zero for the same reason

So: $$P_{avg} = \frac{VI\cos\theta_z}{4\pi}\left[2\pi\right] = \boxed{\frac{VI\cos\theta_z}{2}}$$

When we write P without the subscript, we are meaning P_{avg} unless otherwise specified.

So we see

$$P = \frac{VI\cos\theta_z}{2} = \frac{V}{\sqrt{2}}\cdot\frac{I}{\sqrt{2}}\cos\theta_z$$

$$= V_{eff}\, I_{eff}\cos\theta_z$$

Since $V_{eff} = \frac{1}{\sqrt{2}}V$ for a sinusoid. We may see the formula for power written these two ways.

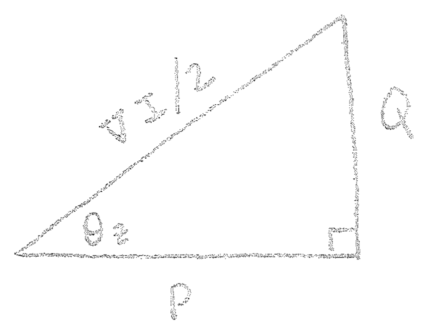
$$P = VI\cos\theta_z \quad \text{and} \quad P = \frac{VI\cos\theta_z}{2}$$

In the first case V and I are <u>PEAK</u> values
while in the second case they are <u>RMS</u> values
of the sinusoid.

<u>Power factor</u>

The quantity θ_z is called <u>power factor</u>. It relates the power in the circuit to the volt-ampere product.

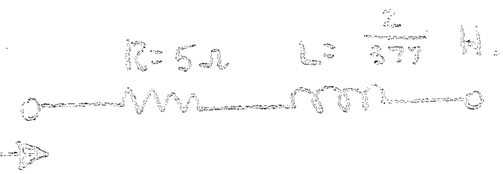
We may draw a <u>POWER TRIANGLE</u>.



Note that θ_z is determined entirely by the nature of the ________.

In the diagram above (i.e., the power triangle) P is the real power in the circuit measured in <u>WATTS</u>, Q is known as the reactive power (not really power) measured in <u>VARS</u>. The product $\frac{VI}{2}$ is called <u>APPARENT POWER</u>; this is measured in <u>volt-amperes</u>.

EXAMPLE (Student to work)



$i = 5 \cos(377t)$ A.

Compute the power in the circuit first by $\frac{I^2 R}{2}$.

$$P = \frac{I^2 R}{2} = \frac{5^2 \times 5}{2} = \boxed{62.5 \text{ W}}$$

Now check this by $P = \frac{VI}{2} \cdot \cos\theta_z$

$Z =$ $\underline{5 + j2 = 5.39 \angle 21.8°}$

$V = IZ =$ $\underline{26.9 \angle 21.8°}$

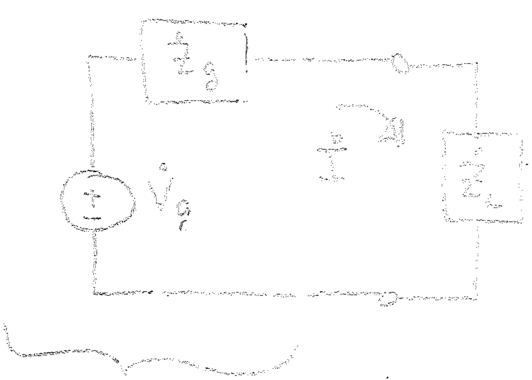
$\theta_z =$ $\underline{\quad 21.8° \quad}$

Power factor, $\cos\theta_z =$ $\underline{\quad 5 \quad}$

$P = \frac{VI}{2} \cos\theta_z =$ __________ (SHOULD BE 62.5 W)

[illegible]

Question: what should Z_L be so that maximum power is extracted from the source?



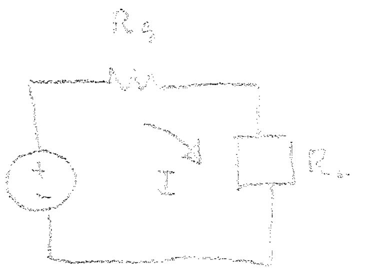
SOURCE

$$\tilde{Z}_L = R_L + j X_L$$

$$P_L = |I|^2 R_L \quad (\text{here } I_{rms} \text{ is rms value})$$

we want to maximize P_L with respect to Z_L

Special case Z_g is real $= R_g + j0$ and $Z_L = R_L + j0$



$$P = I^2 R_L$$

$$I = \frac{V_g}{R_g + R_L}$$

$$P = \frac{V_g^2 R_L}{(R_g + R_L)^2}$$

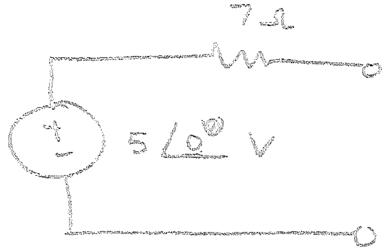
$$\frac{dP}{dR_L} = \frac{(R_g + R_L)^2 V_g^2 - V_g^2 R_L \cdot 2(R_g + R_L)}{(R_g + R_L)^4}$$

$$= \frac{V_g^2 (R_g^2 + 2R_g R_L + R_L^2) - V_g^2 (2R_g R_L + 2R_L^2)}{(R_g + R_L)^4}$$

$$P_{MAX} = \frac{V_g^2}{4 R_g}$$

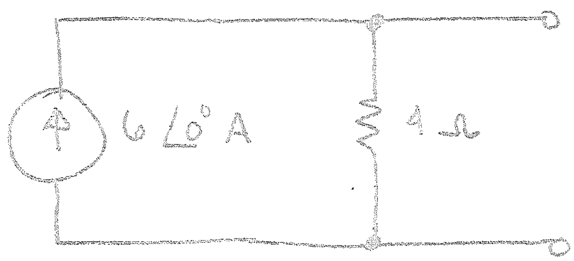
TO GET MAX PWR. WHEN R_L IS FIXED & R_g VARIABLE IS $R_g = 0$

What is the maximum power which may be drawn from the sources below? (V_{RMS})



P_{max} = ______ W.

What is the maximum power which may be drawn



P_{max} = 36 W.

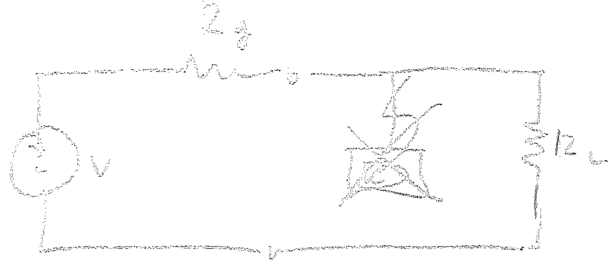
Now if the source impedance is complex, we may ~~[illegible]~~ use a complex load impedance to extract maximum power in the following way. If we let the reactance of the load equal the negative of the reactance of the source, then all that remains in the series path is R_g and R_L

FOR MAX PWR., USE COMPLEX CONJUGATE OF Z_g.

$$P_{MAX} = \frac{V_g^2}{2R_g}$$

$P_{MAX} = \frac{V_g^2}{4R_g}$, $V_{g\,eff}$ OR $P_{MAX} = \frac{V_g^2}{8R_g} \Rightarrow V_g = \text{MAGNITUDE}$

So if $X_L = -X_g$ the circuit reduces to



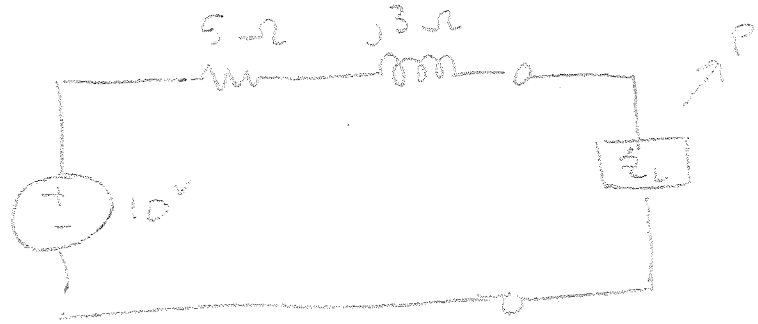
, what should R_L be.

$R_L =$ ______

Thus to get maximum power from a source which has complex impedance. Set $Z_L = R_g - jX_g$

$$Z_L = Z_g^*$$

Ex1



for $P = P_{max}$ what should the value of Z_L be?

$Z_L =$ ______ Ω

Driving point impedance and current in an R-L-C series circuit.

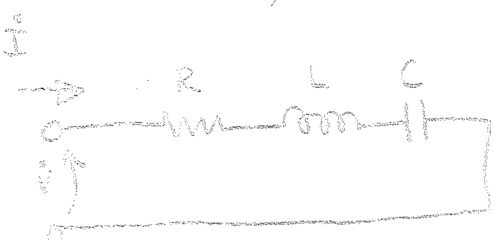
$$Z(\omega) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$\omega_r \triangleq$ (definition) frequency where $\theta_Z = \tan^{-1}\frac{X}{R} = 0$

$$\omega_r L - \frac{1}{\omega_r C} = 0 \qquad \omega_r = \sqrt{\frac{1}{LC}}$$

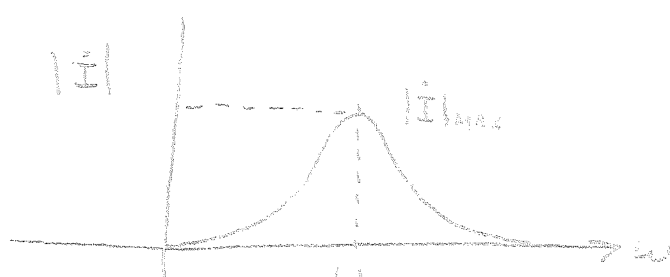
At $\omega > \omega_r$, the impedance is inductive

$\omega < \omega_r$ " " " capacitive



$$\dot{I} = \frac{\dot{V}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

As a function of frequency $|\dot{I}|$ is shown below



Note that $|\dot{I}|$ is maximum when $|\dot{Z}|$ is minimum.

min $|\dot{Z}| = R$, at $\omega = \omega_r$, then

$$\boxed{\omega_p = \omega_r} \qquad \dot{I}_{max} = \boxed{\frac{|\dot{V}|}{R}}$$

NOTES (2)

The half power frequencies ~~occur~~ are those where $|I|^2 = \frac{|I|_{max}^2}{2}$ or when $|Z| = \sqrt{2}\, Z_{min} = \sqrt{2}\, R$

$$\sqrt{2} R = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$2R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$R^2 = \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\pm R = \omega L - \frac{1}{\omega C}$$

using + sign $\quad \omega RC = \omega^2 LC - 1$

$$\omega = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$ one these roots is neg. disregard it

call the positive root the upper half power freq, ω_U

$$\omega_U = \frac{R}{2L} + \sqrt{\frac{1}{LC} + \left(\frac{R}{2L}\right)^2}$$

Using − sign

$$\omega = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$ one of these roots is neg.

use + root, call it the lower half power frequency, ω_L

EE 302
NOTES (3)

$$\omega_L = \sqrt{\frac{1}{LC} + \left(\frac{R}{2L}\right)^2} - \frac{R}{2L}$$

Calling $\frac{1}{LC} = \omega_r^2$ and $\frac{R}{2L} = a$

$$\omega_L = \sqrt{\omega_r^2 + a^2} - a$$

$$\omega_U = \sqrt{\omega_r^2 + a^2} + a$$

Define bandwidth BW as $\omega_U - \omega_L$

$$BW = 2a = \boxed{\frac{R}{L}}$$

Thus we can see that larger R gives larger bandwidth, i.e., the circuit is less selective.

Define $Q_0 = \frac{\omega_r L}{R}$; $\left(\frac{X_L}{R}\right)$

$$Q_0 = \frac{\omega_L}{R/L} = \boxed{\frac{\omega_L}{BW}}$$ ← relates Q_0, BW, and ω_r

Ex. $R = 10\,\Omega$ $L = 0.1$ mH $C = .01\,\mu F$

$$\omega_r = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{10^{-4} \cdot 10^{-8}}} = \sqrt{10^{12}} = \boxed{10^6 \frac{rad}{sec}}$$

(159 kHz)

EE 202
NOTES (4)

$$BW = \frac{R}{L} = \frac{100}{10^{-4}} = \boxed{10^{4} \text{ rad/sec}}$$

$$Q = \frac{BW}{2} = 5000$$

$$Q = \frac{\omega_r}{BW} = \frac{10^6}{10^4} = \boxed{100}$$

Half power frequencies

$$\omega_L = \sqrt{10^{12} + 25 \times 10^6} - 5000$$

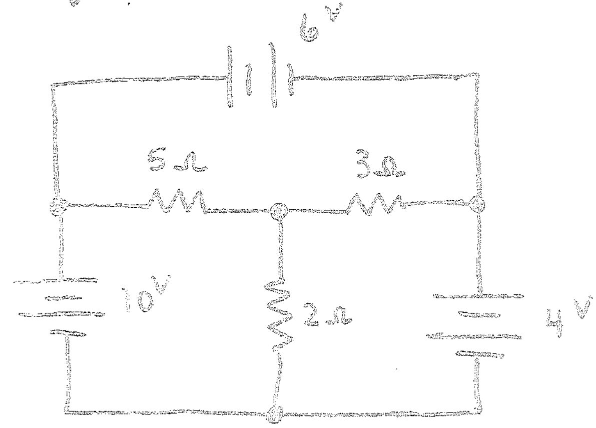
$$\doteq 10^6 - 5000 \text{ rad/sec}$$

$$\omega_U \doteq 10^6 + 5000 \text{ rad/sec}$$

E Sci II

Homework

1. Solve the circuit shown by writing mesh equations.

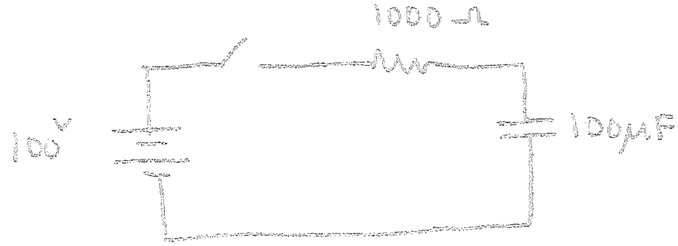


2. $v_1(t) = 169 \sin(377t + 30°)$ V.

$v_2(t) = 47.2 \sin(377t - 41°)$ V

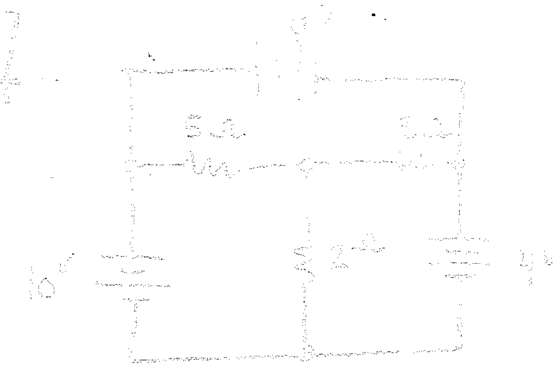
Find $v_1(t) + v_2(t)$ as a sinusoid

3. Find the ~~power~~ energy dissipated by the resistor as the capacitor charges up. C is initially uncharged.



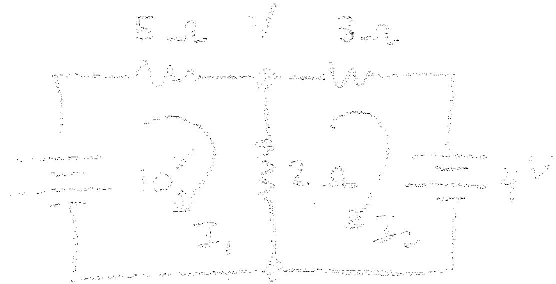
[illegible]

17-1



Note: for preliminary solution of this problem it is seen that only 2 voltage sources are of interest. Third is redundant.

Use 10V and 4V sources



Using mesh currents

$$10 = 7I_1 - 2I_2$$

$$-4 = -2I_1 + 5I_2$$

DIVIDE FIRST EQUATION BY 2, SECOND BY 5

$$\left.\begin{aligned} 5 &= 3.5\,I_1 - I_2 \\ -0.8 &= -0.4\,I_1 + I_2 \end{aligned}\right\} \text{add eqns}$$

$$4.2 = 3.1\,I_1 \qquad I_1 = \frac{4.2}{3.1} = \boxed{1.355\ A}$$

FROM FIRST EQUATION $\quad I_2 = 3.5\,I_1 - 5$

$$= 3.5 \times 1.355 - 5$$

$$= 4.74 - 5 = \boxed{[illegible]}$$

[illegible]

ESC II
Answers

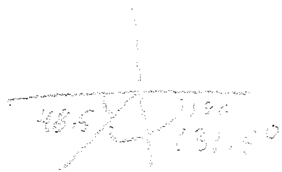
X-2

$$v_1(t) = 169 \sin(377t + 30^\circ) \Rightarrow \dot{V}_1 = 169 \underline{\angle 30^\circ - 90^\circ}$$

$$= 169 \underline{\angle -60^\circ}\ V$$

$$v_2(t) = 47.2 \sin(377t - 41^\circ) \Rightarrow \dot{V}_2 = 47.2 \underline{\angle -131^\circ}\ V$$

$$\dot{V}_1 = 84.5 - j146.5$$



$$\dot{V}_2 = -31.3 - j35.3$$

$$\dot{V}_1 + \dot{V}_2 = 53.2 - j181.8 = 189.5 \underline{\angle -73.7^\circ}$$

$$\Downarrow$$

$$v_1(t) + v_2(t) = \boxed{189.5 \cos(377t - 73.7^\circ)\ V}$$

X-3 (WORKED IN CLASS)

$$RC = 10^{-4} \cdot 10^{3} = [illegible]$$

$$i(t) = \frac{100}{1000} e^{-\frac{t}{RC}} = 0.1 e^{-10t}\ A$$

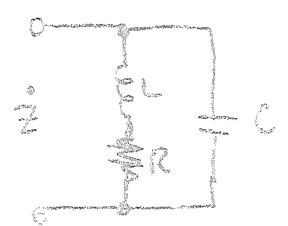
$$p_R(t) = [i(t)]^2 R = 10^3 \times 10^{-2} e^{-20t} \qquad \omega_1 = 10 e^{-10t}$$

$$W = \int_0^\infty p(t)\,dt = 10 \int_0^\infty e^{-20t}\,dt = -\frac{10}{20}(0-1) = \boxed{0.5\ \text{Joule}}$$

EE DEPT – RPI
4-22-70

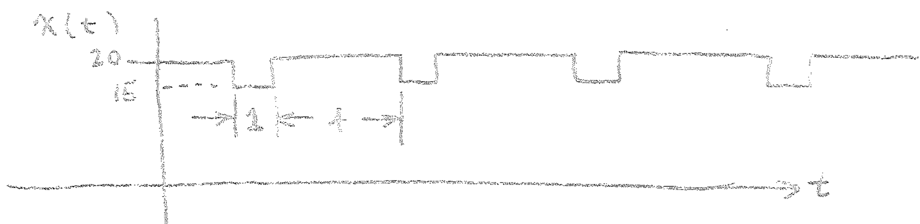
HOMEWORK
DUE 4.27.70

X-14 Find the driving point impedance as a function of ω

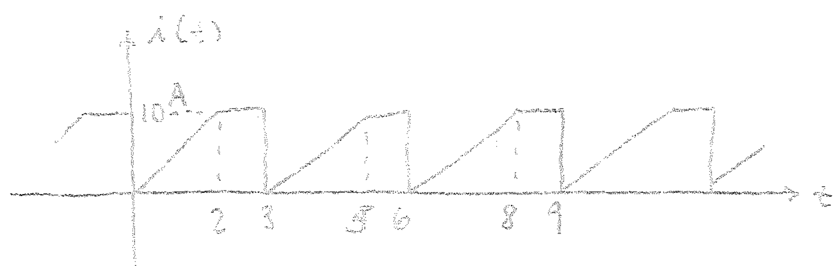


L = 30 mH
C = 0.5 μF
R = 10 Ω

X-15 Find the r.m.s. value of the wave form shown.



X-16 Find the r.m.s. value of i(t)



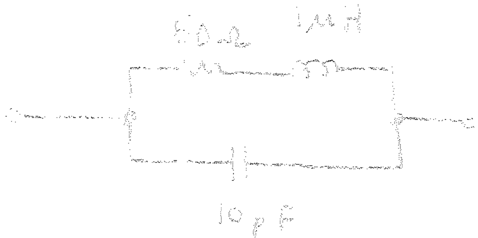
[illegible]

[illegible]

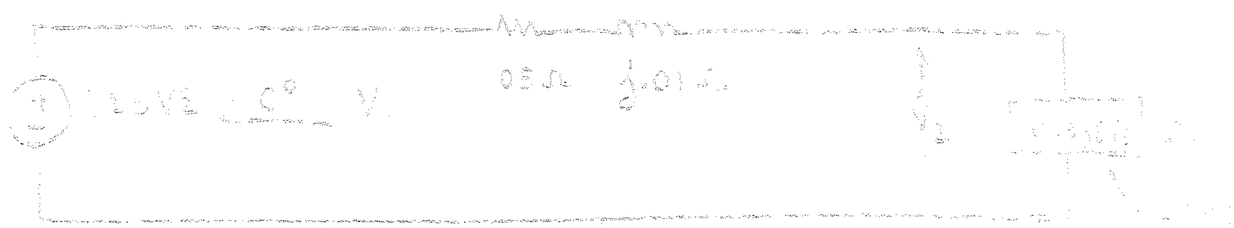
[illegible]

EE 202 — HOMEWORK

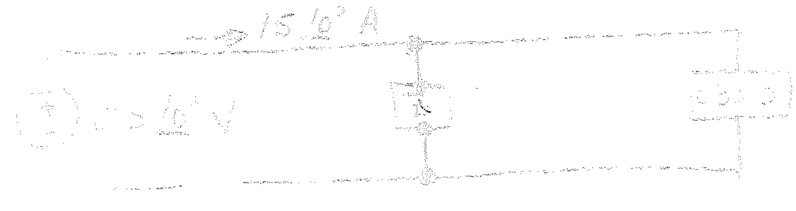
X-10 Find the frequency where the circuit [illegible] has a real impedance



X-11 SOLVE FOR THE POWER [illegible] THE LOAD. FIND V_L



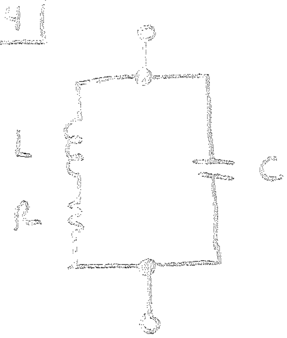
X-12 The load shown dissipates [illegible]. It's power factor is 0.8 and [illegible] is known to be inductive [illegible]



EE DEPT
RPI
4-28-70

EE 202
Answers to homework

X-14

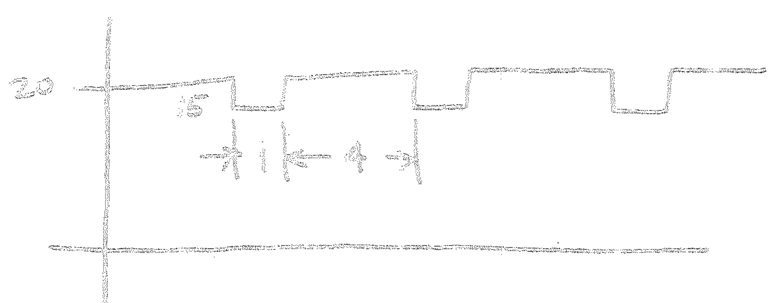


$$Z = \frac{(R + j\omega L)\,\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{R + j\omega L}{1 - \omega^2 LC + j\omega RC}$$

$$= \boxed{\frac{10 + j\omega\,0.03}{1 - \omega^2\,.03 \times 5 \times 10^{-7} + j\omega\,5 \times 10^{-6}}}$$

X-15



$$X_{rms} = \sqrt{\frac{1}{5}\left(20^2 \cdot 4 + 15^2\right)} = \sqrt{\frac{1}{5}\left(225 + 1600\right)}$$

$$= \sqrt{\frac{1}{5}\,1825} = \sqrt{365} = \boxed{19.1}$$

homework answers

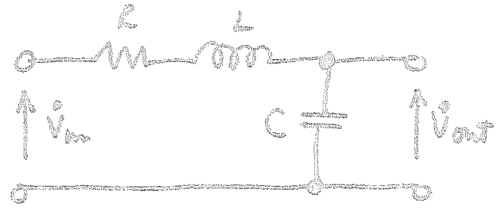
X-16

$$I = \sqrt{\frac{1}{3}\left[\int_0^2 25t^2\,dt + 100\right]}$$

$$= \sqrt{\left\{\frac{1}{3}\left[25 \times 4 + 100\right]\right\}} = \sqrt{\left\{\frac{200}{3}\right\}} = \sqrt{66.7}$$

$$= \boxed{8.15\text{ A.}}$$

X-17



$R = 0.1\ \Omega$

$L = 1\ H$

$C = 0.5\ F$

$$H(\omega) = \frac{1}{1-\omega^2 LC + j\omega RC} = \boxed{\frac{1}{1-\omega^2\, 5\times10^{-7} + j\omega\, 5\times10^{-9}}}$$

$$\omega_p \doteq \omega_r \qquad \omega_r = \sqrt{\frac{1}{5\times10^{-7}}} = \sqrt{\frac{10^7}{5}}$$

$$= \sqrt{2}\times10^3 = \boxed{1414\text{ rad/sec}}$$

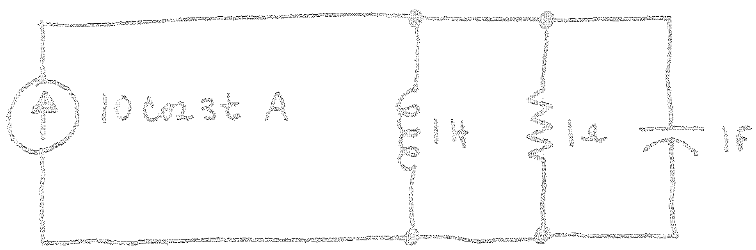
Note: this problem was discussed in detail in class!

EE 202

homework answers

X-18

Scott 16.12



$$Z_{eq} = \frac{1}{j\omega \cdot 1 + 1 + \frac{1}{j\omega 1}} = \frac{j\omega 1}{1-\omega^2+j\omega}$$

$$Z(3) = \frac{j3}{1-9+j3} = \frac{j3}{-8+j3} = \frac{3\angle 90^\circ}{8.33\angle -200.5^\circ}$$

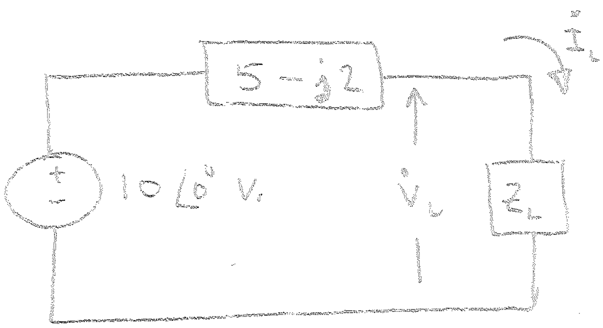
$$= 0.36\angle 290.5^\circ$$

$$\hat{V} = (10\angle 0^\circ)(0.36\angle 290.5^\circ) = 3.6\angle 290.5^\circ = \boxed{3.6\angle -69.5^\circ\ V}$$

4.21.70

EE 202
QUIZ #1

Find the power in the load when $\dot{Z}_L$ is adjusted so that there is maximum power in $\dot{Z}_L$.



(a) P_L = ____

(b) $\dot{V}_L$ = ____

(c) $\dot{I}_L$ = ____

(d) Find the ~~vars~~ reactive power in vars that the load has.

Solution

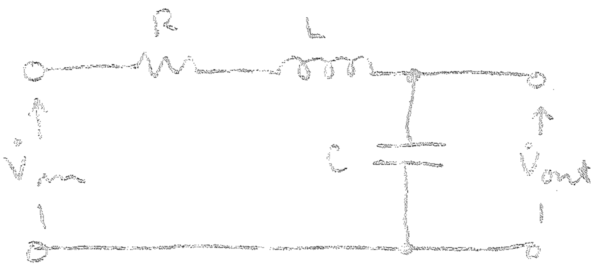
(a) $P_{max} = \dfrac{|V_g|^2}{4 R_g} = \dfrac{100}{20} = \boxed{5 \text{ watts}}$

(c) $\dot{I}_L = \dfrac{\dot{V}_g}{2R_g} = \dfrac{10}{10} = \boxed{1\angle 0^\circ \text{ A}}$

(b) $V_L = Z_L I_L = (5+j2)(1\angle 0^\circ) = \boxed{5 + j2 \text{ V.}}$

(d). $Q_L = |I_L|^2 X_L = 1 \times 2 = \boxed{2 \text{ Vars}}$

X-17



Find $H(\omega) = \dfrac{\dot{V}_{out}}{\dot{V}_{in}}$

$R = .1\ \Omega$

$L = 1\ H$

$C = \frac{1}{2}\ F$

Find the frequency where $|H(\omega)|$ is maximum.

Find the frequencies where $|H(\omega)| = 0.707\ H_{max}$.

Find $|H(\omega)|_{max}$.

X-18 PROBLEM 547.16.12 in SCOTT.

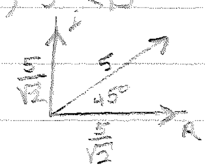
BOB MARKS

Pp. 510-12 4-7-70

put j operator in front always

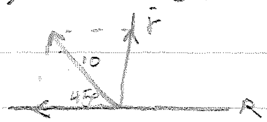
12) a) $10\angle 0^\circ \Rightarrow 10$

b) $5\angle 45^\circ$



$\Rightarrow 3.53 + 3.53j$

e) $10\angle 135^\circ$



$\Rightarrow -7.07 + 7.07j$ ✓

c) $10\angle -90^\circ \Rightarrow -10j$ ✓

d) $5\angle 135^\circ \Rightarrow -3.53 + 3.53j$ ✓

use lower case y for time functions

13) a) $Y = 10\cos t$ ✓

b) $Y = 10\cos(377t + \pi/4)$

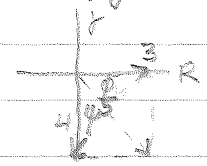
c) $Y = 5\cos(10^3 t - 60^\circ)$ ✓

d) $Y = 5\cos(t + 53^\circ)$

e) $Y = 2.82\cos(\frac{t}{10} + 135^\circ)$ ✓

14) a) $1 + j \Rightarrow \sqrt{2}\cos(\omega t + 45^\circ) = 1.41\cos(\omega t + 45^\circ)$ ✓

b) $3 - j4$



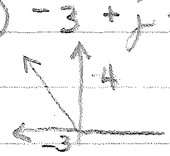
$\psi = 36.9^\circ \quad \phi = -(90 - 36.9) = 53.1$

$\Rightarrow 5\cos(\omega t - 53.1^\circ)$ ✓

c) $-2 - 2j \Rightarrow 2\sqrt{2}\cos(\omega t - 135^\circ)$ ✓

$= 2.83\cos(\omega t - 135^\circ)$

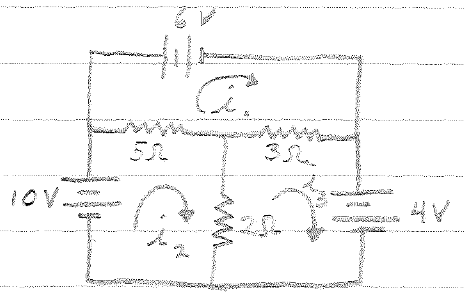
d) $-3 + j4$



$\Rightarrow 5\cos(\omega t + 126.9^\circ)$ ✓

e) $j \Rightarrow \cos(\omega t + 90^\circ)$ ✓

1)



$$6 + 8i_1 - 5i_2 - 3i_3 = 0 \Rightarrow i_1 = \frac{-6}{8} + \frac{5}{8}i_2 + \frac{3}{8}i_3$$

$$-10 - 5i_1 + 7i_2 - 2i_3 = 0 \Rightarrow i_1 = \frac{-10}{5} + \frac{7}{5}i_2 - \frac{2}{5}i_3$$

$$4 - 3i_1 - 2i_2 + 5i_3 = 0 \Rightarrow i_1 = \frac{4}{3} - \frac{2}{3}i_2 + \frac{5}{3}i_3$$

$$\frac{-6}{8} + \frac{5}{8}i_2 + \frac{3}{8}i_3 = -2 + \frac{7}{5}i_2 - \frac{2}{5}i_3 = \frac{4}{3} - \frac{2}{3}i_2 + \frac{5}{3}i_3$$

$$\frac{5}{4} + i_2\left(\frac{5}{8} - \frac{7}{5}\right) + i_3\left(\frac{3}{8} + \frac{2}{5}\right) = 0 = \frac{10}{3} - i_2\left(\frac{2}{3} + \frac{7}{5}\right) + i_3\left(\frac{5}{3} + \frac{2}{5}\right)$$

$$\frac{5}{4} - \frac{31}{40}i_2 + \frac{31}{40}i_3 = 0 = \frac{10}{3} - \frac{31}{15}i_2 + \frac{31}{15}i_3$$

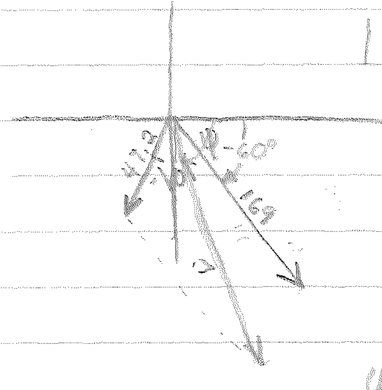
$$50 - 31i_2 + 31i_3 = 0 = 50 - 31i_2 + 31i_3$$

MESH CURRENTS i_1, i_2, AND i_3 MAY BE ANY VALUE

2) $v_1(t) = 169 \sin(377t + 30°)$; $v_2(t) = 47.2 \sin(377t - 41°)$

$v_1(t) = 169 \cos(377t - 60°)\,v$; $v_2(t) = 47.2 \cos(377t - 101°)$

$\dot{V}_1 = 169 < -60°$; $\dot{V}_2 = 47.2 < -101$



$V = V_1 + V_2$

$$|\dot{V}| = 169j \sin -60° + 47.2j \sin -101° \; v$$
$$+ 169 \cos -60° + 47.2 \cos -101° \; v$$
$$= j169 \frac{-\sqrt{3}}{2} - 47.2j \sin 79° \; v$$
$$+ 169\left(\frac{1}{2}\right) - 47.2 \cos 79° \; v$$
$$= -147j - 46.3j + 84.5 - 9.0 \; v$$
$$= -193j + 84.5 \; v$$

$$\psi = \tan^{-1}\frac{193}{84.5} = 2.28$$

$$\operatorname{ctn} \psi = .439 \Rightarrow \psi = -64°$$

$$\therefore \hat{V} = \sqrt{3.72 + .714} \times 10^2 < -64° v = 210 < -64° \; v$$

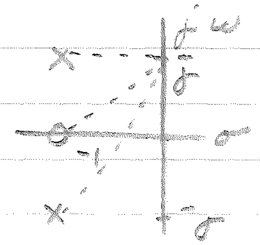
$$v(t) = 210 \cos(377t - 64°)\, V$$

X-27) $Z(s) = \dfrac{s+1}{(s+1+j)(s+1-j)}$

$= \dfrac{s+1}{s^2+s-js+s+1-j+js+j+1}$

$= \dfrac{s+1}{s^2+2s+2}$

b) $Z(s) = \dfrac{s+1}{(s+1+j)(s+1-j)}$

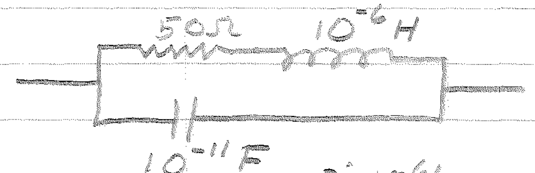


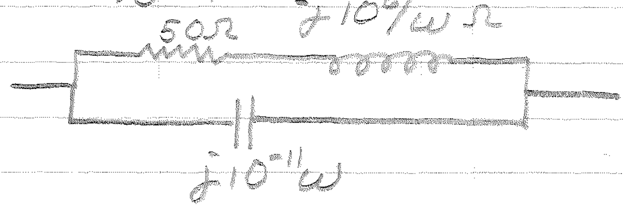
$|Z(s)| = \dfrac{\sqrt{2}}{1 \cdot \sqrt{5}} = .283$

$\phi = 0° - 45° - 71.6° = 116.6°$

BOB MARKS

X-10)





$$Z_{eq} = \frac{j10^{-11}\omega\left(50 - \frac{j10^6}{\omega}\right)}{50 + j\left(10^{-11}\omega - 10^6/\omega\right)}$$

$$= \frac{10^{-5} + j\,50\times10^{-11}\omega}{50 + j\left(10^{-11}\omega - 10^6/\omega\right)}$$

FOR REAL Z_{eq}:

$$\frac{50\times10^{-11}\omega}{10^{-5}} = \frac{10^{-11}\omega - \frac{10^6}{\omega}}{50}$$

$$250\times10^{-9} = 10^{-16}\omega - 10/\omega$$

$$2.50\times10^{-9}\omega - 10^{-16}\omega^2 + 10 = 0$$

$$10^{-16}\omega^2 - 2.50\times10^{-9}\omega - 10 = 0$$

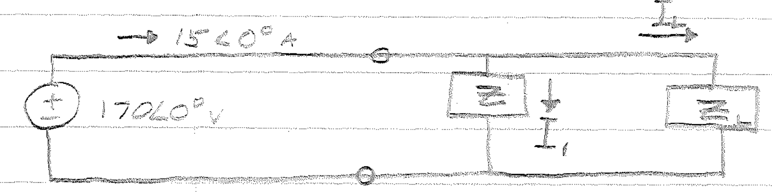
$$\omega = \frac{2.50\times10^{-9} \pm \sqrt{6.25\times10^{-18} + 4\times10^{-15}}}{10^{-32}}$$

$$\sqrt{6.25\times10^{-18} + 4000\times10^{-18}} \doteq 63.2\times10^{-9}$$

$$\omega = \frac{65.7\times10^{-9}}{10^{-32}} = 65.7\times10^{23}$$

BOB MARKS

X-12)



$P = (15)(85) = 1275$

$I_1 = \frac{Z_L}{Z + Z_L}(15)$ $\qquad$ $I_{Z_L} = \frac{Z}{Z_L + Z}\ 15$

$P_Z = \left(\frac{Z_L}{Z + Z_L}\right)(15)(85).8 = 575$ $\qquad$ $P_Z = \frac{Z}{Z_L + Z}(15)(85)(.8) = 700$

$\frac{Z_L}{Z + Z_L}\ 908 = 575$ $\qquad$ $\frac{Z}{Z_L + Z}\ 908 = 700$

$\frac{Z_L}{Z + Z_L} = .633$; $\frac{Z}{Z_L + Z} = .771$

$Z_L = .663 Z + .633 Z_L$; $Z = Z_L .771 + Z .771$

$.367 Z_L = .663 Z$; $Z_L = \frac{.229 Z}{.771}$

$Z_L = 1.8 Z$; $Z_L = .296 Z$

X |

ROBERT MARKS

you need to put [illegible]
[illegible] Answers [illegible]
[illegible]

E Sci. II
HOMEWORK

X-4

$L = 1.0\ H$

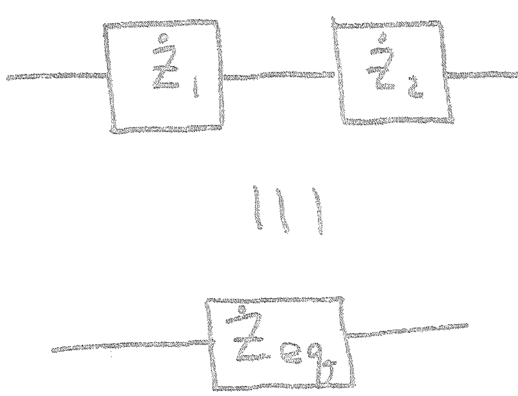
$R = 5.0\ \Omega$

$f = 3.0\ Hz$

$\dot{Z} = \underline{(5 - .0530j)}\ \Omega$ rect.

$= \underline{\cancel{5 \angle .61^\circ\ \Omega}}$ polar

X-5



$Z_1 = 5 - j4\ \Omega$

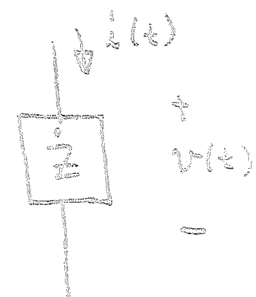
$Z_2 = 8 \angle 30^\circ\ \Omega$

$\dot{Z}_{eq} = \underline{11.92\ \Omega}$

X-6 Draw phasors for

$v(t) = 130 \sin(\omega t + 10^\circ)$ v.

$i(t) = 65 \cos(\omega t - 50^\circ)$ i



$\dot{Z} = \underline{\cancel{8430 \angle -130^\circ}}$

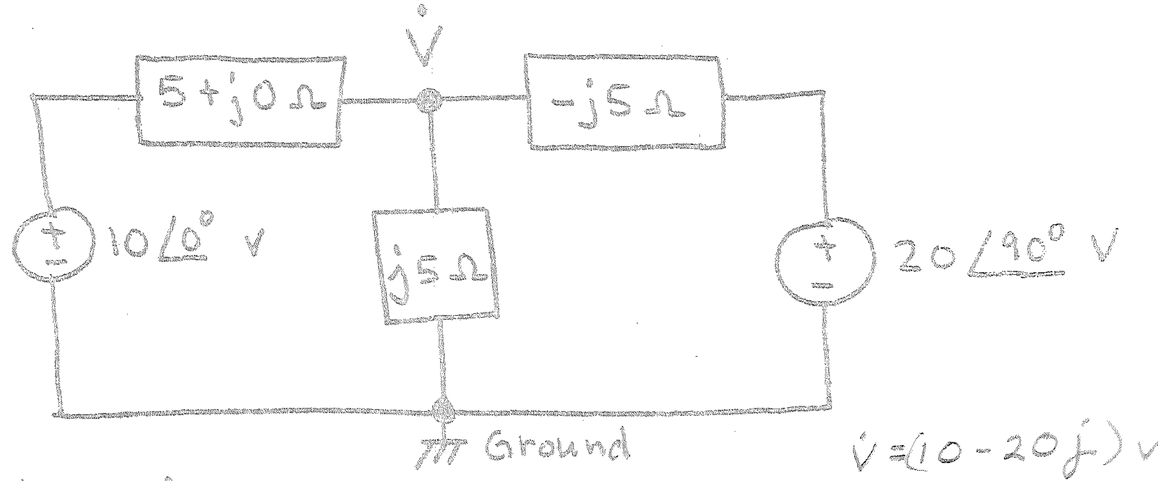
$\boxed{\text{EE 202 HOMEWORK}}$

X-7

$\dot{Z}_{eq}$ = ________________ polar

= ________________ rect.

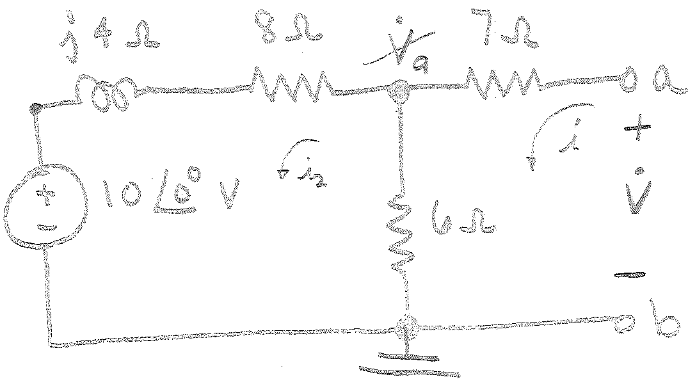
X-8



$\dot{V} = (10-20j)$ V

Find $\dot{V}$, measured against ground.

X-9



Find Thevenins equivalent cct. for terminals a, b

X-4) $\omega = 2\pi f = 6\pi$

$\dot{Z}_L = \frac{1}{j\omega L} = \frac{-j}{(6\pi)(1)}$

$= -.0530\, j$

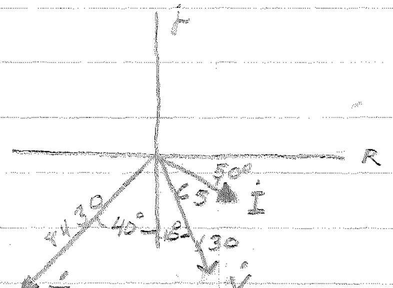
$\dot{Z}_{eq} = (5 - .0530\, j)\,\Omega$

$= 5 \angle -.61°$

X-5) $\dot{Z}_2 = 8 \angle 30° = 6.92 + 4j$

$\dot{Z}_{eq} = 11.92\,\Omega$

X-6)



$\dot{V} = 130 \angle -80°$ V

$\dot{I} = 65 \angle -50°$ A

$\dot{Z} = \dot{V}\dot{I} = 8.43\times 10^3 \angle -130°\,\Omega$

WOW!

X-7) $\frac{1}{\dot{Z}_{eq}} = \frac{1}{\dot{Z}_1} + \frac{1}{\dot{Z}_2} = \frac{\dot{Z}_2 + \dot{Z}_1}{\dot{Z}_2 \dot{Z}_1}$

$\dot{Z}_{eq} = \frac{\dot{Z}_2 \dot{Z}_1}{\dot{Z}_1 + \dot{Z}_2}$

$\dot{Z}_1 = A_1 \angle \psi_1 = A_1 \cos\psi_1 + jA_1 \sin\psi_1$

$\dot{Z}_2 = A_2 \angle \psi_2 = A_2 \cos\psi_2 + jA_2 \cos\psi_2$

$\dot{Z}_1 + \dot{Z}_2 = (A_1 \cos\psi_1 + A_2\cos\psi_2) + j(A_1 \sin\psi_1 + A_2 \sin\psi_2)$

$= (A_1 \sin\psi_1 + A_2 \sin\psi_2) / \sin\tan^{-1}\frac{A_1\sin\psi_1 + A_2\sin\psi_2}{A_2\cos\psi_1 + A_2\cos\psi_2}$

$\angle \tan^{-1}\frac{A_1\sin\psi_1 + A_2\sin\psi_2}{A_2\cos\psi_1 + A_2\cos\psi_2}$

$\dot{Z}_1\dot{Z}_2 = A_1A_2 \angle \psi_1 + \psi_2$

$= A_1A_2\cos(\psi_1+\psi_2) + jA_1A_2\sin(\psi_1+\psi_2)\ \Omega$

$\dot{Z}_{eq} = \frac{A_1A_2 \sin\tan^{-1}\frac{A_1\sin\psi_1 + A_2\sin\psi_2}{A_2\cos\psi_2 + A_1\cos\psi_1}}{A_1\sin\psi_1 + A_2\sin\psi_2} \angle \psi_1 + \psi_2 - \tan^{-1}\frac{A_1\sin\psi_1 + A_2\sin\psi_2}{A_1\cos\psi_1 + A_2\cos\psi_2}$

$\equiv A_3 \angle \psi_3$

$= (A_3\cos\psi_3 + jA_3\sin\psi_3)\ \Omega$

HAIRY ← NO!

X-4) $\omega = 2\pi f = 6\pi$

$\dot{Z}_L = \frac{1}{j\omega L} = \frac{-j}{(6\pi)(1)}$

$= -.0530j$

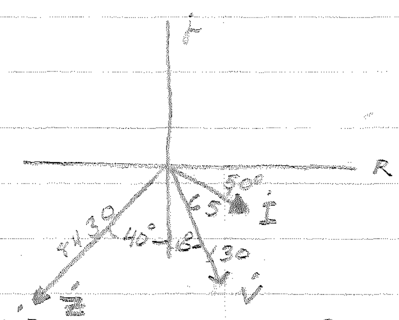
$\dot{Z}_{eq} = (5 - .0530j)\,\Omega$

$= 5 \angle -.61°$

X-5) $\dot{Z}_2 = 8 \angle 30° = 6.92 + 4j$

$\dot{Z}_{eq} = 11.92\,\Omega$

X-6)



$\dot{V} = 130 \angle -80°$ V

$\dot{I} = 65 \angle -50°$ A

$\dot{Z} = \dot{V}\dot{I} = 8.43 \times 10^3 \angle -130°\,\Omega$

WOW!

X-7) $\frac{1}{\dot{Z}_{eq}} = \frac{1}{\dot{Z}_1} + \frac{1}{\dot{Z}_2} = \frac{\dot{Z}_2 + \dot{Z}_1}{\dot{Z}_2 \dot{Z}_1}$

$\dot{Z}_{eq} = \frac{\dot{Z}_2 \dot{Z}_1}{\dot{Z}_1 + \dot{Z}_2}$

$\dot{Z}_1 = A_1 \angle \psi_1 = A_1 \cos\psi_1 + jA_1 \sin\psi_1$

$\dot{Z}_2 = A_2 \angle \psi_2 = A_2 \cos\psi_2 + jA_2 \cos\psi_2$

$\dot{Z}_1 + \dot{Z}_2 = (A_1 \cos\psi_1 + A_2 \cos\psi_2) + j(A_1 \sin\psi_1 + A_2 \sin\psi_2)$

$= (A_1 \sin\psi_1 + A_2 \sin\psi_2) \Big/ \sin\tan^{-1}\frac{A_1 \sin\psi_1 + A_2 \sin\psi_2}{A_2 \cos\psi_1 + A_2 \cos\psi_2}$

$\angle \tan^{-1} \frac{A_1 \sin\psi_1 + A_2 \sin\psi_2}{A_2 \cos\psi_1 + A_2 \cos\psi_2}$

$Z_1 Z_2 = A_1 A_2 \angle \psi_1 + \psi_2$

$= A_1 A_2 \cos(\psi_1 + \psi_2) + jA_1 A_2 \sin(\psi_1 + \psi_2)\quad\Omega$

$\dot{Z}_{eq} = \frac{A_1 A_2 \sin\tan^{-1}\frac{A_1 \sin\psi_1 + A_2 \sin\psi_2}{A_2 \cos\psi_2 + A_1 \cos\psi_1}}{A_1 \sin\psi_1 + A_2 \sin\psi_2} \angle \psi_1 + \psi_2 - \tan^{-1}\frac{A_1 \sin\psi_1 + A_2 \sin\psi_2}{A_1 \cos\psi_1 + A_2 \cos\psi_2}$

$\equiv A_3 \angle \psi_3$

$= (A_3 \cos\psi_3 + jA_3 \sin\psi_3)\,\Omega$

HAIRY ← NO!

EE 302

Answers to homework

$\boxed{X-4}$

$$\dot{Z} = 5 + j\,2\pi \cdot 3 \cdot 1 = \boxed{5 + j\,18.85\ \Omega}$$

$$= \boxed{19.5\ \angle 75.15^\circ\ \Omega}$$

$\boxed{X-5}$

$$Z_2 = 8\cos 30^\circ + j\,8\sin 30^\circ$$

$$= 6.92 + j\,4$$

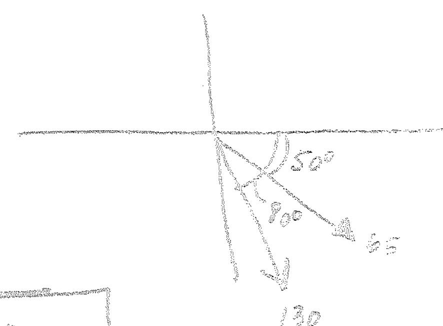
$$\dot{Z}_{eq} = 5 - j\,4 + 6.92 + j\,4 = \boxed{11.92\angle 0^\circ\ \Omega}$$

$\boxed{X-6}$

$$v(t) = 130\sin(\omega t + 10^\circ)\ \text{V}$$

$$= 130\cos(\omega t - 80^\circ)\ \text{V}$$

$$i(t) = 65\cos(\omega t - 50^\circ)\ \text{V}$$



$$\dot{Z} = \frac{130\angle -80^\circ}{65\angle -50^\circ} = \boxed{2.0\angle -30^\circ\ \Omega}$$

$\boxed{\text{EE 202 Answers}}$

$\boxed{X-7}$ Using $\dot{Z}_1$ and $\dot{Z}_2$ from X-5

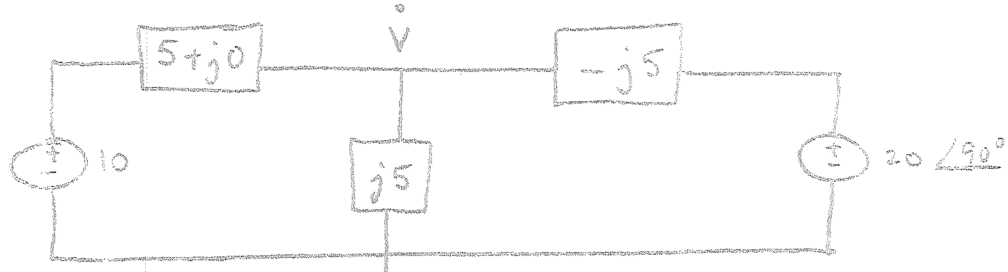
$$Z_1 = 5 - j4\ \Omega \qquad Z_2 = 8\angle 30^\circ\ \Omega$$

$$= \cancel{5.32}\ \angle -38.7^\circ \qquad = 6.92 + j4$$

6.4

$$\dot{Z}_{eq} = \frac{(5.32\angle -38.7^\circ)(8\angle 30^\circ)}{11.92} = \boxed{\overset{4.57}{\cancel{3.35}}\angle -8.7^\circ\ \Omega}$$

$\boxed{X-8}$



$$\frac{10-\dot{V}}{5} + \frac{j20 - V}{-j5} + \frac{-V}{j5} = 0$$

$$2 - 0.2\dot{V} - 4 - \cancel{j0.2\dot{V}} \ \cancel{+j0.2}\dot{V} = 0$$

$$-0.2\dot{V} = 2 \implies \dot{V} = \frac{-2}{.2} = \boxed{-10\angle 0^\circ\ V}$$

$\boxed{\text{EE 202 Answers}}$

$\boxed{\text{X-9}}$

$$\dot{V}_T = \frac{6}{14 + j4}\, 10\angle 0^\circ$$

$$= \frac{60\angle 0^\circ}{14.55\angle 15.95^\circ} = \boxed{4.13\angle -15.95^\circ \text{ V.}}$$

$$\dot{Z}_T = 7 + \frac{6(8+j4)}{14+j4} = 7 + \frac{48+j24}{14.55\angle 15.95^\circ}$$

$$= 7 + \frac{53.6\angle 26.6^\circ}{14.55\angle 15.95^\circ} = 7 + 3.68\angle 10.65$$

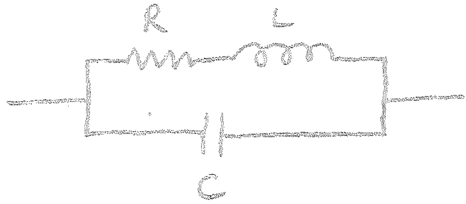
$$= 7 + 3.62 + j0.68 = \boxed{10.62 + j0.68}$$

EE DEPT
RPI
4.28.70

E.E. 202

ANSWERS TO HOMEWORK

$\boxed{K-10}$



$R = 50\,\Omega$

$L = 1\,\mu H$

$C = 10\,pF$

$$Z = \frac{\left(\frac{1}{j\omega C}\right)(R + j\omega L)}{\frac{1}{j\omega C} + R + j\omega L} = \frac{R + j\omega L}{1 - \omega^2 LC + j\omega RC}$$

impedance is real when angle of numerator = angle of denominator

$$\frac{\omega L}{R} = \frac{\omega RC}{1 - \omega^2 LC}$$

$$R^2 C = L - \omega^2 L^2 C$$

$$\omega^2 L^2 C = -R^2 C + L$$

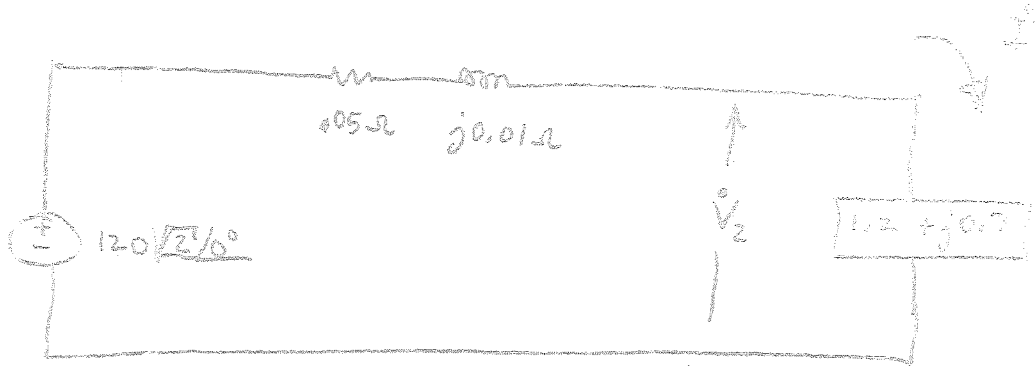
$$\omega^2 = \frac{-R^2 C + L}{L^2 C} = \frac{-2500 \times 10^{-11} + 10^{-6}}{10^{-12} \cdot 10^{-11}}$$

$$= 10^{-6} \cdot 10^{23} = 10^{17}$$

$$\omega = 10^8 \sqrt{10} = \boxed{3.16 \times 10^8 \ \frac{\text{rad}}{\text{sec}}}$$

EE 202
ANSWERS

X-11



$$\dot{I} = \frac{120\sqrt{2}\angle 0^\circ}{1.25 + j\,0.71} = \frac{120\sqrt{2}\angle 0^\circ}{1.435\angle 29.6^\circ}$$

$$= 118.5\angle -29.6^\circ \text{ A.}$$

$$P_L = \frac{|I|^2 R_L}{2} = \frac{118.5^2 \times 1.2}{2} = \boxed{8400 \text{ W}}$$

$$\dot{V}_2 = \dot{I}\,\dot{Z}_L = (118.5\angle -29.6^\circ)(1.2 + j0.7)$$

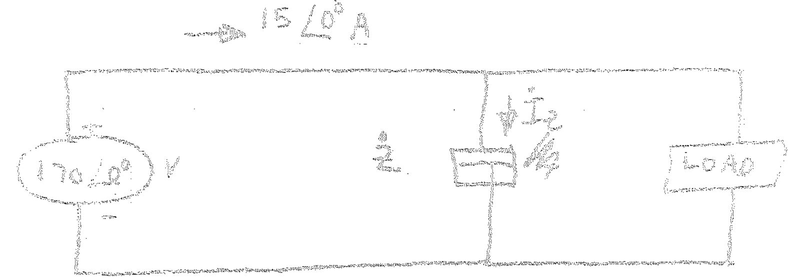
$$\underbrace{}_{1.385\angle 30.3^\circ}$$

$$= \boxed{164\angle 0.7^\circ \text{ V.}}$$

EE 202

ANSWERS TO HOMEWORK

X-12



700 W, @ 0.8 P.F. INDUCTIVE

Power triangle for source

$P_s = 170 \times 15 = 2550$ W

$Q_s = 0$ VARS

LOAD

$P_L = 700$ W. $\theta_L = \cos^{-1} 0.8 = 36.9°$

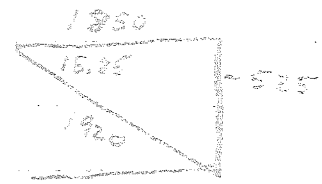
$Q_L = 875 \sin 36.5°$ $VA_L = 875$ V-A

$= 525$ VARS

Power Δ for Z.

$P_Z = 2550 - \cancel{525}^{\,700} = \cancel{2025}^{\,1850}$ watts

$Q_Z = -525$ VARS



$\theta_Z = \tan^{-1} \dfrac{-525}{1850} = -15.85°$

VA = 1920 VA

$|I_Z| = \dfrac{1920}{170} = 11.3$ A. $|Z| = \dfrac{170}{11.3} = 15.05\ \Omega$

$$Z = \boxed{15.05 \angle -15.85°\ \Omega}$$

BOB MARKS

-1%

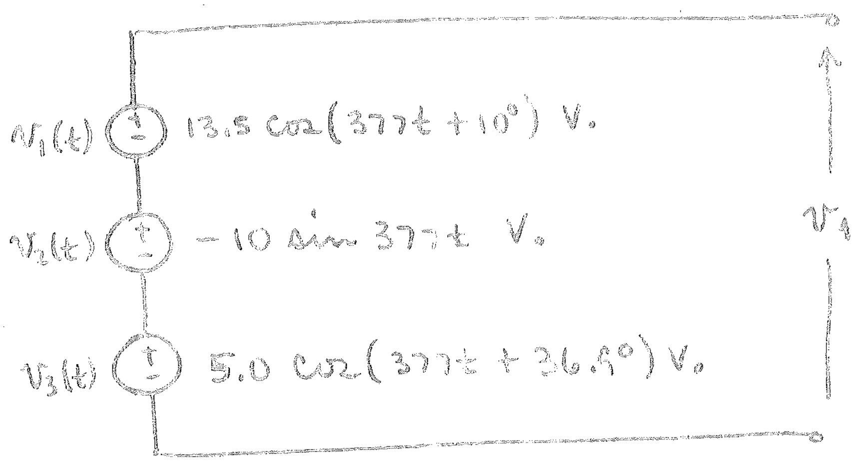
-1%

Department of Electrical Engrg.
Rose Polytechnic Institute
Terre Haute, Indiana
April 28, 1970

9

EE 202
TEST 1
CLOSED BOOKS - 50 Minutes

1. Find the time equation for v(t).



$v_4(t) = \underline{23.1 \cos(377t + 41.6°)}$

UNITS

$\omega = 377$

$\mathbf{V}_1 = 13.5 \angle 10° = 13.3 + j2.34$

$\mathbf{V}_2 = -10 \angle -90° = \qquad j10$

$\mathbf{V}_3 = 5 \angle 36.9° = \underline{4.0 + j3}$

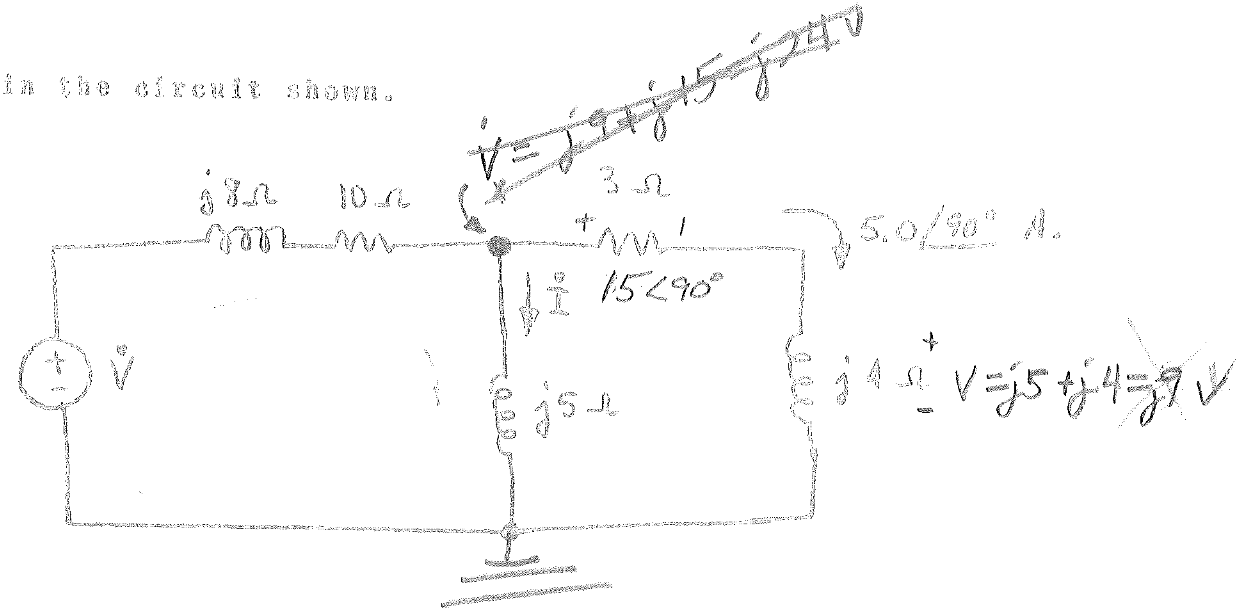
$17.3 + j15.34$

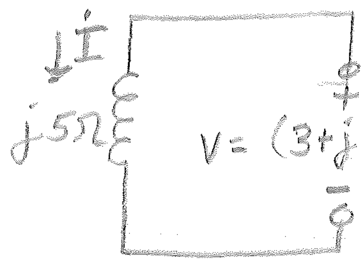
$\mathbf{V}_4 = 23.1 \angle 41.6°$ V

$v_4(t) = 23.1 \cos(377t + 41.6°)$ VOLTS

10

2. Find $\dot{I}$ in the circuit shown.





$V = (3+j4)j5 = j15 - 20$

$$\dot{I} = \frac{\dot{V}}{\dot{Z}} = \frac{-20 + j15}{j5}$$

$$= \frac{25 \angle (180 + 36.9)^\circ}{5 \angle 90^\circ}$$

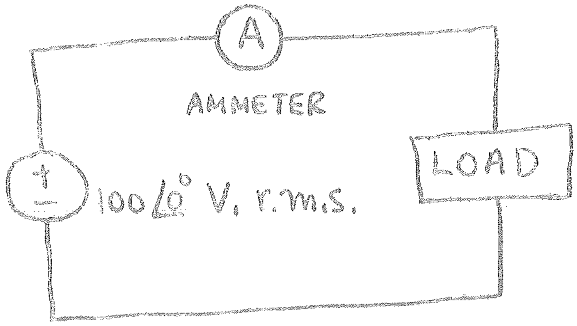
$$= \frac{25 \angle 216.9^\circ}{5 \angle 90.0^\circ}$$

$$\dot{I} = 5 \angle 126.9^\circ \text{ AMPS}$$

quadrant

9

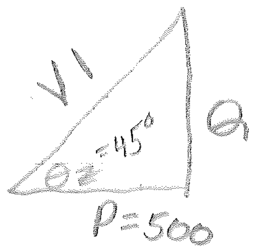
3. The power that the load dissipates is 500 watts. Its power factor is 0.707 and known to be inductive. Find the reading of the a-c ammeter and the complex load impedance. The ammeter has zero impedance.



500 WATTS

$\cos\theta_z = .707$ INDUCTIVE

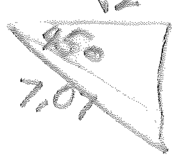
$\theta_z = 45^\circ$



$$P = 500 = VI\cos\theta_z = I(.707)(100)$$

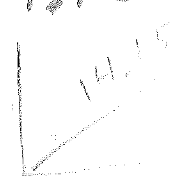
$$I = \frac{500}{70.7} = 7.07 < -\theta_z$$

$$\frac{7.07}{\sqrt{2}} = 5$$



→ AMMETER READS ~~5~~ 7.07 AMPS

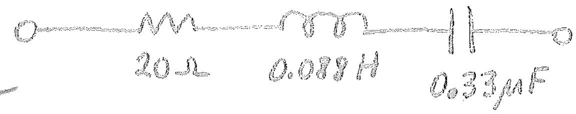
$$\dot{Z} = \frac{\dot{V}}{\dot{I}} = \frac{100\angle 0^\circ}{7.07\angle -45^\circ} = 14.15\angle 45^\circ$$



→ $\dot{Z} = (10 + j10)\,\Omega$

8

4. For the circuit shown find



a) minimum value of $|\dot{Z}|$ = 20Ω

b) frequency in Hz where minimum value of $|\dot{Z}|$ occurs.

c) frequency or frequencies where $|\dot{Z}|$ equals $\sqrt{2}$ (minimum value of $|\dot{Z}|$).

d) Q_0 of this resonant circuit

e) bandwidth in Hz of this resonant circuit.

a) $|Z|_{MIN} = R = 20\Omega$

$\cancel{b)\ Z_{eq} = 20 + j8.8\times10^{-2}\omega + \frac{1}{j\omega(.33\times10^{-6})}}$

$\cancel{b)\ Z = 20 + j8.8\times10^{-2}\omega + \frac{1}{j(.33\times10^{-6})\omega}}$

$\cancel{= 20 + j(8.8\times10^{-2}\omega - 3\times10^{6}/\omega)}$

$\cancel{8.8\times10^{-2}\omega^2 = 3\times10^{6}}$

$\cancel{\omega = \sqrt{\frac{3\times10^{6}}{8.8\times10^{-2}}} = \sqrt{34}\times10^{2} = 58.4\ \frac{RAD}{SEC}}$

b) $\omega_r = \sqrt{\frac{1}{LC}}$

$= \sqrt{\frac{3\cdot10^{2}\cdot10^{6}}{8.8}} = \sqrt{34}\times10^{4}$

$= .585\times10^{4}\ \frac{RAD}{SEC}\ \frac{CYCLE}{2\pi RAD} = .093\times10^{4}\ HZ = 930\ HZ$

c) $BW = \frac{R}{L} = \frac{20}{8.8\times10^{-2}} = 2.27\times10^{2} = 227$

$\omega_U \doteq 934 + 113.5 = 1047.5\ \frac{RAD}{SEC}$

$\omega_L \doteq 934 - 113.5 = 820.5\ \frac{RAD}{SEC}$

d) $BW = \frac{\omega_R}{Q_0} \Rightarrow Q_0 = \frac{\omega_r}{BW} = \frac{5.87\times10^{3}}{2.27\times10^{2}} = 25.9$

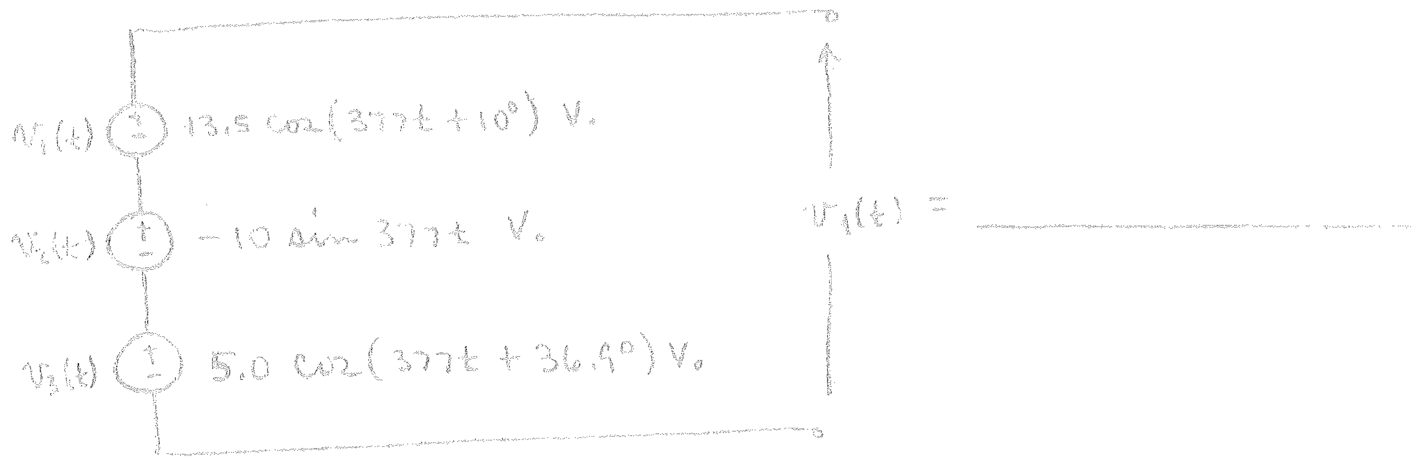
e) $BW = 227\ \frac{RAD}{SEC}\ \frac{CYCLE}{2\pi\ RAD} = 36.1\ HZ$

10

[illegible]
Rose Polytechnic Institute
Terre Haute, Indiana
April [illegible]

EE 202
TEST 1
CLOSED BOOKS - 50 Minutes

1. Find the time equation for v(t).



Solution

$$\dot{V}_1 = 13.5 \angle 10^\circ = 13.3 + j\,2.26$$

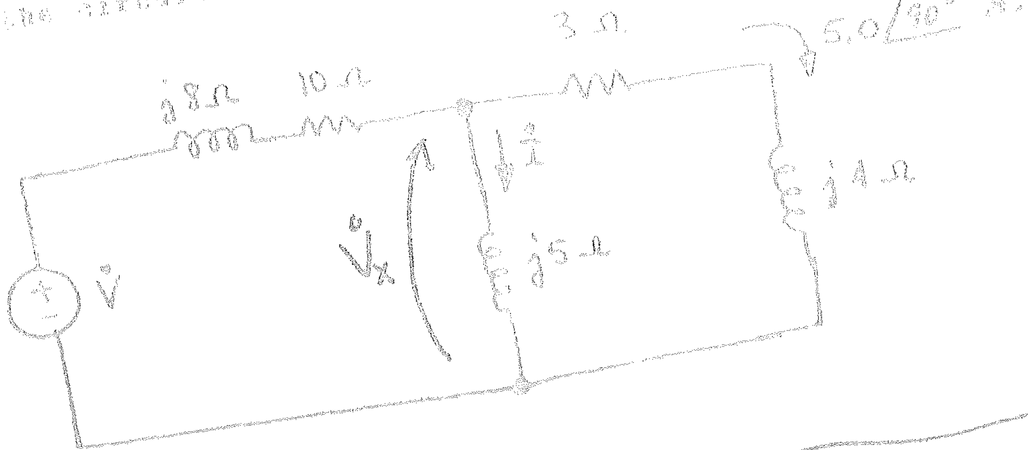
$$\dot{V}_2 = 10 \angle 90^\circ = 0 + j\,10$$

$$V_3 = 5 \angle 36.9^\circ = 4 + j\,3$$

$$\dot{V}_4 = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 = 17.3 + j\,15.26 = 23.1 \angle 41.4^\circ \ \text{V}$$

$$v_4(t) = \boxed{23.1 \cos(377t + 41.4^\circ)\ \text{V.}}$$

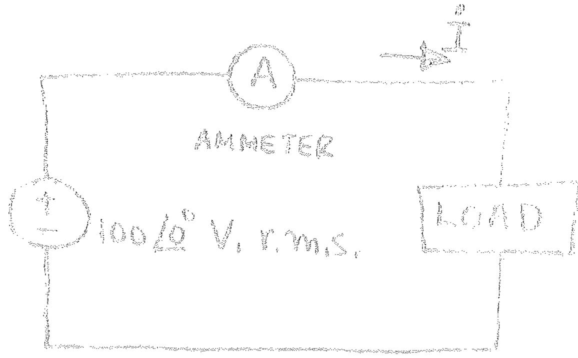
2. Find $\dot{I}$ in the circuit shown.



$$\dot{V}_x = (5\angle 90^\circ)(3+j4)$$

$$\dot{I} = \frac{\dot{V}_x}{j5} = \frac{(5\angle 90^\circ)(3+j4)}{5\angle 90^\circ} = \boxed{3+j4\ \text{A}}$$

[illegible] the power factor is 0.707 and known to be inductive. Find the reading of the a-c ammeter and the complex load impedance. The ammeter has zero impedance.



$$P = |\dot{V}||\dot{I}|\cos\theta = 100 \cdot |\dot{I}| \cdot 0.707 = 500$$

$$|\dot{I}| = \frac{500}{100 \times .707} = \frac{5}{.707} = \boxed{7.07\ A}$$

$$|\dot{Z}| = \frac{|\dot{V}|}{|\dot{I}|} = \frac{100}{7.07} = 14.14\ \Omega$$

$$\dot{Z} = \boxed{14.14\ \angle 45^\circ} = \boxed{10 + j10\ \Omega}$$

a) minimum value of $|\dot{Z}|$ $\qquad$ $L = 0.088H$ $\quad$ $C = 0.33\mu F$

b) frequency in Hz where minimum value of $|\dot{Z}|$ occurs.

c) frequency or frequencies where $|\dot{Z}|$ equals $\sqrt{2}$ (minimum value of $|\dot{Z}|$).

d) Q_0 of this resonant circuit

e) bandwidth in Hz of this resonant circuit.

(a) $|\dot{Z}|_{MIN} = R = \boxed{20\,\Omega}$

(b) $|\dot{Z}| = |\dot{Z}|_{MIN}$ when $X_L = -X_C$

$$\omega L = \frac{1}{\omega C}$$

$$\omega = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{8.8\times10^{-2}\cdot 3.3\times10^{-7}}}$$

$$= \cancel{\sqrt{3.44\times10^{6}}} = \sqrt{34.4\times10^{6}}$$

$$= 5.87\times10^{3} = 5870\ \frac{rad}{sec}$$

$$f_r = \frac{5870}{2\pi} = \boxed{934\ Hz}$$

(c) $\pm 20 = 0.088\omega - \dfrac{1}{3.3\times10^{-7}\omega}$

mult $\quad 20\times3.3\times10^{-7}\omega = 0.088\times3.3\times10^{-7}\omega^2 - 1$

$$6.6\omega = 0.29\omega^2 - 10^7 \Rightarrow \omega^2 - 228\omega - 3.45\times10^7 = 0$$

$$\omega = \frac{228 \pm \sqrt{228^2 + 4\times3.45\times10^7}}{2} = 114 \pm \sqrt{114^2 + 3.45\times10^7}$$

4 Cont

$$\omega = 114 \pm \sqrt{13000 + 3.45 \times 10^7} = 114 \pm \sqrt{34.5 \times 10^6}$$

$$= 114 \pm 5870 = 5984 \ \frac{\text{rad}}{\text{sec}} = \boxed{953 \text{ Hz}}$$ upper $\frac{1}{2}$ power freq.

lower - sign.

$$\omega^2 + 228\omega - 3.45 \times 10^7 = 0$$

$$\omega = \frac{-228 \pm \sqrt{(\quad)}}{2} = -228 + 5870$$

$$= 5042 \text{ rad/sec} = \boxed{899 \text{ Hz}}$$

lower $\frac{1}{2}$ power freq.

(d) $$Q_0 = \frac{\omega_0 L}{R} = \frac{5870 \times .088}{20} = \boxed{25.8}$$

(e) $$BW = \frac{\omega_0}{Q_0} = \frac{5870}{25.8} = 227 \text{ rad/sec}$$

$$\text{or} \quad \frac{934}{25.8} = \boxed{34.8 \text{ Hz}}$$

EE DEPT RPI
TERRE HAUTE, IND.
MAY 4, 1970

EE 202
HOMEWORK

X-20 Draw the Bode diagram for the magnitude of the $H(\omega)$ shown.

$$H(\omega) = \frac{1}{1 + j\omega/5}$$

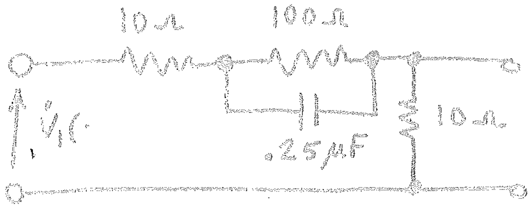
X-21 Same as above

$$H(\omega) = \frac{(1 + j\omega/10)}{(1 + j\omega/100)}$$

X-22 Same as above

$$H(s)\Big|_{s=j\omega} = \frac{s+5}{s^2+12s+20}$$

X-23



Draw the Bode diagram of $|H(\omega)|$ vs ω on log-log axes for $H(\omega) = V_2/V_1$

EE 202 HOMEWORK
CONTINUED

(X-24)

$$H(s) = \frac{10s(s+40)}{(s+1)(s+200)}$$

Sketch $\left| H(s)|_{s=j\omega} \right|$ as fct. of ω on log-log axes.

(X-25) $$H(\omega) = \frac{1+j\frac{\omega}{10}}{(j\omega)(1+j\frac{\omega}{10})^2(1+j\frac{\omega}{100})}$$

(X-26) $$H(\omega) = \frac{(1+j\omega)(1+j\omega 3)}{(1+j\frac{\omega}{5})(1+j10\omega)}$$

Find $|H(100)|$, $|H(1)|$, $|H(3)|$

[illegible], MD
MAY 11, 1980

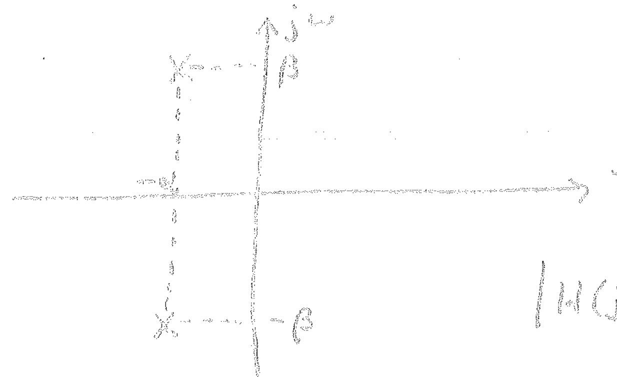
$\boxed{\text{EE 202 NOTES}}$

Second order response constants, formulas

$$H(s) = \frac{1}{s^2 + 2\alpha s + \omega_0^2} = \frac{1}{(s^2 + 2\alpha s + \alpha^2) + (\omega_0^2 - \alpha^2)}$$

$$= \frac{1}{(s+\alpha)^2 + \beta^2} \qquad \text{assume } \omega_0 > \alpha$$

$$= \frac{1}{(s+\alpha+j\beta)(s+\alpha-j\beta)} \qquad \beta = \sqrt{\omega_0^2 - \alpha^2}$$



$$|H(j\omega)|_{max} = \boxed{\frac{1}{2\alpha\beta}}$$

$$\omega_{MAX} = \sqrt{\omega_0^2 - 2\alpha^2} = \sqrt{\beta^2 - \alpha^2}$$

$$\omega_U = \sqrt{\omega_M^2 + 2\alpha\beta} \qquad \omega_L = \sqrt{\omega_M^2 - 2\alpha\beta}$$

$$BW = \omega_U - \omega_L$$

Approx. formulas; $\omega_0 \gg \alpha$
$\omega_M \doteq \omega_0$
$\omega_U \doteq \omega_0 + \alpha$
$\omega_L \doteq \omega_0 - \alpha$
$H_{MAX} = 1/2\alpha\beta$

EE DEPT · RPI
TERRE HAUTE, IND.
MAY 14, 1970

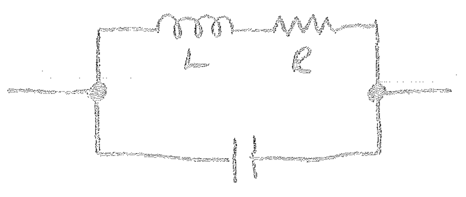
EE 202
MORE HOMEWORK

DUE 5.17.70

X-27 SCOTT'S 633.18.31

X-28 SCOTT'S 634.18.34

X-29 Find $Z(s)$



$L = 88\,\text{mH}$
$C = 4\,\mu\text{F}$
$R = 20\,\Omega$

(a) Find $Z(s)$
(b) Find $|Z(j\omega)|_{max}$
(c) " ω_{MAX} where $|Z(j\omega)|$ is max.

$\boxed{X-17}$

$$H(s) = \frac{1/LC}{s^2 + \frac{R}{L}\,s + \frac{1}{LC}} = \frac{\omega_0^2}{(s+\alpha-j\beta)(s+\alpha+j\beta)}$$

$$\omega_0^2 = \frac{1}{LC} = 2$$

$$\alpha = \frac{R}{2L} = \frac{0.1}{2} = \boxed{.05 \text{ sec}^{-1}}$$

$$\beta = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{2 - 25\times10^{-4}} = \sqrt{1.9975} = \sqrt{2 - .0025}$$

$$= \sqrt{2}\cdot\sqrt{1 - .0012} \doteq \sqrt{2}\,(1 - .0006) = \sqrt{2}\,(.9994)$$

$$= \boxed{1.413294 \text{ rad/sec}}$$

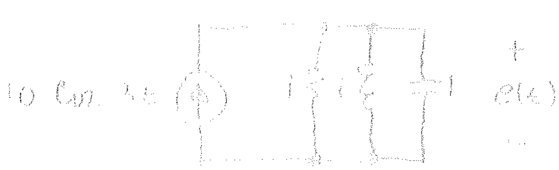
$$\omega_p = \sqrt{\omega_0^2 - 2\alpha^2} = \boxed{1.41244468 \text{ rad/sec}}$$

$$\omega_U = \sqrt{\omega_0^2 + 2\alpha\beta} = \boxed{1.463294 \text{ rad/sec}}$$

$$\omega_L = \sqrt{\omega_0^2 - 2\alpha\beta} = \boxed{1.363294 \frac{\text{rad}}{\text{sec}}}$$

$$|H|_{max} = \frac{\omega_0^2}{2\alpha\beta} = \frac{2}{2\times.05\times1.413294} = \boxed{14.15058}$$

$\boxed{X-18}$ (Prob. 5.17.16.12) in SCOTT



$$I = 10\angle 0^\circ$$

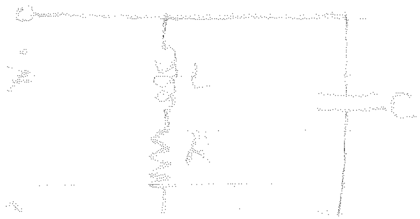
$$\hat{Z} = \frac{1}{\frac{1}{1} + j3 + \frac{1}{j3}} = \frac{j3}{1-9+j3} = \frac{3\angle 90^\circ}{-8+j3}$$

$$\hat{Z} = \frac{3\angle 90^\circ}{8.55\angle 159.4^\circ} = 0.351\angle -69.4^\circ$$

$$\hat{V} = 3.51\angle -69.4^\circ \text{ v} \quad \Rightarrow \quad \boxed{v(t) = 3.51\cos(3t - 69.4^\circ) \text{ volts}}$$

Bob Marks

X·19



$L = 20\times10^{-3}\,H$

$C = 5\times10^{-7}\,F$

$R = 10\,\Omega$

$$Z_{eq} = \frac{\frac{1}{j\omega L} + R\, j\omega C}{\frac{1}{j\omega L} + R + j\omega C}$$

$$= \frac{\frac{R}{L} + j\omega RC}{R + j\left(\omega C - \frac{1}{\omega L}\right)}$$

AT DRIVING POINT:

at zero phase driving point

$$\frac{\omega RC}{C} = \frac{\omega C - \frac{1}{\omega L}}{R}$$

$$\omega R^2 L = \omega C - \frac{1}{\omega L}$$

$$\omega^2 R^2 L^2 = \omega^2 LC - 1$$

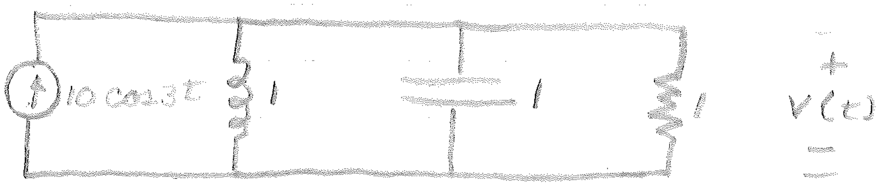
$$\omega^2 (R^2 L^2 - LC) = 1$$

$$\omega = \sqrt{\frac{1}{R^2 L^2 - LC}}$$

$$= \sqrt{\frac{1}{10^2 \cdot 9 \cdot 10^{-4} - 2 \cdot 10^{-2} \cdot 5 \cdot 10^{-7}}}$$

$$= \frac{1}{3} \cdot 10^2 = 33.\cancel{3}\ \text{RAD/SEC}$$

X-18)



$$V = \frac{\dfrac{Z_c Z_L}{Z_c + Z_L}}{\dfrac{Z_c Z_L}{Z_c + Z_L} + Z_R} I$$

$$= \frac{Z_c Z_L I}{Z_c Z_L + Z_R (Z_c + Z_L)}$$

$$= \frac{\left(j\omega \frac{1}{j\omega}\right) I}{1 + \left(j\omega + \frac{1}{j\omega}\right.}$$

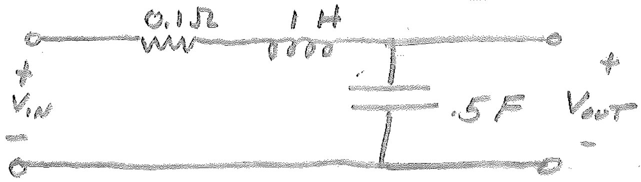
$$\dot{I} = 10\angle 0^\circ \ ; \ \omega = 3$$

$$\dot{V} = \frac{10\angle 0^\circ}{1 + j(3 - .333)} = \frac{10\angle 0^\circ}{1 + j(2.667)} = \frac{10\angle 0^\circ}{2.85\angle 69.45^\circ}$$

$$\dot{V} = 3.51\angle -69.45^\circ$$
$$= 3.51 \cos(3t - 69.45^\circ) \text{ volts}$$

2 ✓

X-17)



$$\dot{H}(\omega) = \frac{Z_C}{Z_C + Z_L + Z_R}$$

$$= \frac{j\,.5\omega}{j\,.5\omega + \frac{1}{j\omega} + .10}$$

$$= \frac{-.5\omega}{-.5\omega + 1 + .1j\omega}$$

$$= \frac{1}{1 - \frac{1}{.5\omega} - \frac{.1j}{.5}}$$

$$\frac{1}{\dot{H}(\omega)} = 1 - \frac{1}{.5\omega} - j\,.2$$

$$y = \left|\frac{1}{\dot{H}(\omega)}\right| = \sqrt{\left(1 - \frac{1}{.5\omega}\right)^2 + .04}$$

$$U = 1 - \frac{1}{.5\omega} \qquad \frac{dU}{d\omega} = \frac{1}{.5\omega^2}$$

$$V = U^2 + .04 \qquad \frac{dV}{dU} = 2U = 2 - \frac{4}{\omega}$$

$$y = V^{\frac{1}{2}} = \frac{1}{2} V^{-\frac{1}{2}} = \frac{1}{2\sqrt{U^2 + .04}}$$

$$= \frac{1}{2\sqrt{\left(1 - \frac{1}{.5\omega}\right)^2 + .04}}$$

$$\frac{dy}{d\omega} = \frac{\frac{1}{.5\omega^2}\left(2 - \frac{4}{\omega}\right)}{2\sqrt{\left(1 - \frac{1}{.5\omega}\right)^2 + .04}} = 0$$

$$\frac{2 - \frac{4}{\omega}}{\omega^2\sqrt{\left(1 - \frac{1}{.5\omega}\right)^2 + .04}} = 0$$

$$\frac{-2\omega + 4}{\omega^3\sqrt{\left(1 - \frac{1}{.5\omega}\right)^2 + .04}} = 0$$

$\dot{H}(\omega)$ MAX AT $\omega = 2$ X

b) $$\frac{\dot{H}(\omega)}{H_{MAX}(\omega)} = .707 = \frac{\left| 1 - \frac{1}{.5\omega} - j.2 \right|}{2}$$

$$1.414 = \sqrt{\left(1 - \frac{1}{.5\omega}\right)^2 - .04}$$

$$2 = 1 - \frac{4}{\omega} + \frac{1}{.25\omega^2} - .04$$

$$2\omega = \omega - 4 + 4/\omega - .04\omega$$

$$\cancel{2}\omega^2 = \cancel{\omega^2} - 4\omega + 4 - .04\omega^2$$

$$1.96\omega^2 + 4\omega - 4 = 0$$

$$\omega = \frac{-4 \pm \sqrt{16 + 31.4}}{2.92}$$

$$= \frac{-4 \pm \sqrt{77.4}}{2.92}$$

$$= \frac{-4 + 8.8}{2.92} = 1.64 \frac{RAD}{SEC}$$

c) $$\Gamma = 1 - \frac{1}{.5\omega} - j.2$$

$$\omega = 2$$

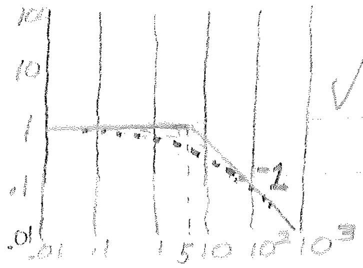
$$\Gamma = 0 - j.2 = .2\angle{-90^\circ}$$

$$\dot{H}(\omega) = 5\angle 90^\circ$$

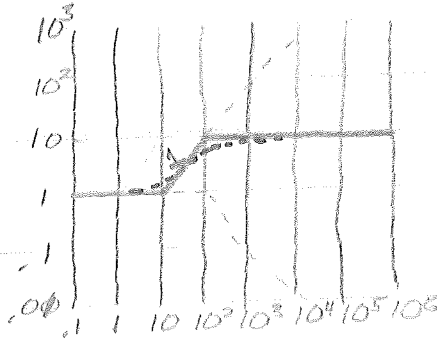
ARG!

BOB MARKS
EE-202

X-20) $H(\omega) = \frac{1}{1+j\frac{\omega}{5}}$



X-21) $H(\omega) = \frac{1+j\,\omega/10}{1+j\,\omega/100}$

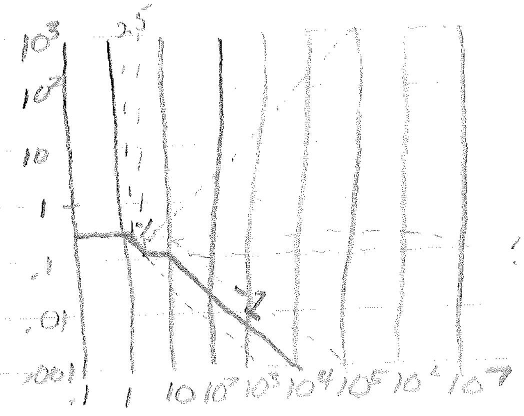


X-22) $H(s)\big|_{s=j\omega} = \frac{s+5}{(s+10)(s+2)}$

$H(\omega) = \frac{5+j\omega}{(10+j\omega)(2+j\omega)}$

$= \frac{5\left(1+j\frac{\omega}{5}\right)}{10\left(1+j\frac{\omega}{10}\right)2\left(1+j\frac{\omega}{2}\right)}$

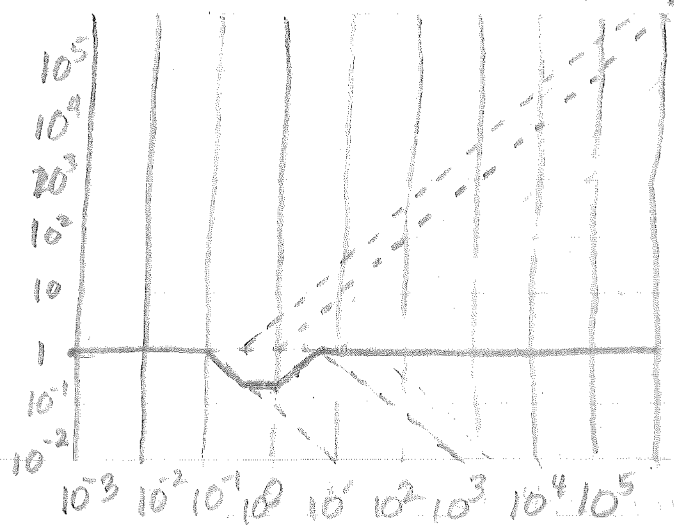
$= .25\,\frac{\left(1+j\frac{\omega}{5}\right)}{\left(1+j\frac{\omega}{10}\right)\left(1+j\frac{\omega}{2}\right)}$



2

X-26)

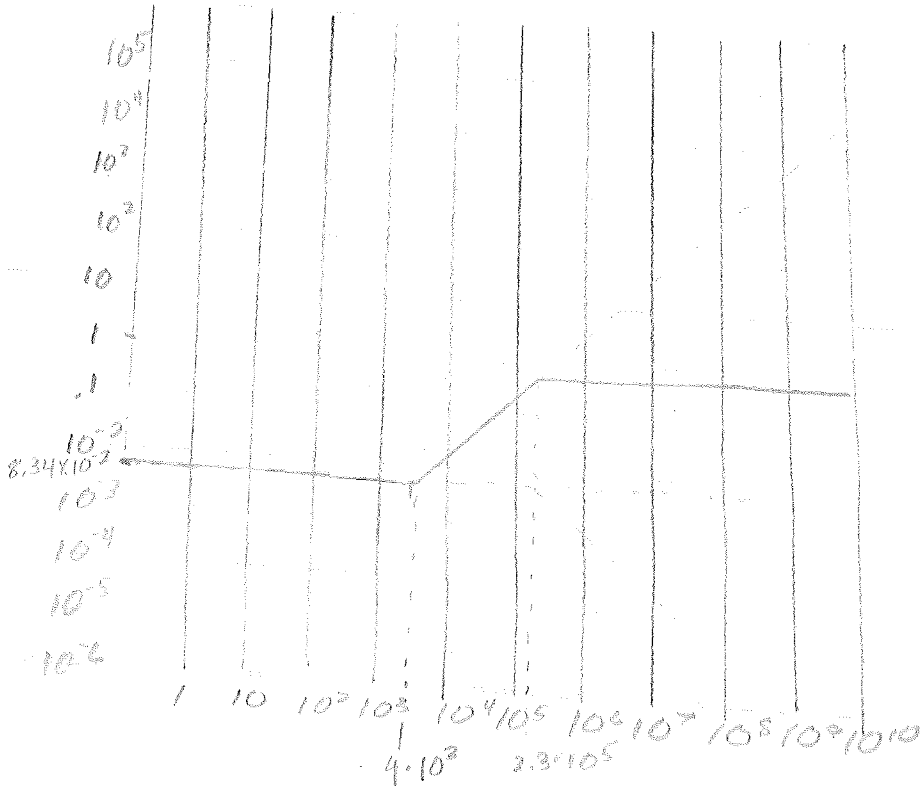
$$H(\omega) = \frac{(1+j\omega)\left(1+j\frac{\omega}{1/3}\right)}{\left(1+j\frac{\omega}{5}\right)\left(1+j\frac{\omega}{1/10}\right)}$$



a) $H(100) = 0$

b) $5 = \frac{1}{H(1)} \Rightarrow H(1) = 1/5$

c) $\frac{3}{1} = \frac{H(3)}{1/5} \Rightarrow H(3) = \frac{3}{5}$

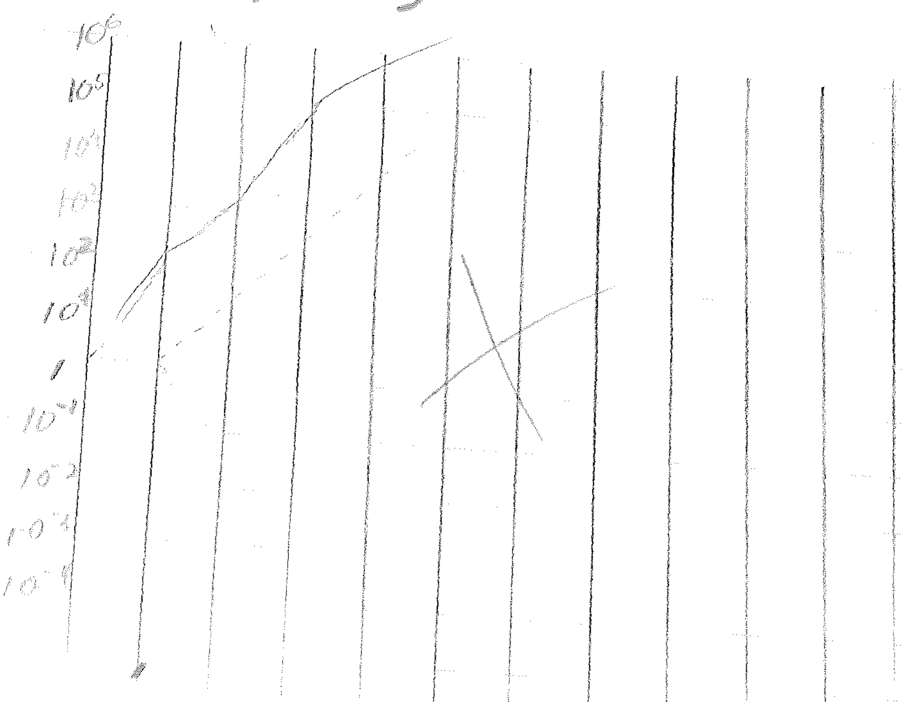


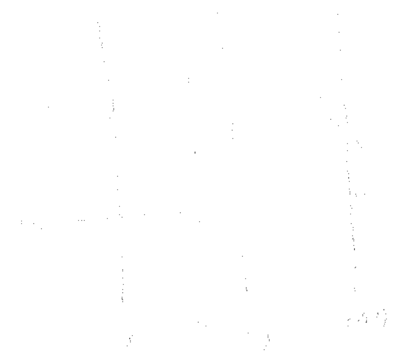
2

X-24) $H(s) = \dfrac{10s(s+40)}{(s+1)(s+200)}$

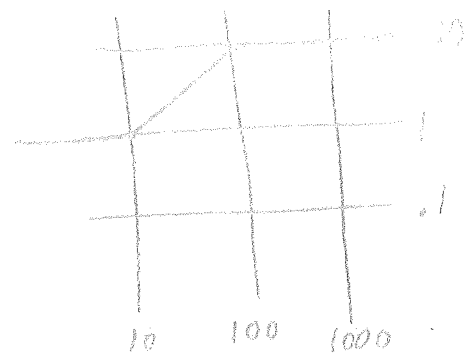
$$H(\omega) = \frac{10\omega j(\omega j+40)}{(\omega j+1)(\omega j+200)} = \frac{10\,\omega j\;40\left(1+j\frac{\omega}{40}\right)}{(1+\omega j)\,200\left(1+j\frac{\omega}{200}\right)}\;2$$

$$= \frac{j\frac{\omega}{1/2}\left(1+j\frac{\omega}{40}\right)}{(1+\omega j)\left(1+j\frac{\omega}{200}\right)}$$





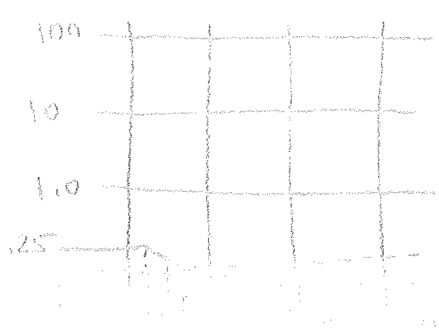
$$H(\omega) = \frac{1+j\frac{\omega}{10}}{1+j\frac{\omega}{100}}$$



$$H(s) = \frac{S+5}{S^2+12S+20} = \frac{S+5}{S^2+12S+36-16}$$

$$\frac{S+5}{(S+6)^2-4^2} = \frac{S+5}{(S+6-4)(S+6+4)} = \frac{S+5}{(S+2)(S+10)}$$

$$H(j\omega) = \frac{j\omega+5}{(j\omega+2)(j\omega+10)} = \frac{5}{2 \times 10} \; \frac{1+j\frac{\omega}{5}}{\left(1+j\frac{\omega}{2}\right)\left(1+j\frac{\omega}{10}\right)}$$



$$\frac{.25}{X} = \frac{5}{2}$$

$$X = \frac{.25}{5}$$

EE 202

SOLUTIONS

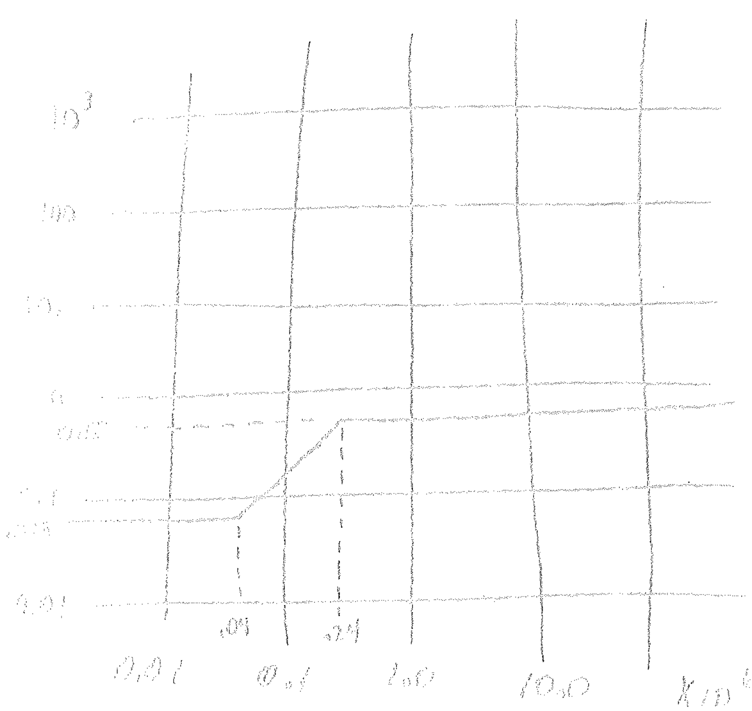
$$\frac{V_o}{V_{in}} = H(\omega) = \frac{10}{20 + \dfrac{100\left(\frac{10^6}{j\omega .25}\right)}{100 + \frac{10^6}{j\omega .25}}}$$

$$= \frac{10\left(100 + \frac{10^6}{j\omega .25}\right)}{20\left(100 + \frac{10^6}{j\omega .25}\right) + \frac{400}{j\omega}} = \frac{1000 + \frac{40 \times 10^6}{j\omega}}{2000 + \frac{480 \times 10^6}{j\omega}}$$

$$= \frac{10^6 \cdot 40 + j\omega 1000}{10^6 \cdot 480 + j\omega 2000} = \frac{1}{12} \; \frac{1 + j\omega(25) \cdot 10^{-6}}{1 + j\omega(4.17) \cdot 10^{-6}}$$

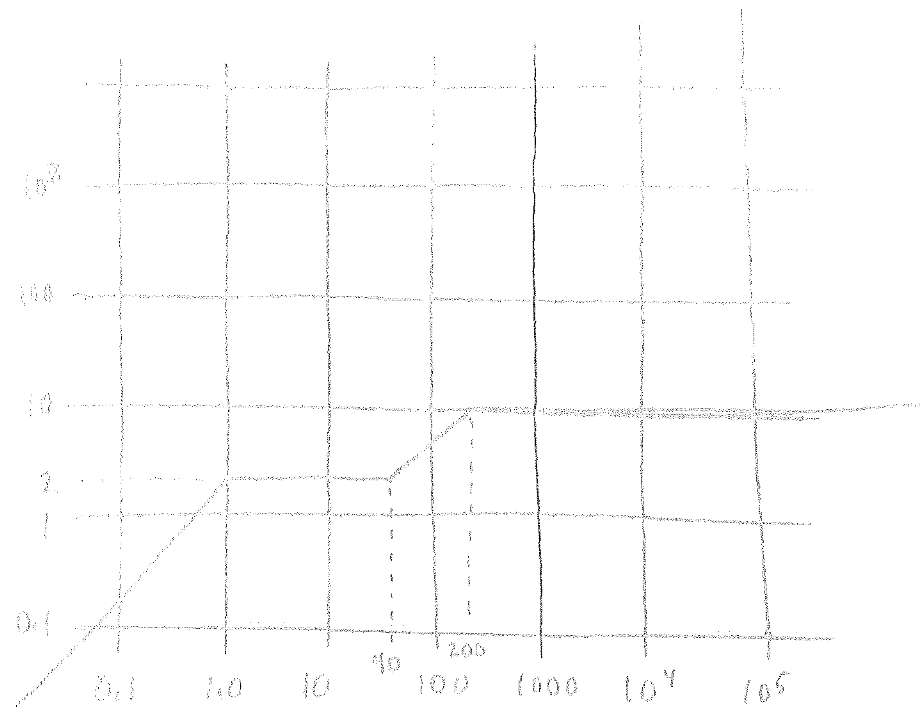
$$= \frac{1}{12} \; \frac{1 + j\dfrac{\omega}{.04 \times 10^6}}{1 + j\dfrac{\omega}{0.24 \times 10^6}}$$

.083

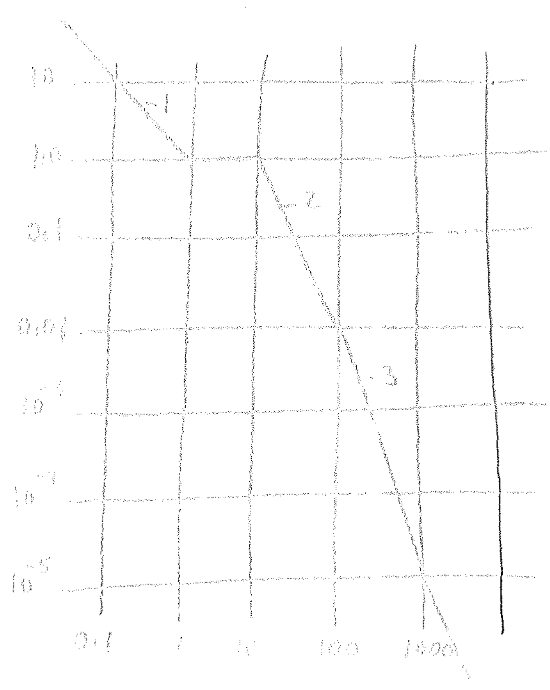


$$H(s) = \frac{\cdots}{(s+1)(s+200)} = \frac{\cdots}{200(1+s)(1+\frac{s}{200})}$$

$$= \frac{2(j\omega)(1+j\frac{\omega}{40})}{(1+j\frac{\omega}{1})(1+j\frac{\omega}{200})}$$



X.15 $H(\omega) = \dfrac{1+\frac{j\omega}{1}}{j\omega(1+j\frac{\omega}{10})^2(1+j\frac{\omega}{100})}$



$$H(100) = \frac{[\text{illegible}]}{(1+j20)(1+j1000)} \doteq \frac{[\text{illegible}]}{(20\angle 89.65)(1000\angle 89^\circ)}$$

$$\doteq \boxed{1.5\angle -.4^\circ}$$

$$H(\;) = \frac{(1+j1)(1+j3)}{(1+j0.2)(1+j10)} = \frac{(1.414\angle 45^\circ)(3.16\angle 71.56^\circ)}{(1.02\angle 11.1^\circ)(10.05\angle 84.3^\circ)}$$

$$= \cancel{4.356}\ \boxed{0.436\angle 20.95^\circ}$$

$$H(\;) = \frac{(1+j3)(1+j9)}{(1+j.6)(1+j60)} = \frac{(3.16\angle 71.55^\circ)(9.05\angle 83.65^\circ)}{(1.165\angle 31^\circ)(60\angle 89.05^\circ)}$$

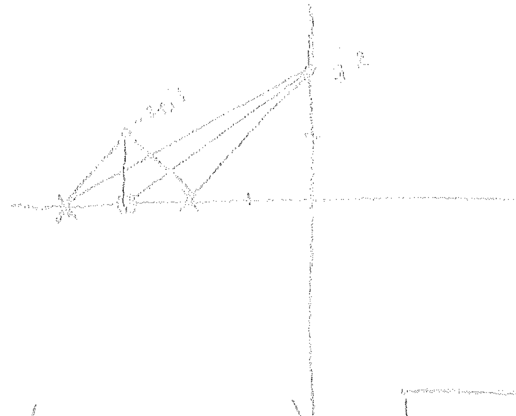
$$= \boxed{0.408\angle 35.15^\circ}$$

RPI
MAY 10, 1970

EE 202
SOLUTIONS

X-27 (numbered X-24)

$$H(s) = \frac{s+3}{(s+4)(s+2)}$$



(a)

(b) $H(-3+j1) = \frac{1}{(\sqrt{2})^2} \angle(90-45-135) = \boxed{0.5 \angle -90^\circ}$

(c) $H(j2) = \frac{\sqrt{13}}{(2\sqrt{2})(\sqrt{20})} \angle \tan^{-1}\frac{2}{3} - \tan^{-1}\frac{2}{2} - \tan^{-1}\frac{1}{2}$

$0.285 \angle 34.7 - 45 - 26.6^\circ = \boxed{0.285 \angle -36.9^\circ}$

[illegible] Solutions

[illegible] (assume $K = 2\pi$ [illegible])

$$H(s) = \frac{10^6}{[illegible]} = \frac{A}{(s+\alpha+j\beta)(s+\alpha-j\beta)}$$

$\alpha = 5000$

$\beta = 10^6$

(a) $|H|_{max} = \frac{A}{2\alpha\beta} = \frac{10^6}{2\times 5000\times 10^6} = \boxed{10^{-4}}$

(b) $\omega_{max} = \sqrt{\beta^2-\alpha^2} = \sqrt{10^{12}-25\times 10^6} = 10^6\sqrt{1-\frac{25}{10^6}}$

$= 10^6\left(1-\frac{12.5}{10^6}\right) = 10^6 - 12.5 = \boxed{999987.5 \ \frac{\text{rad}}{\text{sec}}}$

(c) $\omega_2 = \sqrt{\omega_{max}^2 + 2\alpha\beta} = \boxed{1,004,987.51 \ \text{rad/sec}}$

(d) $\omega_1 = \boxed{994,974.9 \ \frac{\text{rad}}{\text{sec}}}$

(e) $\omega_0 = \sqrt{\alpha^2+\beta^2} = \boxed{1,000,012.5 \ \frac{\text{rad}}{\text{sec}}}$

(f) $Q = \frac{\omega_{max}}{\omega_2-\omega_1} = \boxed{99.996}$

[illegible] (... mounted [illegible])

$L = 4\,\mu H$ [illegible]

$C = 25\,pF$ [illegible]

$R = [illegible]\,\Omega$

$$\frac{V_o}{V_i} = \frac{1/sC}{R + sL + 1/sC} = \frac{1}{s^2LC + sRC + 1} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$= \frac{\omega_0^2}{s^2 + 2\alpha s + \omega_0^2} \qquad \omega_0^2 = \frac{1}{LC} = [illegible] = 10^{17} \qquad \alpha = \frac{R}{2L} = [illegible] = 2.5 \times 10^{6}$$

$$\boxed{\frac{10^{17}}{s^2 + [illegible] \times 10^{6}\, s + 10^{17}}}$$

$\alpha = \boxed{2.5 \times 10^{6}\ \frac{rad}{sec}}$ $\qquad \omega_0 = \sqrt{10}\ 10^{8} = \boxed{3.16 \times 10^{8}\ \frac{rad}{sec}}$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \cancel{2.5\times10^{6}}\ \sqrt{10^{17} - 2.5 \times 10^{6}}$$

$$= 3.16 \times 10^{8} \sqrt{1 - \frac{2.5}{10^{11}}} = 3.16 \times 10^{8} \sqrt{1 - \frac{2.5}{10^{10}}}$$

$$= 3.16 \times 10^{8} \left(1 - 1.25 \times 10^{-10}\right) \simeq \omega_0 = \boxed{3.16 \times 10^{8}\ \frac{rad}{sec}}$$

$$\frac{\omega_0^2}{2\alpha\,[illegible]} = \frac{10^{17}\ \ 3.16\ 10^{[illegible]}}{2 \times 1.25 \times 10^{6} \times 3.16 \times 10^{8}} = \frac{316}{50} = \boxed{[illegible]}$$

$$\omega_{max} = \sqrt{\omega_0^2 - 2\alpha^2} = \sqrt{10^{17} - 2 \times 6.25 \times 10^{12}} = \sqrt{10^{17}}\sqrt{1 - [illegible]}$$

$$= \sqrt{10^{17}}\sqrt{1 - .0002} \doteq \sqrt{10^{17}}\,(1 - .0002) = [illegible] \times 3.16 \times 10^{8}$$

$$= \boxed{3.1[illegible] \times 10^{8}\ [illegible]}$$

BOB MARKS
EE-202

Department of Electrical Engineering
Rose Polytechnic Institute
Terre Haute, Indiana
May 6, 1970

EE 202

Homework

10

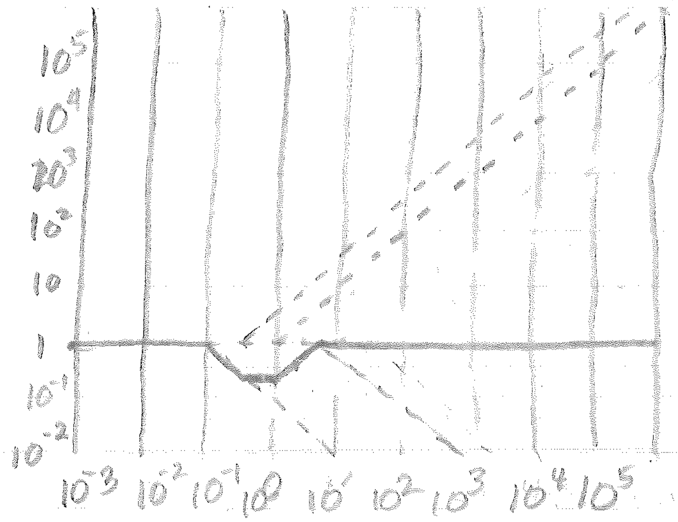
C-1. Plot on log-log graph paper $|H(\omega)|$ for

$$H(\omega) = \frac{(1 + j\omega)(1 + j\frac{\omega}{100})(1 + j\frac{\omega}{10,000})}{(1 + j\frac{\omega}{10})(1 + j\frac{\omega}{1000})(1 + j\frac{\omega}{10^5})}$$

Plot for 10 equally spaced frequencies per decade for a frequency range from 0.1 Hz to 1 MHz. Also plot $\Theta_H(\omega)$ for the same frequencies on semi-log.

X-26)

$$H(\omega) = \frac{(1+j\omega)\left(1+j\frac{\omega}{1/3}\right)}{\left(1+j\frac{\omega}{5}\right)\left(1+j\frac{\omega}{1/10}\right)}$$



a) $H(100) = 0$

b) $5 = \frac{1}{H(1)} \Rightarrow H(1) = 1/5$

c) $\frac{3}{1} = \frac{H(3)}{1/5} \Rightarrow H(3) = \frac{3}{5}$

```
// JOB T      08069
// FORTRAN IV
*IOCS(1132 PRINTER,CARD)
* EXTENDED PRECISION
      WRITE(3,30)
   30 FORMAT(2X,'BODE DIAGRAM DATA-EE202',/)
      WRITE(3,31)
   31 FORMAT(7X,'FREQ',7X,'LOG FREQ',5X,'ANG FREQ',5X,'LOG A FREQ',6X,'H
     5(W)',6X,'LOG H(W)', 6X,'AH(W)-RAD',5X,'DEGREES')
      C=ALOG(10.)
      RT=10.**(1./10.)
      F=0.1
      T=10.
      PI=3.1415926535898
      DO 50 J=1,71
      M=J-1
      W=F*2.*PI
      WS=W**2.
      HW=(((1.+WS)*(1.+WS/(T**4.))*(1.+WS/(T**8.)))/((1.+WS/100.)*(1.+WS
     2/(T**6.))*(1.+WS/(T**T))))**.5
      AHW=ATAN(W)+ATAN(W/100.)+ATAN(W/10000.)-ATAN(W/10.)-ATAN(W/1000.)-
     2ATAN(W/100000.)
      AID=AHW*180./PI
      TLF=ALOG(F)/C
      TLW=ALOG(W)/C
      TLHW=ALOG(HW)/C
      WRITE(3,20) M,F,TLF,W,TLW,HW,TLHW,AHW,AID
   20 FORMAT(I3,1X,E12.5,1X,E12.5,1X,E12.5,1X,E12.5,1X,E12.5,1X,E12.5,1X
     5,E12.5,1X,E12.5)
      F=F*RT
   50 CONTINUE
      STOP
      END
// XEQ
```

PAGE 1 5-14-70 RPI MONITOR VER 2 MOD 06

// JOB T 08069

// FORTRAN IV
*IOCS(1132 PRINTER,CARD)
* EXTENDED PRECISION

FEATURES SUPPORTED
EXTENDED PRECISION
IOCS

CORE REQUIREMENTS FOR
COMMON 0 VARIABLES 78 PROGRAM 430

END OF COMPILATION

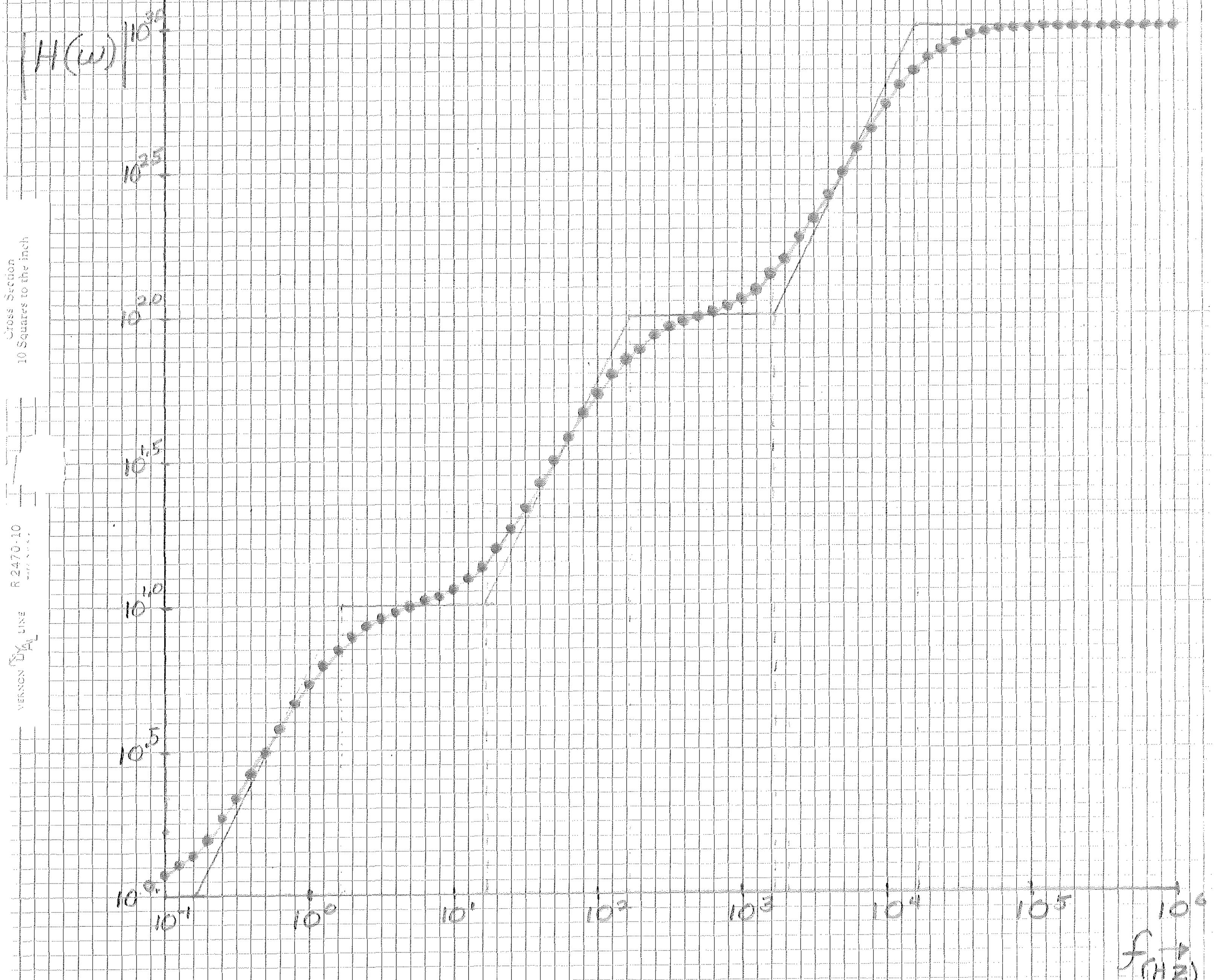
// XEQ

BODE DIAGRAM DATA-EE202

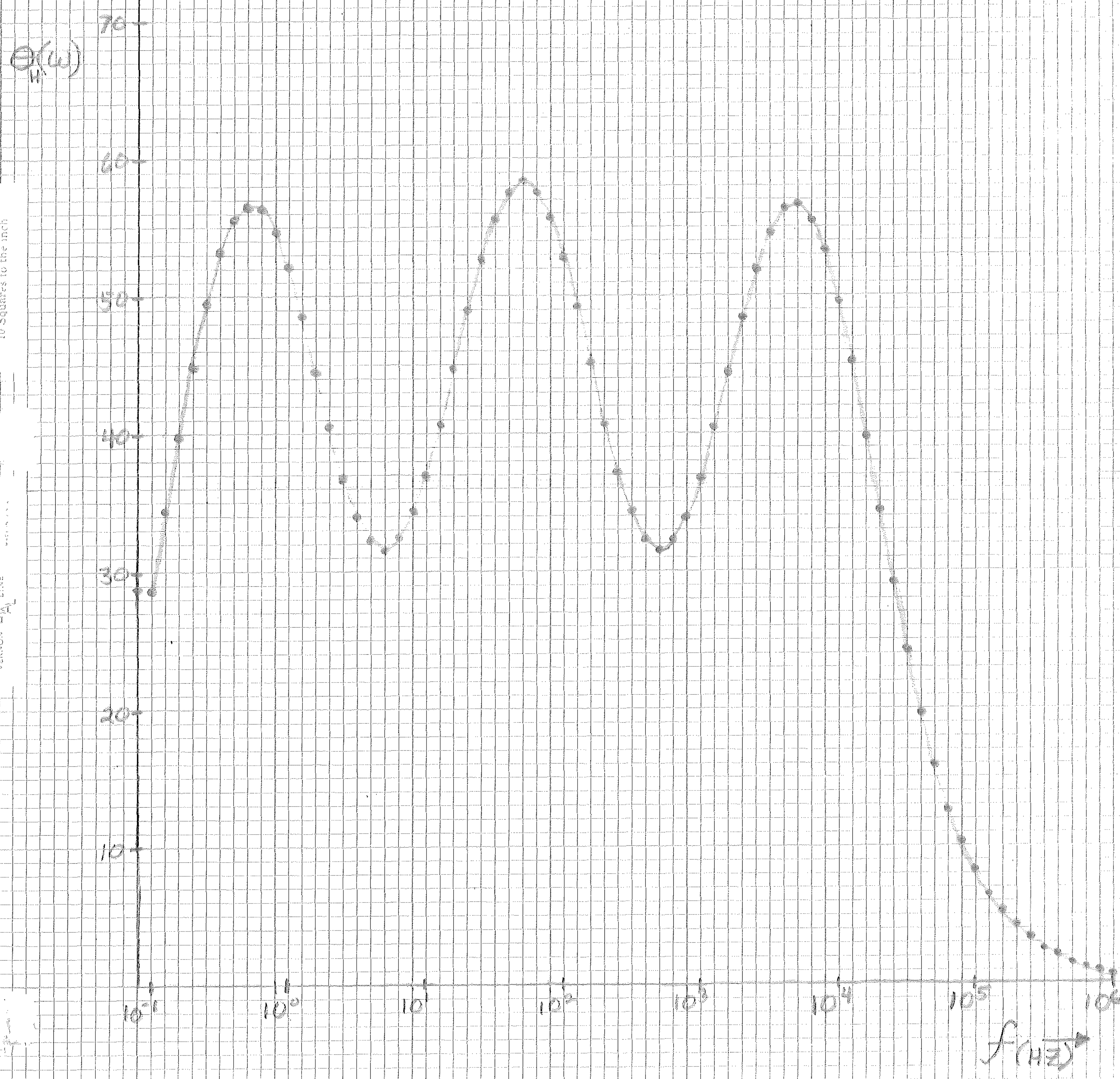
	FREQ	LOG FREQ	ANG FREQ	LOG A FREQ	H(W)	LOG H(W)	AH(W)-RAD	DEGREES
0	0.10000E 00	-0.10000E 01	0.62831E 00	-0.20182E 00	0.11787E 01	0.71406E-01	0.50394E 00	0.28873E 02
1	0.12589E 00	-0.90000E 00	0.79100E 00	-0.10182E 00	0.12710E 01	0.10417E 00	0.59748E 00	0.34233E 02
2	0.15848E 00	-0.80000E 00	0.99581E 00	-0.18201E-02	0.14043E 01	0.14748E 00	0.69309E 00	0.39711E 02
3	0.19952E 00	-0.69999E 00	0.12536E 01	0.98179E-01	0.15913E 01	0.20175E 00	0.78416E 00	0.44929E 02
4	0.25118E 00	-0.59999E 00	0.15782E 01	0.19817E 00	0.18457E 01	0.26618E 00	0.86384E 00	0.49494E 02
5	0.31622E 00	-0.49999E 00	0.19869E 01	0.29817E 00	0.21821E 01	0.33888E 00	0.92643E 00	0.53081E 02
6	0.39810E 00	-0.39999E 00	0.25013E 01	0.39817E 00	0.26141E 01	0.41733E 00	0.96810E 00	0.55468E 02
7	0.50118E 00	-0.29999E 00	0.31490E 01	0.49817E 00	0.31529E 01	0.49872E 00	0.98685E 00	0.56542E 02
8	0.63095E 00	-0.19999E 00	0.39644E 01	0.59817E 00	0.38037E 01	0.58021E 00	0.98228E 00	0.56280E 02
9	0.79432E 00	-0.99999E-01	0.49909E 01	0.69817E 00	0.45599E 01	0.65896E 00	0.95545E 00	0.54743E 02
10	0.10000E 01	0.32357E-08	0.62831E 01	0.79817E 00	0.53976E 01	0.73220E 00	0.90901E 00	0.52082E 02
11	0.12589E 01	0.10000E 00	0.79100E 01	0.89817E 00	0.62725E 01	0.79744E 00	0.84754E 00	0.48560E 02
12	0.15848E 01	0.20000E 00	0.99581E 01	0.99817E 00	0.71264E 01	0.85287E 00	0.77760E 00	0.44553E 02
13	0.19952E 01	0.30000E 00	0.12536E 02	0.10981E 01	0.79031E 01	0.89780E 00	0.70702E 00	0.40509E 02
14	0.25118E 01	0.40000E 00	0.15782E 02	0.11981E 01	0.85677E 01	0.93286E 00	0.64366E 00	0.36879E 02
15	0.31622E 01	0.50000E 00	0.19869E 02	0.12981E 01	0.91168E 01	0.95984E 00	0.59405E 00	0.34036E 02
16	0.39810E 01	0.60000E 00	0.25013E 02	0.13981E 01	0.95762E 01	0.98119E 00	0.56271E 00	0.32240E 02
17	0.50118E 01	0.70000E 00	0.31490E 02	0.14981E 01	0.99925E 01	0.99967E 00	0.55216E 00	0.31636E 02
18	0.63095E 01	0.80000E 00	0.39644E 02	0.15981E 01	0.10425E 02	0.10181E 01	0.56325E 00	0.32271E 02
19	0.79432E 01	0.90000E 00	0.49909E 02	0.16981E 01	0.10947E 02	0.10393E 01	0.59525E 00	0.34105E 02
20	0.10000E 02	0.10000E 01	0.62831E 02	0.17981E 01	0.11642E 02	0.10660E 01	0.64580E 00	0.37001E 02
21	0.12589E 02	0.11000E 01	0.79100E 02	0.18981E 01	0.12611E 02	0.11007E 01	0.71052E 00	0.40710E 02
22	0.15848E 02	0.12000E 01	0.99581E 02	0.19981E 01	0.13974E 02	0.11453E 01	0.78305E 00	0.44865E 02
23	0.19952E 02	0.13000E 01	0.12536E 03	0.20981E 01	0.15863E 02	0.12003E 01	0.85566E 00	0.49026E 02
24	0.25118E 02	0.14000E 01	0.15782E 03	0.21981E 01	0.18421E 02	0.12653E 01	0.92063E 00	0.52748E 02
25	0.31622E 02	0.15000E 01	0.19869E 03	0.22981E 01	0.21794E 02	0.13383E 01	0.97151E 00	0.55663E 02
26	0.39810E 02	0.16000E 01	0.25013E 03	0.23981E 01	0.26120E 02	0.14169E 01	0.10038E 01	0.57515E 02
27	0.50118F 02	0.17000E 01	0.31490E 03	0.24981E 01	0.31514E 02	0.14985E 01	0.10151E 01	0.58163E 02
28	0.63095E 02	0.18000E 01	0.39644E 03	0.25981E 01	0.38025E 02	0.15800E 01	0.10046E 01	0.57560E 02
29	0.79432E 02	0.19000E 01	0.49909E 03	0.26981E 01	0.45590E 02	0.16588E 01	0.97303E 00	0.55750E 02
30	0.10000E 03	0.20000E 01	0.62831E 03	0.27981E 01	0.53969E 02	0.17321E 01	0.92277E 00	0.52870E 02
31	0.12589E 03	0.21000E 01	0.79100E 03	0.28981E 01	0.62720E 02	0.17974E 01	0.85821E 00	0.49171E 02
32	0.15848E 03	0.22000E 01	0.99581E 03	0.29981E 01	0.71260E 02	0.18528E 01	0.78574E 00	0.45019E 02
33	0.19952E 03	0.23000E 01	0.12536E 04	0.30981E 01	0.79029E 02	0.18977E 01	0.71307E 00	0.40856E 02
34	0.25118E 03	0.24000E 01	0.15782E 04	0.31981E 01	0.85676E 02	0.19328E 01	0.64794E 00	0.37124E 02
35	0.31622E 03	0.25000E 01	0.19869E 04	0.32981E 01	0.91167E 02	0.19598E 01	0.59679E 00	0.34193E 02
36	0.39810F 03	0.26000E 01	0.25013E 04	0.33981E 01	0.95761E 02	0.19811E 01	0.56405E 00	0.32318E 02
37	0.50118E 03	0.27000E 01	0.31490E 04	0.34981E 01	0.99924E 02	0.19996E 01	0.55219E 00	0.31638E 02

38	0.63095E 03	0.28000E 01	0.39644E 04	0.35981E 01	0.10425E 03	0.20180E 01	0.56195E 00	0.32197E 02
39	0.79432E 03	0.29000E 01	0.49909E 04	0.36981E 01	0.10947E 03	0.20392E 01	0.59256E 00	0.33951E 02
40	0.10000E 04	0.30000E 01	0.62831E 04	0.37981E 01	0.11641E 03	0.20660E 01	0.64158E 00	0.36759E 02
41	0.12589E 04	0.31000E 01	0.79100E 04	0.38981E 01	0.12611E 03	0.21007E 01	0.70454E 00	0.40367E 02
42	0.15848E 04	0.32000E 01	0.99581E 04	0.39981E 01	0.13973E 03	0.21453E 01	0.77499E 00	0.44403E 02
43	0.19952E 04	0.33000E 01	0.12536E 05	0.40981E 01	0.15861E 03	0.22003E 01	0.84510E 00	0.48420E 02
44	0.25118E 04	0.34000E 01	0.15782E 05	0.41981E 01	0.18418E 03	0.22652E 01	0.90700E 00	0.51967E 02
45	0.31622E 04	0.35000E 01	0.19869E 05	0.42981E 01	0.21789E 03	0.23382E 01	0.95408E 00	0.54665E 02
46	0.39810E 04	0.36000E 01	0.25013E 05	0.43981E 01	0.26112E 03	0.24168E 01	0.98169E 00	0.56246E 02
47	0.50118E 04	0.37000E 01	0.31490E 05	0.44981E 01	0.31498E 03	0.24982E 01	0.98709E 00	0.56556E 02
48	0.63095E 04	0.38000E 01	0.39644E 05	0.45981E 01	0.37996E 03	0.25797E 01	0.96919E 00	0.55530E 02
49	0.79432E 04	0.39000E 01	0.49909E 05	0.46981E 01	0.45534E 03	0.26583E 01	0.92834E 00	0.53190E 02
50	0.10000E 05	0.40000E 01	0.62831E 05	0.47981E 01	0.53864E 03	0.27313E 01	0.86644E 00	0.49643E 02
51	0.12589E 05	0.41000E 01	0.79100E 05	0.48981E 01	0.62527E 03	0.27960E 01	0.78730E 00	0.45108E 02
52	0.15848E 05	0.42000E 01	0.99581E 05	0.49981E 01	0.70913E 03	0.28507E 01	0.69653E 00	0.39908E 02
53	0.19952E 05	0.43000E 01	0.12536E 06	0.50981E 01	0.78421E 03	0.28944E 01	0.60096E 00	0.34432E 02
54	0.25118E 05	0.44000E 01	0.15782E 06	0.51981E 01	0.84639E 03	0.29275E 01	0.50724E 00	0.29063E 02
55	0.31622E 05	0.45000E 01	0.19869E 06	0.52981E 01	0.89436E 03	0.29515E 01	0.42056E 00	0.24096E 02
56	0.39810E 05	0.46000E 01	0.25013E 06	0.53981E 01	0.92928E 03	0.29681E 01	0.34399E 00	0.19709E 02
57	0.50118E 05	0.47000E 01	0.31490E 06	0.54981E 01	0.95357E 03	0.29793E 01	0.27862E 00	0.15964E 02
58	0.63095E 05	0.48000E 01	0.39644E 06	0.55981E 01	0.96993E 03	0.29867E 01	0.22416E 00	0.12843E 02
59	0.79432E 05	0.49000E 01	0.49909E 06	0.56981E 01	0.98070E 03	0.29915E 01	0.17953E 00	0.10286E 02
60	0.10000E 06	0.50000E 01	0.62831E 06	0.57981E 01	0.98769E 03	0.29946E 01	0.14336E 00	0.82141E 01
61	0.12589E 06	0.51000E 01	0.79100E 06	0.58981E 01	0.99218E 03	0.29965E 01	0.11426E 00	0.65467E 01
62	0.15848E 06	0.52000E 01	0.99581E 06	0.59981E 01	0.99504E 03	0.29978E 01	0.90955E-01	0.52113E 01
63	0.19952E 06	0.53000E 01	0.12536E 07	0.60981E 01	0.99686E 03	0.29986E 01	0.72346E-01	0.41451E 01
64	0.25118E 06	0.54000E 01	0.15782E 07	0.61981E 01	0.99801E 03	0.29991E 01	0.57516E-01	0.32954E 01
65	0.31622E 06	0.55000E 01	0.19869E 07	0.62981E 01	0.99874E 03	0.29994E 01	0.45711E-01	0.26190E 01
66	0.39810E 06	0.56000E 01	0.25013E 07	0.63981E 01	0.99920E 03	0.29996E 01	0.36322E-01	0.20811E 01
67	0.50118E 06	0.57000E 01	0.31490E 07	0.64981E 01	0.99950E 03	0.29997E 01	0.28858E-01	0.16534E 01
68	0.63095E 06	0.58000E 01	0.39644E 07	0.65981E 01	0.99968E 03	0.29998E 01	0.22925E-01	0.13135E 01
69	0.79432E 06	0.59000E 01	0.49909E 07	0.66981E 01	0.99980E 03	0.29999E 01	0.18212E-01	0.10434E 01
70	0.10000E 07	0.60000E 01	0.62831E 07	0.67981E 01	0.99987E 03	0.29999E 01	0.14467E-01	0.82891E 00

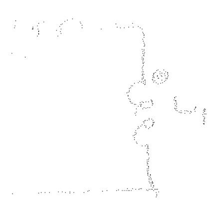
EXECUTION TIME 0047 SEC



|H(ω)|
$10^{3.0}$
$10^{2.5}$
$10^{2.0}$
$10^{1.5}$
$10^{1.0}$
$10^{0.5}$
10^{0}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
f (HZ)



70
60
50
40
30
20
10
$\theta_H(\omega)$
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
f (Hz)



$L_1 = 2\ H$

$L_2 = 5\ H$

$L_3 = 6\ H$

$L_4 = 3\ H$

$M_{14} = 2H$

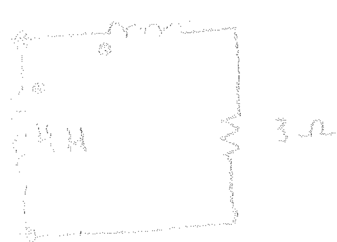
$M_{23} = 5.2\ H$

---

the circuit shown.

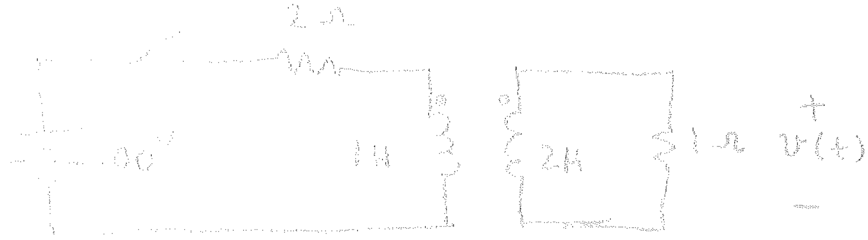
$\omega = 2$ rad/sec

$M = 2\ H$



X – 40

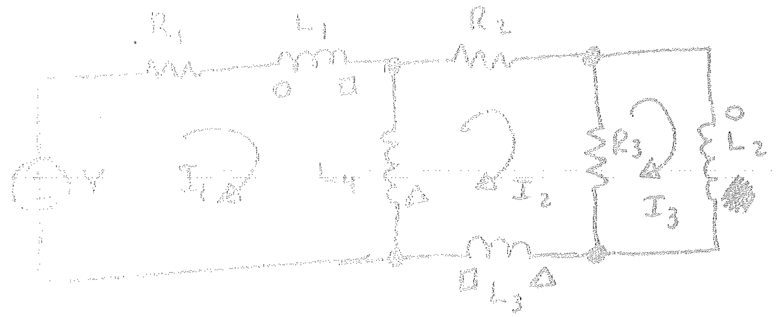
The switch is closed at $t = 0$. Solve for $v(t)$



$M = \sqrt{2}\ H.$

X – 41

Write the equations for the mesh currents



$M_{12} \neq 0$

$M_{34} \neq 0$

$M_{23} \neq 0$

[illegible]

$$a_n = \frac{2a}{T}\,\frac{\sin\frac{n\pi a}{T}}{\frac{n\pi a}{T}}$$ where $a = [illegible]$, $T = 10^{-3}$ [illegible]

$$a_n = \boxed{2 \times 10^{-3}\;\frac{\sin 10^{-3} n\pi}{10^{-3} n\pi}} \Rightarrow a_0 = 2 \times 10^{-3}$$

$$a_n = \frac{2}{n\pi}\,\sin 10^{-3} n\pi = \frac{.636}{n}\,\sin n(.0031415)$$

n	$\frac{.636}{n}$	$.0031415\,n$	$\sin\theta$	a_n
1	[illegible]	.0031415	← same	1.9999671
2	[illegible]	.00[illegible]283	0	1.9999868
3	.212	.00[illegible]	0	1.9999704
4	[illegible]	.01[illegible]57	0	1.9999473
5	.1273	.0[illegible]57	0	1.999177
6	.106	.01885	0	1.999881
7	[illegible]	.02[illegible]	0	1.999839
8	[illegible]	.02[illegible]57	0	1.999789
9	[illegible]	.0283	0	1.999783
10	.0636	.0314	0	1.999671

NOTE: NEEDED COMPUTER TO FIGURE THESE!!

$-e^{0}(0+1)\Big]$

$2n\pi)-1\Big]$

$2n\pi)(1+j2n\pi)-1\Big]$

$1+2n\pi\sin 2n\pi\Big]$

$\boxed{\dfrac{\sin 2n\pi}{2n\pi}\Big)}$

$)=\left(\frac{1}{2}\right)\cdot 2=\boxed{1}$

$\Rightarrow$ odd fct. except for d-c value

$$\frac{2\cos 2n\pi}{(2n\pi)}$$

always $-\frac{1}{n\pi}$

$$\boxed{-\frac{1}{n\pi}}$$

$\omega_0 = 2000$ rad/sec instead of 2 r/s as stated above. However this doesn't effect the components

Passing through filter is d-c + fundamental

$$v_o(t) = \boxed{\frac{1}{2} - \frac{1}{\pi}\sin 2000t}$$